Sec 4.6:
Related Rates

Today we are going to look at how to solve problems where things are changing. so far in math, we have only been able to look at problems where things are stagnant - but how often does that really happen in life?? Here are some steps to help us through these problems.

1. Draw a figure if possible.
2. Assign variables and list your given information and what you want to find.
3. Write an equation that relates the given info and what you are looking for.
4. Differentiate both side with respect to time (using implicit differentiation.)
5. Substitute the given info and solve for the unknown.

Let's look at the algebra first before we get into the problem situations.

Find $\frac{dy}{dx}$ for $y = x^2 + 3$ when $x = 1$ and $\frac{dx}{dt} = 2$

Find $\frac{dx}{dt}$ when $x = 1$ and $\frac{dy}{dt} = 2$ if $y = \sqrt{9 - x^2}$

Air is being pumped into a spherical balloon at a rate of 3 cu. in per second. Find the rate of change of the radius is 2 inches.

Picture:

Relating Equation:

Differentiate and Solve:

Find:

When:

Given:

A pebble is dropped into a calm pond, causing ripples in the shape of concentric circles. The radius of the outer ripple is increasing at a constant rate of 1 ft/sec. When the radius is 4 ft, find the rate at which the area of the distrubed water is changing.

Picture:

Relating Equation:

Differentiate and Solve:

Find:

When:

Given:

A conical container's height is three times its radius. Find the rate of change of its volume when the radius is 6, if the radius is decreasing at a rate of 2 in/min.

Picture:

Relating Equation:

Differentiate and Solve:

Find:

When:

Given:

Water runs out of a conical tank at the constant rate of 2 cu. ft per min. The radius at the top of the tank is 5 feet, and the height of the tank is 10 feet. How fast is the water level sinking when the water is 4 ft deep?

Picture:

Relating Equation:

Differentiate and Solve:

Find:

When:

Given:

A 16 foot ladder is leaning against the wall of a house. The foot of the ladder is moving away from the wall at a constant rate of 3 ft per second. How fast is the top of the ladder moving when the foot of the ladder is 12 ft from the wall?

Picture:

Relating Equation:

Differentiate and Solve:

Find:

When:

Given:

A 13-ft ladder is leaning against a wall. Suppose that the base of the ladder slides away from the wall at the constant rate of 3 ft/sec. How fast is the top of the ladder sliding down the wall when the base of the ladder is 5 ft from the wall?

Picture:

Relating Equation:

Differentiate and Solve:

Find:

When:

Given:

A fish is reeled in at a rate of 2 ft/sec from a bridge that is 16 ft above the water. At what rate is the angle between the line and the water changing when there are 20 ft of line out?

Picture:

Relating Equation:

Differentiate and Solve:

Find:

When:

Given:

A 6-foot man walks away from a 16 foot lamp at a constant rate of 2 ft/sec. When he is 30 feet away from the base of the lamp, at what rate is the lenght of his shadow changing?

Picture:

Relating Equation:

Differentiate and Solve:

Find:

When:

Given:

At what rate is the distance from the lamp to the far end of his shadow changing?

A 5'6" woman walks at a rate of 5 ft/sec toward a lightpole 12 ft. tall. When the woman is 10 feet from the base of the light, at what rate is the lenght of her shadow changing?

Picture:

Relating Equation:

Differentiate and Solve:

Find:

When:

Given:

At what rate is the distance from the lamp to the far end of her shadow changing?