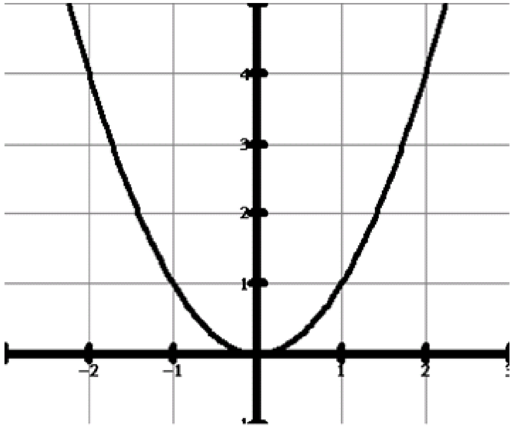
Section 2.1: Limits

What is a Limit??

* The intended y-value of a function
* The "destination" of the graph
* How the outputs (answers) of a function behave as the inputs approach some particular value

I think it's easiest to look at some graphs first.

$$f(x) = x^2$$

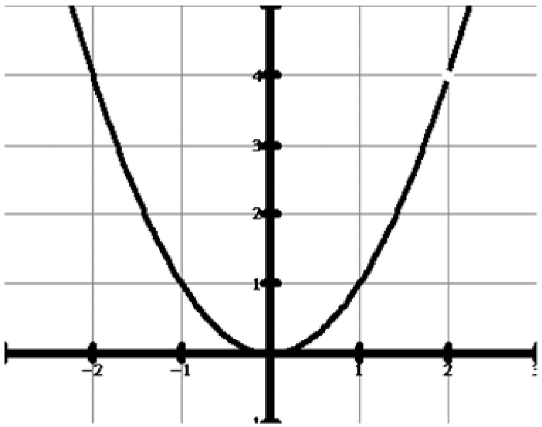


$\lim_{x \to 2} f(x) =$ ______ $f(2) =$ ________

"When x approaches _____, the _____ -value is _____"

Is this graph continuous? ______

$$f(x) = x^2 \quad for\, x \neq 2$$

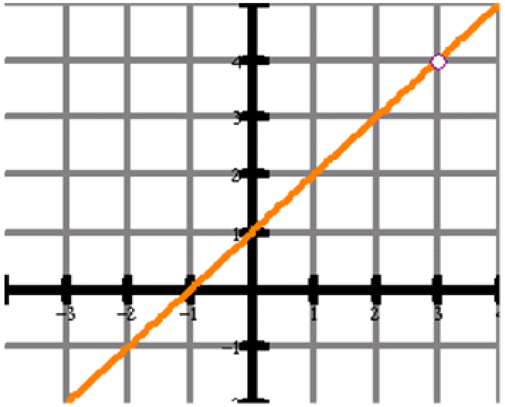


$\lim_{x \to 2} f(x) =$ ______ $f(2) =$ ________

"When x approaches _____, the _____ -value approaches _____"

Is this graph continuous? ______

$f(x)$

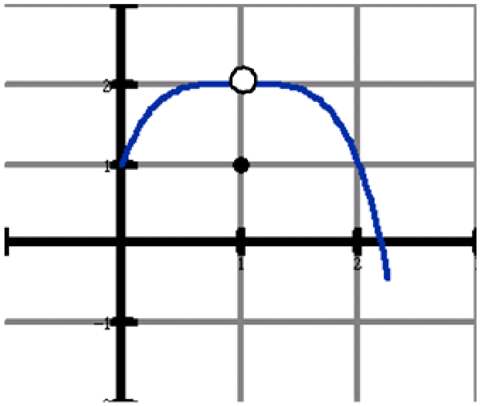


$\lim_{x \to 3} f(x) =$ ______

$f(3) =$ ______

continuous? ______

$f(x)$



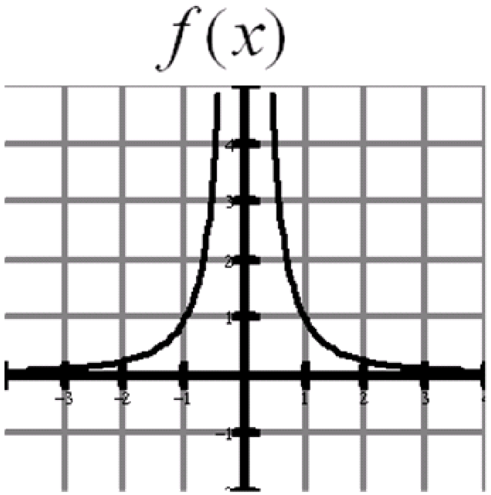
$\lim_{x \to 1} f(x) =$ ______

$f(1) =$ ______

continuous? ______

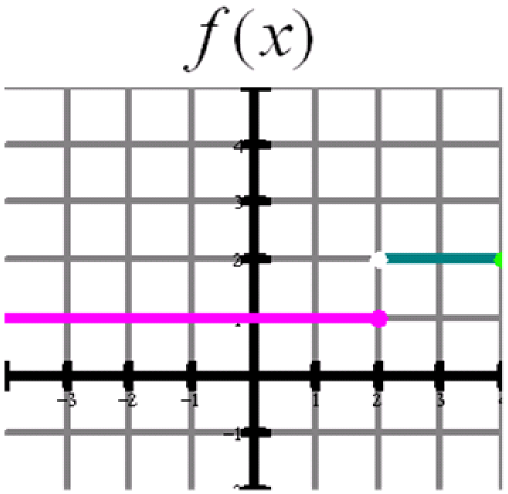
Does the function have to have a y-value at the designated x-value to have a limit?

If it does have a y-value, does it have to be a "part of" the graph?



$$\lim_{x \to 0} f(x) = \_\_\_\_\_\_$$

$$f(0) = \_\_\_\_\_\_$$



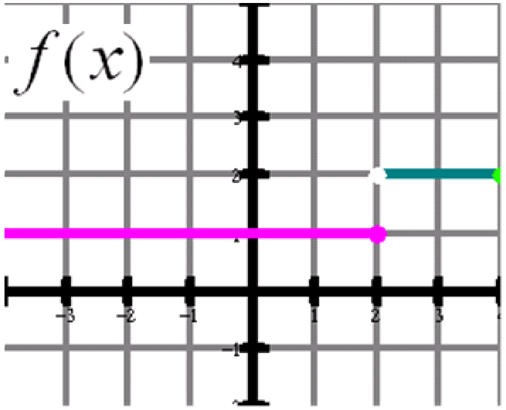
$$\lim_{x \to 2} f(x) = \_\_\_\_\_\_$$

$$f(2) = \_\_\_\_\_\_$$

Are these graphs continuous?

If a graph is not continuous, can it have a limit?

Let's look at this guy a little more. . .



What is our "destination" if we only follow the left part of the graph to the spot where x = 2? _______

What is our "destination" if we only follow the right part of the graph to the spot where x = 2? _______

These are called right-hand and left-hand limits and look like this:

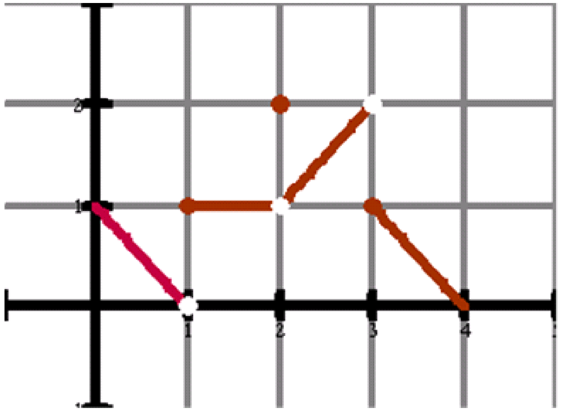
$$\lim_{x \to 2^-} f(x) = \_\_\_\_\_ \qquad \lim_{x \to 2^+} f(x) = \_\_\_\_\_$$

Very Important! For a limit to exist at a number, the right-hand and left-hand limits must be equal.

So, does the limit at x = 2 exist?

$$\lim_{x \to 2} f(x) = \_\_\_\_\_$$

Let's try a few more *one-sided* limits.



Find the following:

$$\lim_{x\to 0^+} f(x) = \_\_\_\_\_\_$$

$$\lim_{x\to 1^-} f(x) = \_\_\_\_\_\_$$

$$\lim_{x\to 3^+} f(x) = \_\_\_\_\_\_$$

$$\lim_{x\to 0^-} f(x) = \_\_\_\_\_\_$$

$$\lim_{x\to 1^+} f(x) = \_\_\_\_\_\_$$

$$\lim_{x\to 4^+} f(x) = \_\_\_\_\_\_$$

$$\lim_{x\to 2^+} f(x) = \_\_\_\_\_\_$$

$$\lim_{x\to 3^-} f(x) = \_\_\_\_\_\_$$

$$\lim_{x\to 4^-} f(x) = \_\_\_\_\_\_$$

$$\lim_{x\to 2^-} f(x) = \_\_\_\_\_\_$$

$$\lim_{x\to 2} f(x) = \_\_\_\_\_\_$$

$$\lim_{x\to 3} f(x) = \_\_\_\_\_\_$$

$$\lim_{x\to 1} f(x) = \_\_\_\_\_\_$$

$$\lim_{x\to 4} f(x) = \_\_\_\_\_\_$$

$$\lim_{x\to 0} f(x) = \_\_\_\_\_\_$$

$$f(3) = \_\_\_\_\_\_$$

$$f(2) = \_\_\_\_\_\_$$

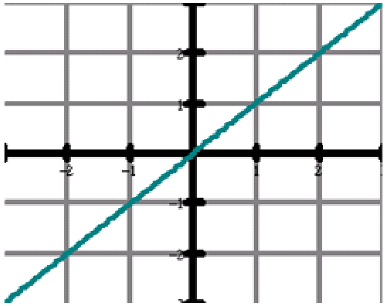
$$f(0) = \_\_\_\_\_\_$$

$$f(4) = \_\_\_\_\_\_$$

$$f(1) = \_\_\_\_\_\_$$

Let's look at and explore some properties of limits. . . what fun! :)

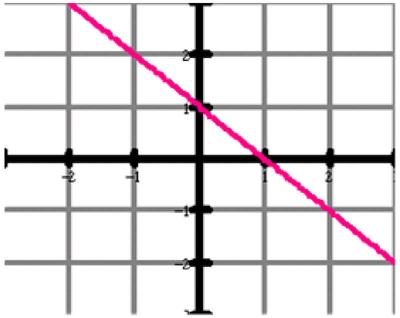
$$f(x) = x$$



$$\lim_{x \to -1} f(x) = \_\_\_\_$$

$$\lim_{x \to 2} f(x) = \_\_\_\_$$

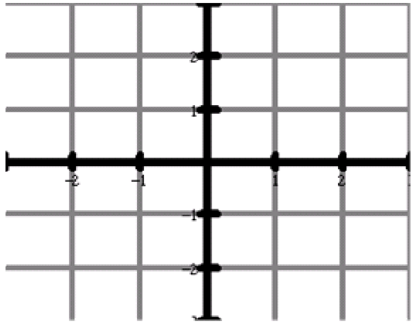
$$g(x) = -x + 1$$



$$\lim_{x \to -1} g(x) = \_\_\_\_$$

$$\lim_{x \to 2} g(x) = \_\_\_\_$$

$$f(x) + g(x)$$



$$\lim_{x \to -1} \left[f(x) + g(x) \right] = \_\_\_\_$$

$$\lim_{x \to 2} \left[f(x) + g(x) \right] = \_\_\_\_$$

So. . .Does it matter what the x-value is? _____ In general,

$$\lim_{x \to c} f(x) + \lim_{x \to c} g(x) = \lim_{x \to c} \left[f(x) + g(x) \right]$$

There are lots of other great limit properties on pg. 61-62 in your book. Look at them sometime, but for now let's just work some examples.

Suppose you are given the following information:

$$\lim_{x \to a} f(x) = 3 \qquad \lim_{x \to a} g(x) = 2$$

Describe what transformation is happening and then find:

$$\lim_{x \to a} [4f(x)] = \_\_\_\_$$ transformation: ____________________

$$\lim_{x \to a} [f(x)g(x)] = \_\_\_\_$$ transformation: ____________________

$$\lim_{x \to a} \left[\sqrt{g(x)}\right] = \_\_\_\_$$ transformation: ____________________

But what if you don't have a graph?

Evaluate these limits algebraically:

$$\lim_{x \to 3} (4x) = \_\_\_\_\_\_\_\_$$

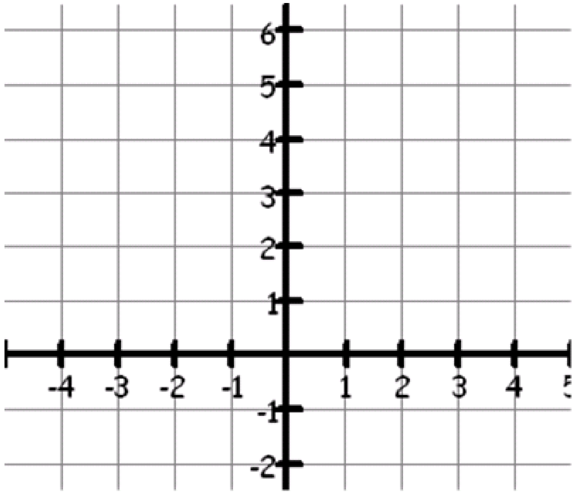
$$\lim_{x \to 1} \left(e^{x}\right) = \_\_\_\_\_\_\_\_$$

$$\lim_{x \to 0^{-}} \left(\sqrt{x+3}\right) = \_\_\_\_\_\_\_\_$$

$$\lim_{x \to 2^{+}} \left(\frac{1}{x^{2}}\right) = \_\_\_\_\_\_\_\_$$

How about some piecewise functions. . .

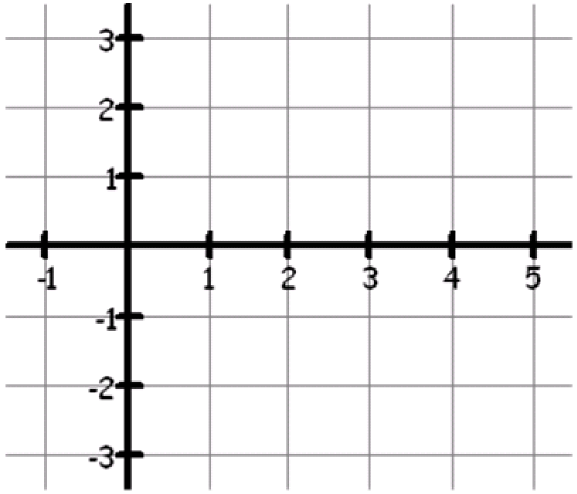
$$f(x)=\begin{cases} x^2+1 & x\ge 1 \\ 2x+3 & x<1 \end{cases}$$



$\lim_{x\to 1^-} f(x) =$ ______ $\lim_{x\to 1^+} f(x) =$ ______

$\lim_{x\to 2} f(x) =$ ______ $\lim_{x\to 1} f(x) =$ ______

$$f(x)=\begin{cases} -x+1 & x\le 2 \\ -2 & x>2 \end{cases}$$



$\lim_{x\to 2^-} f(x) =$ ______

$\lim_{x\to 2^+} f(x) =$ ______

$\lim_{x\to 2} f(x) =$ ______

Evaluate the following limit.

$$\lim_{x \to -1} \frac{x^2 - 1}{x + 1} = \_\_\_\_\_\_$$

Algebraically:

Graphically: