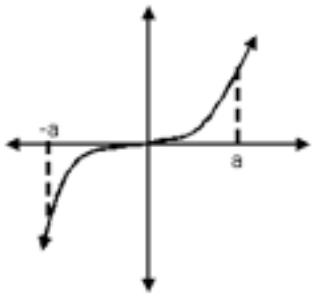
Section 5.3:
Properties of Definite Integrals

Properites of Integrals:

Just in case you want to see a formal definition. . .

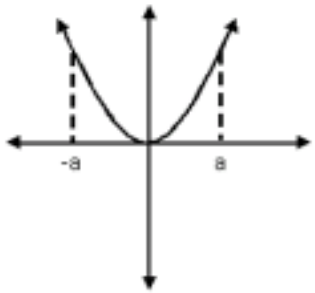
$\int_a^a f(x)dx = 0$	(Area at a single point is zero.)
$\int_b^a f(x)dx = -\int_a^b f(x)dx$	(Area going backwards is negative.)
$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$	(Area of two parts equals the whole area.)
$\int_a^b kf(x)dx = k\int_a^b f(x)dx$	(If k is a constant then it can be pulled out.)
$\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$	(Area of two functions can be split up.)
$f(x) \geq 0$ on $[a,b]$ then $\int_a^b f(x)dx \geq 0$	(If function is always positive, so is the area.)
$f(x) \leq g(x)$ on $[a,b]$ then $\int_a^b f(x)dx \leq \int_a^b g(x)dx$	(If f is always below g, then the area under f is always less than the area under g.)

Odd Functions



A = _____

Even Functions



A = _____

Let's start with an easy one. (Don't ask why they are numbered 22 - 30. . . just go with it!)

Suppose $\int_{-2}^{5} f(x)\,dx = 18$, $\int_{-2}^{5} g(x)\,dx = 5$, $\int_{-2}^{5} h(x)\,dx = -11$, and $\int_{-2}^{8} f(x)\,dx = 0$, find

22. $\int_{-2}^{5} (f(x) + g(x))\,dx$

23. $\int_{-2}^{5} [f(x) + g(x) - h(x)]\,dx$

24. $\int_{5}^{-2} 4g(x)\,dx$

25. $\int_{-2}^{5} (g(x) + 2)\,dx$

26. $\int_{-2}^{5} (f(x) - 6)\,dx$

27. $\int_{0}^{7} h(x-2)\,dx$

28. $\int_{-4}^{3} g(x+2)\,dx$

29. $\int_{5}^{8} f(x)\,dx$

30. $\int_{1}^{8} [f(x-3) + 3]\,dx$

How about these?

$\int_0^6 f(x)dx = 14$ $\int_0^3 f(x)dx = 10$ $\int_3^6 g(x)dx = -4$

$\int_3^6 f(x)dx$

$\int_6^3 2g(x)dx$

$\int_3^6 [f(x) - g(x)]dx$

$\int_0^6 g(x)dx - \int_0^3 g(x)dx$

Now they're getting harder! Find $\int_3^6 f(x)dx$

$f(x)$ is odd and $\int_{-3}^{6} f(x)dx = 10$

$f(x)$ is even, $\int_{-3}^{3} f(x)dx = 6$, and $\int_{-6}^{6} f(x)dx = 18$

$\int_3^6 [3f(x)+4]dx = 24$

$\int_3^5 [2f(x)]dx = 15$ and $\int_6^5 f(x)dx = 2$

And finally, how about these??

Is $\int_0^1 e^{-x}\,dx$ greater than, equal to, or less than 1?

True or False?

$$\int_{-\pi}^{\pi} [\sin x]\,dx = \int_{-\pi}^{\pi} [3x]\,dx$$

True or False?

$$\int_{-1}^{1} \left| \frac{\cos(x+2)}{1+\tan^2 x} \right| dx = 0$$