Section 2.3

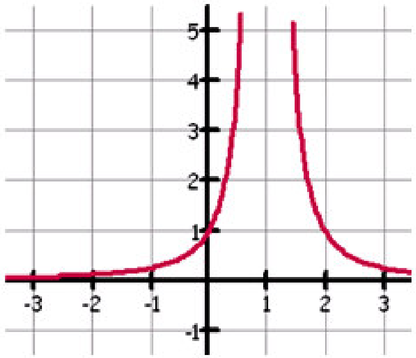
Continuity:
An Analytical and Graphical Approach

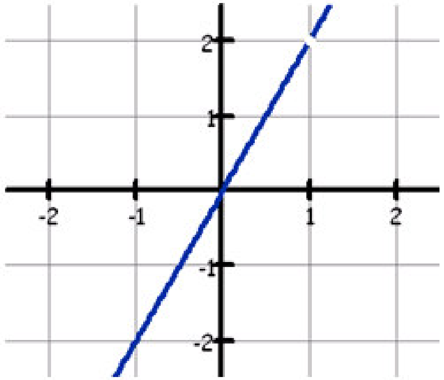
What does the word

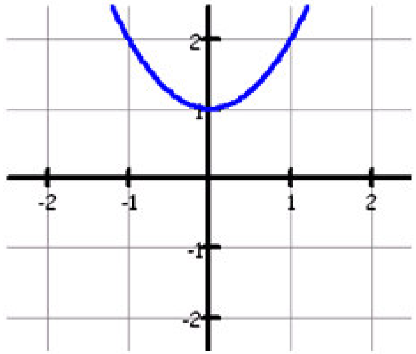
continuous

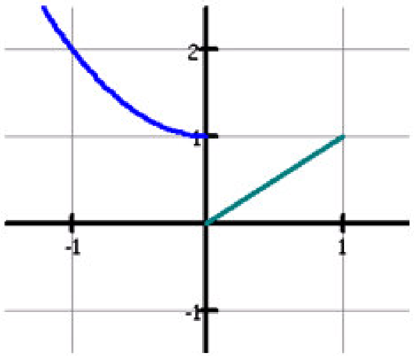
mean?

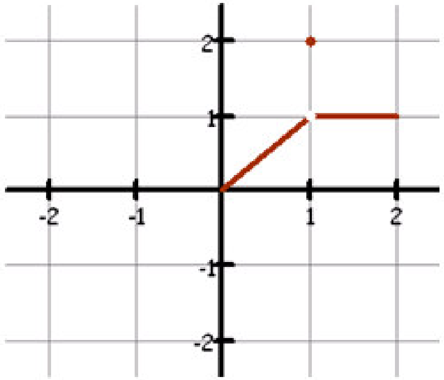
Decide if the graphs are conitnuous or not and explain why/why not.











From our study of functions in PreCalculus we can make the following conclusions:

a) All polynomials are continuous over their entire domain.

b) Fractions in the form of $y = \frac{f(x)}{g(x)}$ are discontinuous wherever $y = g(x) = 0$

c) Radicals in the form $y = \sqrt[\text{odd root}]{f(x)}$ are continuous everywhere.

d) Radicals in the form of $y = \sqrt[\text{even root}]{f(x)}$ are discontinuous where $f(x) < 0$

e) All exponential functions are continuous over their entire domain

Find the x-values (if any) at which the function is not continuous.

$$f(x) = 3x^3 + 5x - 7$$

$$f(x) = \frac{x-3}{x^2 - 9}$$

$$f(x) = \sqrt[3]{x^2 + 2x - 1}$$

$$f(x) = \sqrt[4]{x^2 - x - 6}$$

$$f(x) = x + 1 + \cos(x)$$

$$f(x) = e^{-x} + 2x + 1$$

Definition of

Continuity at a Point

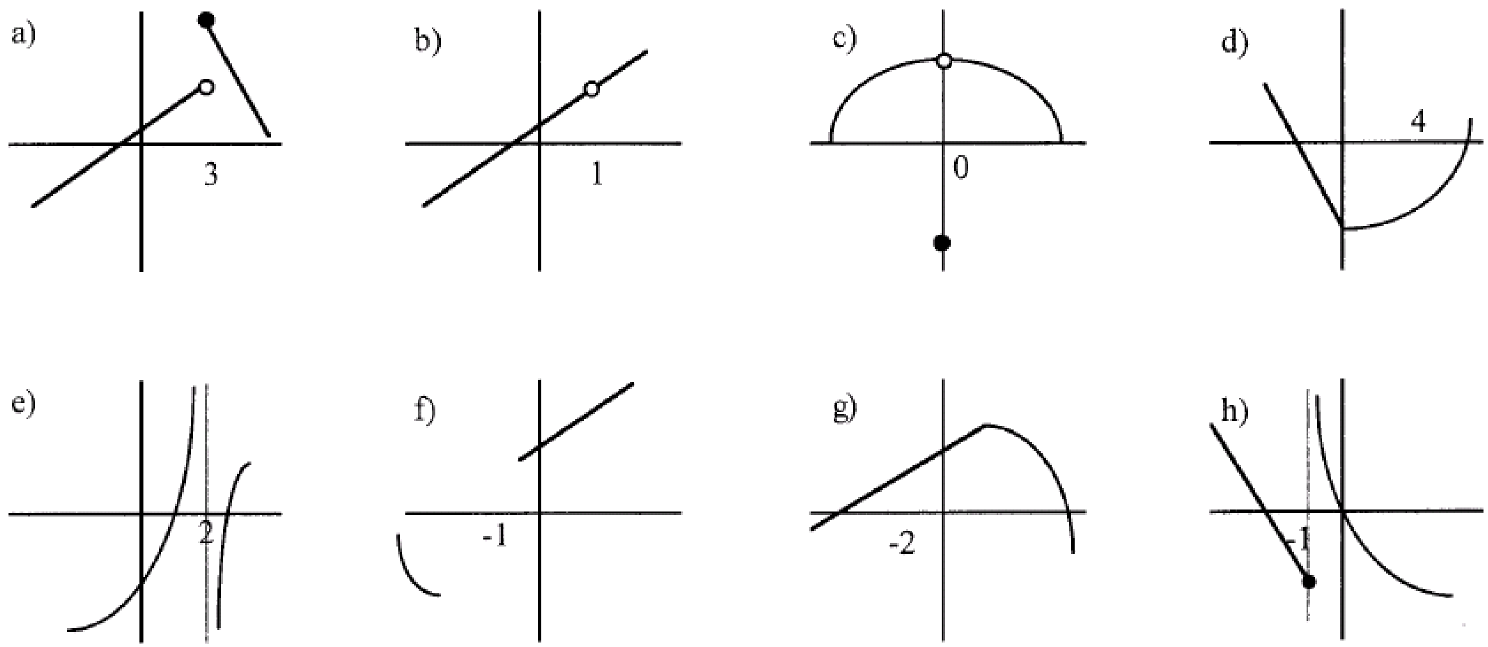
A function is continuous at c if the following conditions are met:

1. $f(c)$ is defined

2. $\lim_{x \to c} f(x)$ exists

3. $\lim_{x \to c} f(x) = f(c)$

Determine if the function is continuous at the marked value. If it is not, then determine which of the rules of continuity the function fails.



Justify that the function is/is not continuous at x = 2.

$$f(x)=\begin{cases} x+4 & x<2 \\ x^2+2 & x\geq 2 \end{cases}$$

$$f(x)=\begin{cases} x^2-3 & x\geq 1 \\ 1-x & x<1 \end{cases}$$

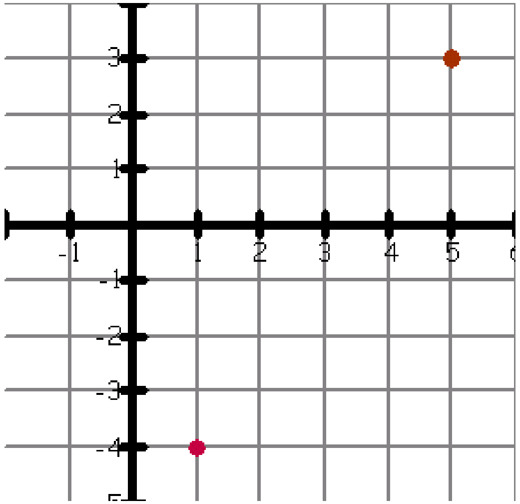
Find the constant a so that the function is continuous everywhere.

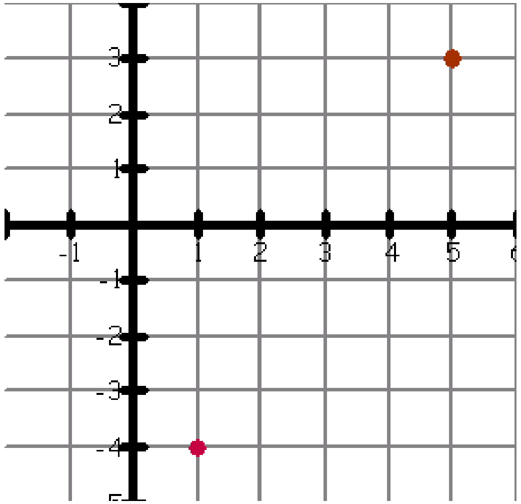
$$f(x) = \begin{cases} 2x^2 & x \leq 4 \\ ax & x > 4 \end{cases}$$

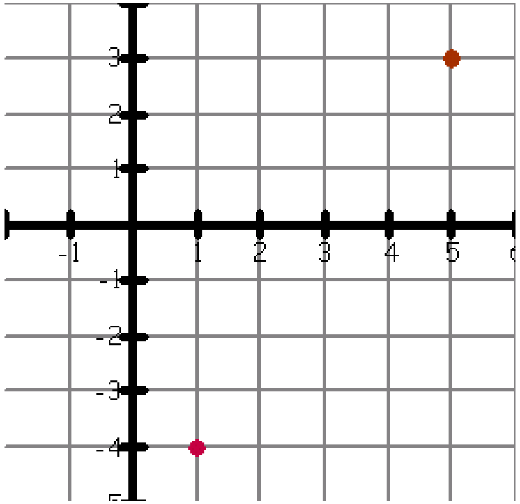
$$f(x) = \begin{cases} 3x+2 & x \geq 1 \\ 2a - x & x < 1 \end{cases}$$

Let's look at a consequence of continuous functions.

Consider a continuous function $f(x)$, which contians the points (1, -4) and (5, 3). You are not given an equation that defines $f(x)$ -- only these two points. Draw three possible graphs of $f(x)$.



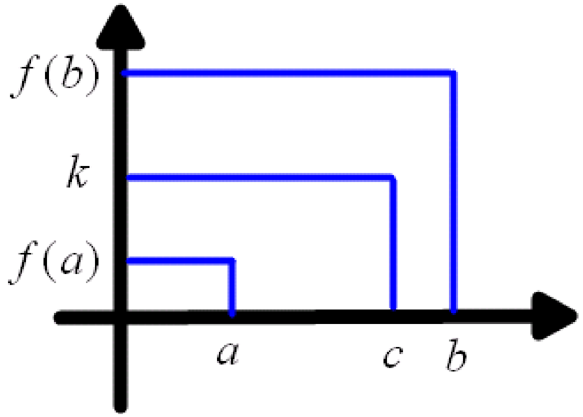




Which of the following height(s), is the function guaranteed to reach between x = 1 and x = 5?

A. -5 B. -1 C. 2 D. 5 E. 0

Based on the graphs you just drew and the diagram below, complete the statements below.



If a continuous function has a height of $f(a)$ when x = _____ and a height of ______ when x = b, then the function $f(x)$ will pass through every single height between $f(\_\_\_\_)$ and $f(\_\_\_\_)$ on the interval [a, b].

Intermediate Value Theorem:

If f is ________________ on the closed interval ________ and k is any number between _______ and _______, then there is at least one number c in [a, b] such that $f(c) =$ ______.

1. Given a function $h(x)$ continuous on [3, 7] with $h(3)=1$ and $h(7)=9$, which of the following <u>must</u> be true?
   I. There exists a real number p such that $h(p)=5$ for $1<p<9$.
   II. $h(5)=5$
   III. There exists a real number p such that $h(p)=2$ for $3<p<7$.

2. Given the continuous function $f(x)=\ln(-x)+\cos x$, show that there exists a $c\in[-\pi,-\frac{\pi}{2}]$ such that $f(c)=0.240$ Find c.