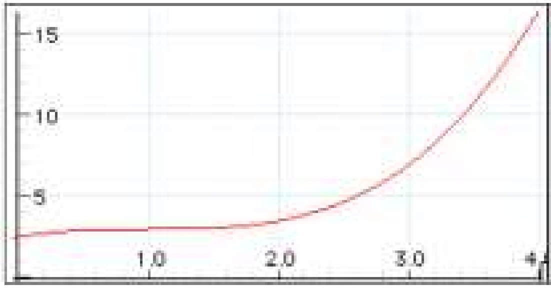
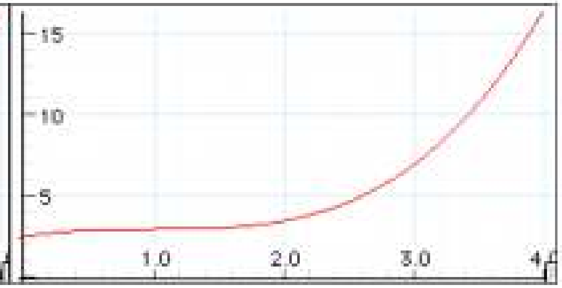
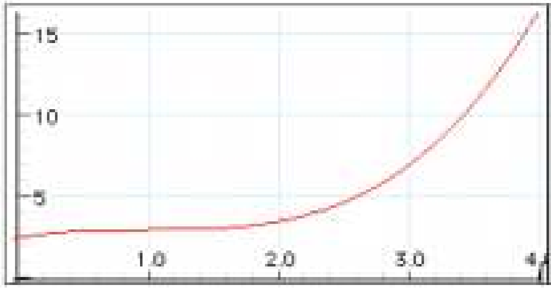
# Sec 5.1, 5.5, & 5.2: Estimating Area & Definite Integrals



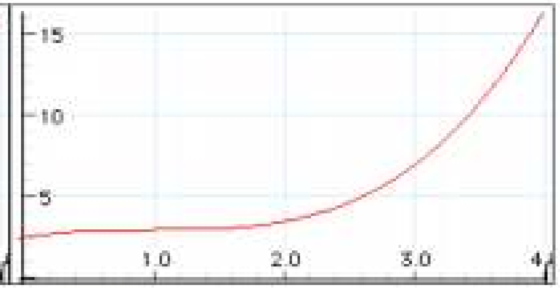

Estimate of area ______________




2 Rectangles: _______ + ________ = ______




4 Rectangles: ____ + ____ + ____ + ____ = ______



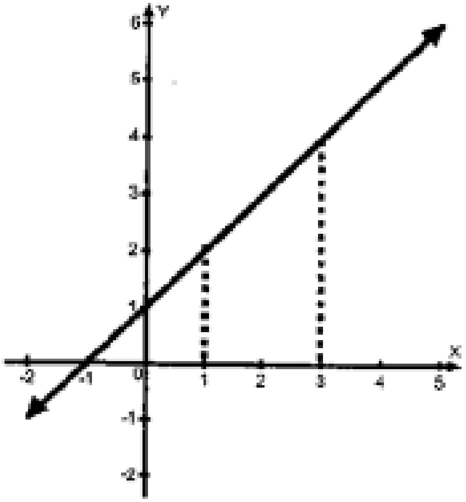

8 Rectangles: ____ + ____ + ____ + ____ + ____ + ____ + ____ + ____ = ______

Which one is a better estimate?

How can we make it even better?

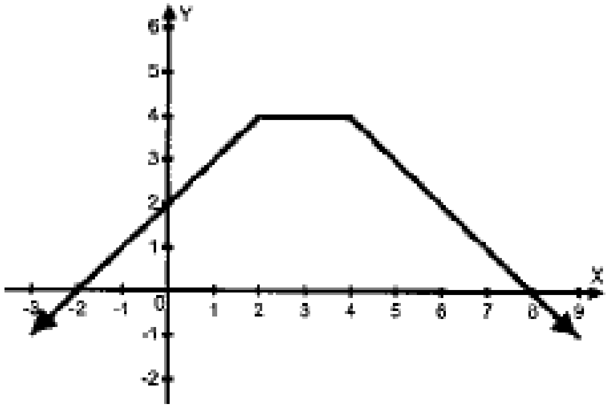
Let's look at some geometric shapes to find their area.

$y = x + 1 \qquad for\ 1 \le x \le 3$



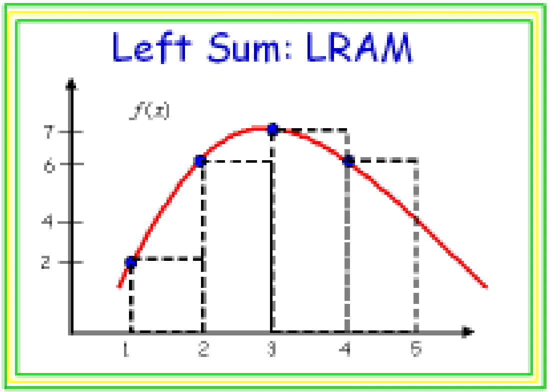

Find the area under the curve from 1 to 3.

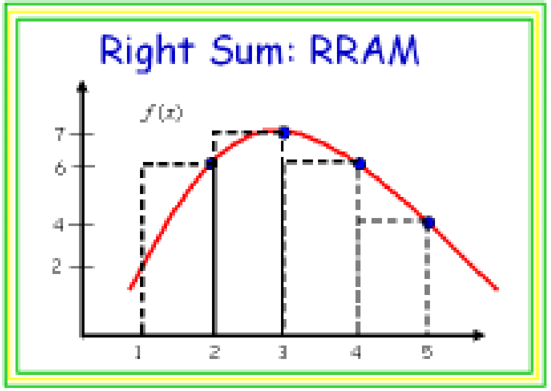
$$f(x) = \begin{cases} x+2 & for\ 0 \le x < 2 \\ 4 & for\ 2 \le x < 4 \\ -x+8 & for\ 4 \le x \le 6 \end{cases}$$

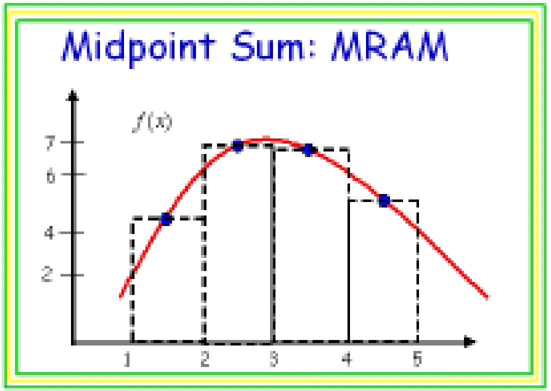


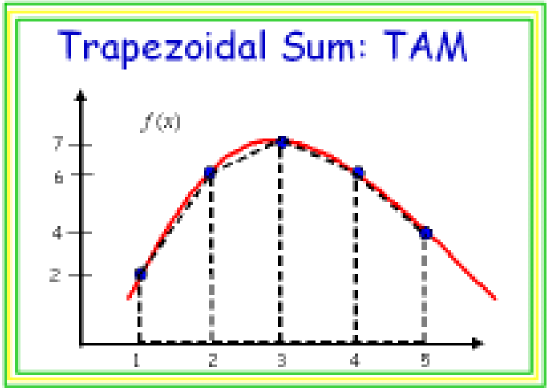

Find the area under the curve from 0 to 6.

Not everything is so nice, though. Many curves actually curve! How are we going to deal with those graphs?? I'm glad you asked.

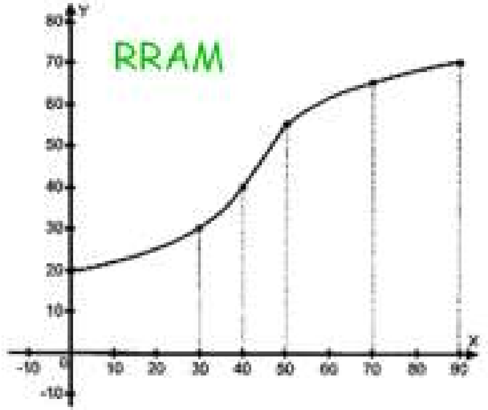


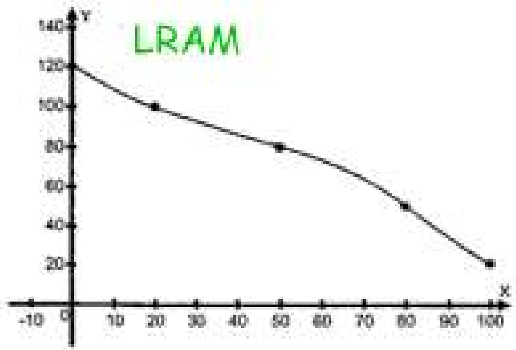



Approximate the area under each curve for the interval given using the method indicated. Show you calculations!



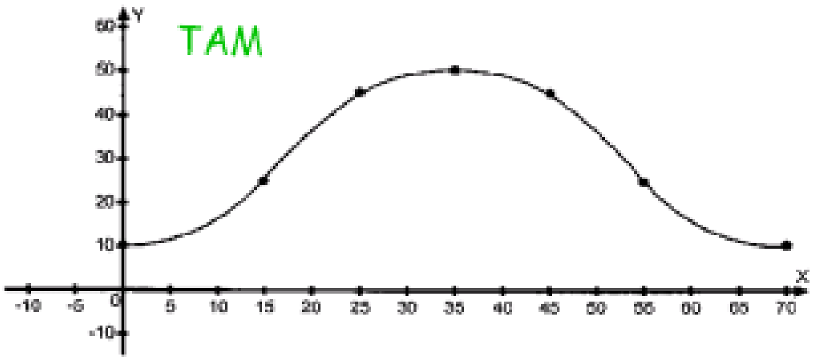

| $x$ | $f(x)$ |
|---|---|
| 0 | 20 |
| 30 | 30 |
| 40 | 40 |
| 50 | 55 |
| 70 | 65 |
| 90 | 70 |




| $x$ | $f(x)$ |
|---|---|
| 0 | 120 |
| 20 | 100 |
| 50 | 80 |
| 80 | 50 |
| 100 | 20 |

Approximate the area under each curve for the interval given using the method indicated. Show you calculations!

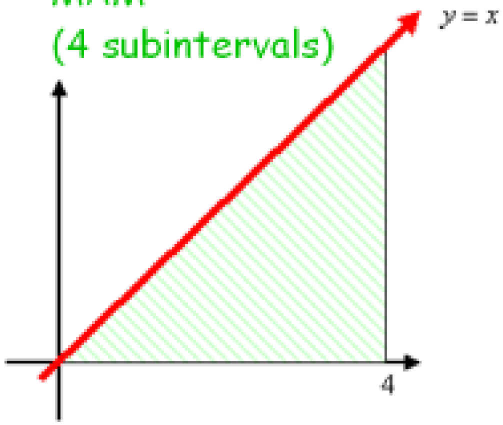
TAM




| $x$ | $f(x)$ |
|---|---|
| 0 | 10 |
| 15 | 25 |
| 25 | 45 |
| 35 | 50 |
| 45 | 45 |
| 55 | 25 |
| 70 | 10 |

MAM

(4 subintervals)

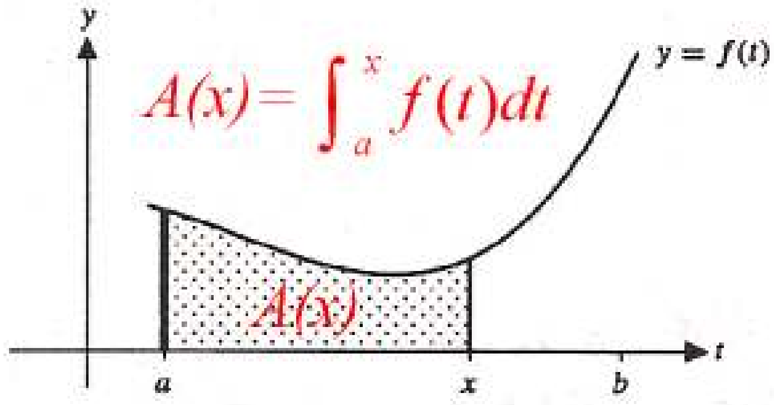


Hopefully we see that the more rectangles you create, the better the approximation will be. Ultimately, we would like the number of rectangles to go to infinity (which really can't happen) so here's where our limits come in to play (I know you are excited about this!)

The true area can be found by: $\lim_{n \to \infty} \sum_{i=1}^{n} A_i$

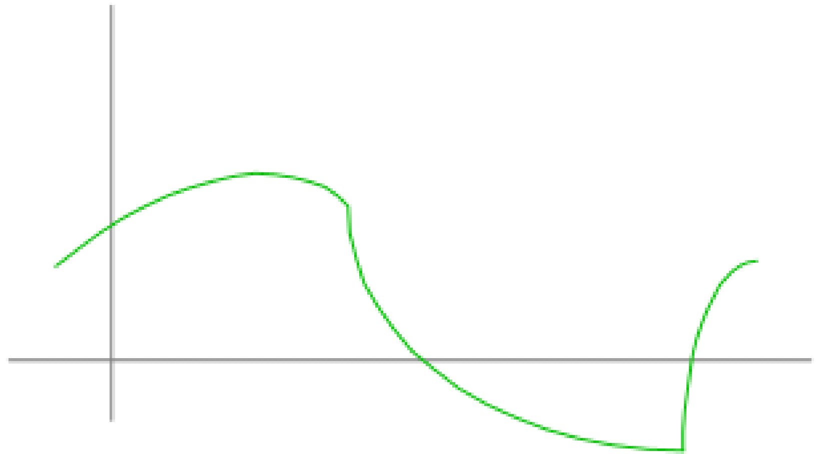
But do you really want to sit down and add up a bunch of areas?? I don't and neither does anyone else, so. . . they came up with an easier way to find the area under a curve.

What do all the parts mean??



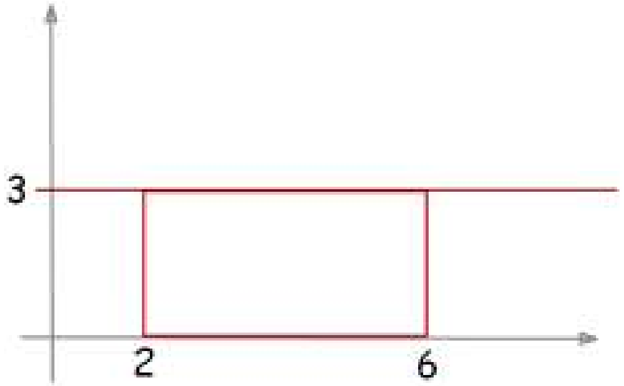
In general, the area under a curve can be found by evaluating a definite integral. We will look more at that in a minute, but first let's talk about signed area. Area under a curve can be either positive or negative, and it's important to know which one. The table below summarizes your options.

| | $f(x) > 0$ (curve above axis) | $f(x) < 0$ (curve below axis) |
|---|---|---|
| $dx > 0$ (left to right) ($a < b$) | | |
| $dx < 0$ (right to left) ($b < a$) | | |



So how are we going to use this integral to find area? Let's look.

Find the area of the rectangle below.




Given $f(x) = 3$, find the following.

$$\int f(x)\,dx = \int \_\_\_\_ \,dx$$

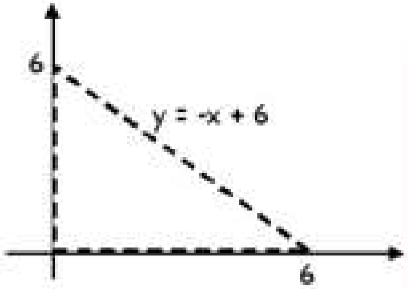
Now evaluate the antiderivative at 2 and 6.

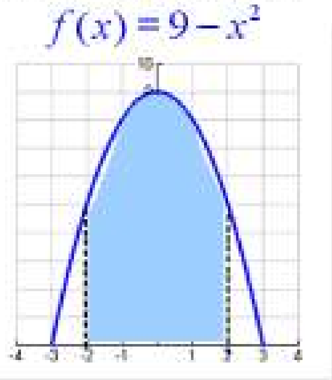
*How can you use those answers to come up with the area of the rectangle to the left?*

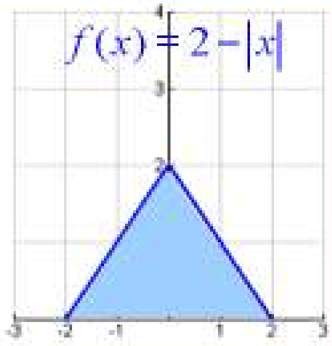
**Fundamental Theorem of Calculus**

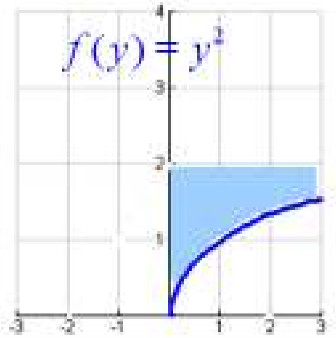
$$\int_a^b f'(x)dx = f(b) - f(a)$$

How about these? Set up a definite integral and then calculate the area under the curve.



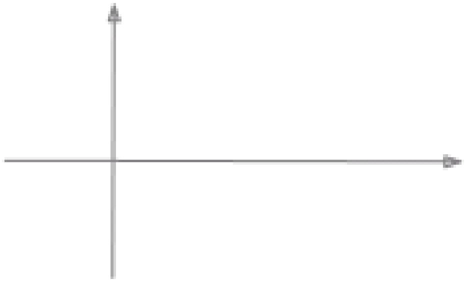



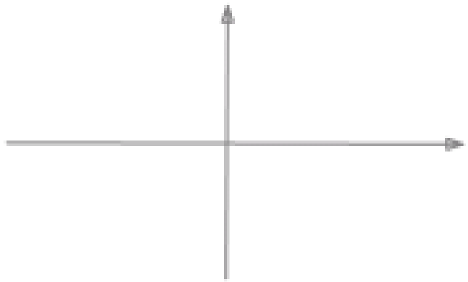



Sketch the region whose area is given by the definite integral and then find the area.

$$\int_0^7 \sqrt{x+2}\ dx$$



$$\int_{-2}^{3} x\ dx$$



$$\int_{-2}^{2} y^3\ dy$$