Section 2.2
Limits Involving Infinity

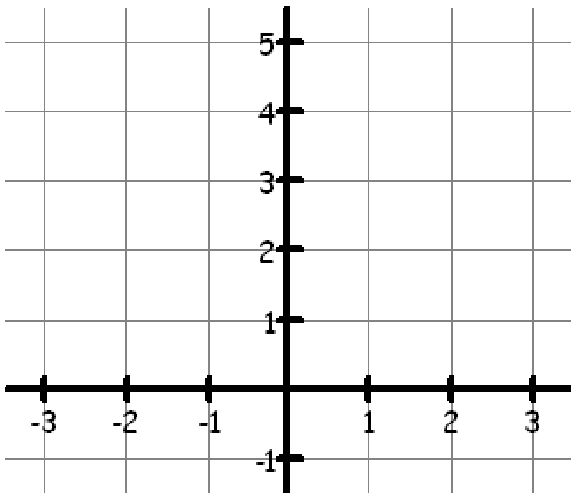
Evaluate the following limits.

$$\lim_{x \to \infty}\left(3+\frac{2}{x^2}\right)= \_\_\_\_\_\_$$

$$\lim_{x \to -\infty}\left(3+\frac{2}{x^2}\right)= \_\_\_\_\_\_$$

Graph the function and find it's horizontal asymptote.

$$f(x)=3+\frac{2}{x^2}$$



How about this expression. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

Logic would tell us that as x gets large, $1+\frac{1}{x}$ gets close to ________, and thus the limit as x approaches infinity, $\left(1+\frac{1}{x}\right)^x$ should be ________. But when you play with infinity, logic doesn't always work. Let's use our calculator to see what is happening to the function for large values of x.

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

End Behavior Model

An end behavior model is one that approximates what the function is "doing" at each end of the graph. It is found by looking at what part of the function will "dominate" as the x-values get really big.

Find (a) End Behavior Model

(b) $\lim_{x \to \infty} f(x)$

(c) $\lim_{x \to -\infty} f(x)$

(d) Horizontal asymptote(s), if any

$$f(x) = \frac{2x^2 + 3x}{x + 4}$$

For simple functions in the form $f(x) = \frac{polynomial}{polynomial}$

Top Heavy: (degree of top > degree of bottom)

$\lim_{x \to \infty} f(x) =$ ________

$\lim_{x \to \infty} \frac{3x^3 + 2x - 5}{2x^2 - 3} =$ ________

Bottom Heavy: (degree of top < degree of bottom)

$\lim_{x \to \infty} f(x) =$ ________

$\lim_{x \to \infty} \frac{5x^2 + x - 7}{x^4 - 2x + 3} =$ ________

Equal: (degree of top = degree of bottom)

$\lim_{x \to \infty} f(x) =$ ________

$\lim_{x \to \infty} \frac{2x^2 - 6x - 9}{3x^2 + x - 3} =$ ________

Not all functions fit that nice pattern though, so we need to look at end behavior models.

Find (a) End Behavior Model

(b) $\lim_{x \to \infty} f(x)$

(c) $\lim_{x \to -\infty} f(x)$

(d) Horizontal asymptote(s), if any

$$f(x) = \frac{x}{\sqrt{x^2+1}}$$

Find (a) End Behavior Model

(b) $\lim_{x \to \infty} f(x)$

(c) $\lim_{x \to -\infty} f(x)$

(d) Horizontal asymptote(s), if any

$$f(x) = \frac{4x - \cos x}{x}$$

Find (a) End Behavior Model

(b) $\lim_{x \to \infty} f(x)$

(c) $\lim_{x \to -\infty} f(x)$

(d) Horizontal asymptote(s), if any

$$f(x) = \frac{\ln\left(x^3\right)}{\ln\left(x\right)}$$

Find (a) End Behavior Model

(b) $\lim_{x \to \infty} f(x)$

(c) $\lim_{x \to -\infty} f(x)$

(d) Horizontal asymptote(s), if any

$$f(x) = e^x - x$$

Rank the following functions in terms of "size" as x gets larger.

$\log x$ $\quad x \quad$ x^n $\quad \frac{1}{x} \quad$ x^x $\quad b^x$

smallest → largest

Evaluate the following limits.

$$\lim_{x\to\infty}\frac{x^3}{2^x} = \_\_\_\_\_\_\_\_$$

$$\lim_{x\to\infty}\frac{x}{\log x} = \_\_\_\_\_\_\_\_$$

Behavior at Vertical Asymptotes

If a function has a vertical asymptote, the the left- and right-hand limits at the asymptote will be either positive or negative infinity.

Find the vertical asymptote(s) and the the left- and right-hand limit at each one.

$$f(x) = \frac{1}{x-6}$$

Find the vertical asymptote(s) and the the left- and right-hand limit at each one.

$$f(x) = \frac{2x}{x^2 + 3x - 4}$$

Find the vertical asymptote(s) and the the left- and right-hand limit at each one.

$$f(x) = \tan(x)$$

Group Activity. . .

Sketch the graph of a function that satisfies the stated condition. Include any asymptotes.

Function 1: $y = f(x)$		Function 2: $y = g(x)$	
$\lim_{x \to 1} f(x) = 2$		$\lim_{x \to 2} f(x) = -1$	
$\lim_{x \to 5^-} f(x) = \infty$	$\lim_{x \to -2^+} f(x) = -\infty$	$\lim_{x \to 4^+} f(x) = -\infty$	$\lim_{x \to 4^-} f(x) = \infty$
$\lim_{x \to 5^+} f(x) = \infty$	$\lim_{x \to -2^-} f(x) = \infty$	$\lim_{x \to -\infty} f(x) = 2$	$\lim_{x \to \infty} f(x) = \infty$
$\lim_{x \to -\infty} f(x) = 0$	$\lim_{x \to \infty} f(x) = -1$		