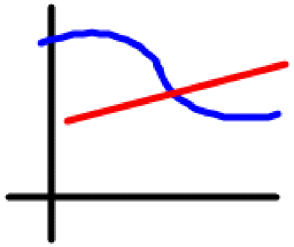
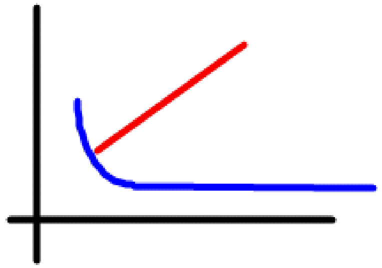
Tangent Lines, Local Linearity and Differentiability

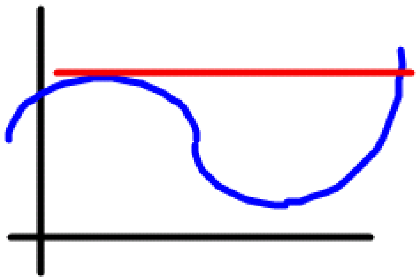
In Geometry, we said that a tangent line is tangent to a circle if it intersects the circle at one point. However, for more general curves, we need a better definition. We will work on developing that definition, but for now let's look at some common misconceptions.



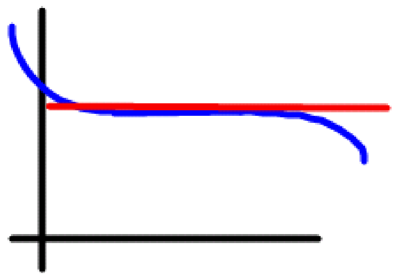
"A line is tangent to a curve if it crosses the curve at exactly one point."



"A line is tangent to a curve if it touches the curve at one point but does not cross the curve."



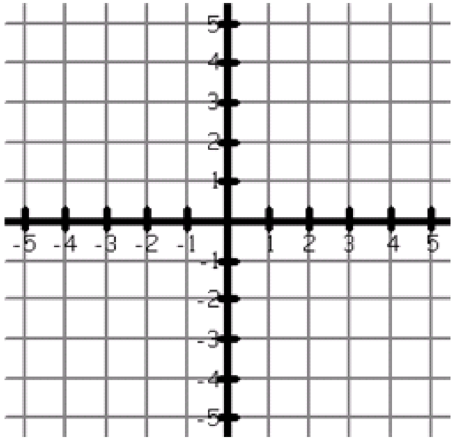
"A tangent line to a curve must cross the curve only once."



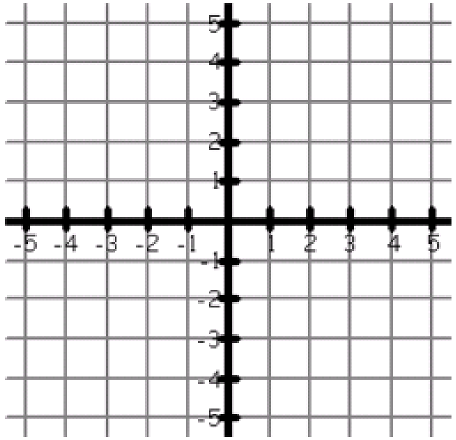
"A tangent line to a curve is a line that just 'grazes' the curve at a point but does not cross the curve."

Let's practice drawing some tangent lines at the indicated point.

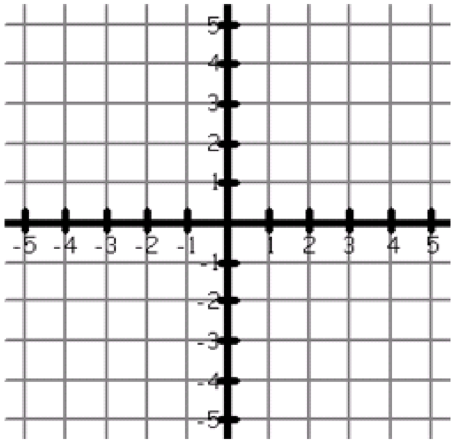
$y = -2x \; (x = -1)$



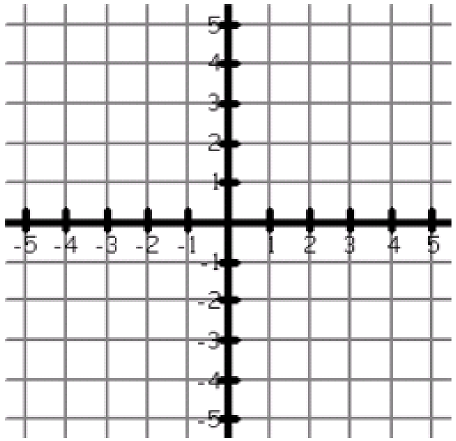
$y = 1 - x^2 \quad (x = 0)$



$y = e^x \quad (x = 0)$



$y = |x - 3| \quad (x = 3)$



When we are drawing tangent lines, we are saying that the curve can be approximated by that line in a close neighborhood around the given point. When we can zoom in on a point of a graph and see a staight line, we say that the function has **local linearity** at that point.

At $x=0$, the local linearity of the functions below will fall in one of the three categories listed. Place the functions in the correct category.

$y=1+\sin x$	$y=\frac{1}{2}\ln\left(x^2+1\right)$	$y=\sqrt{x}$	$y=\sin x$	$y=x^2$	$y=x\sin x$
$y=\tan x$	$y=\sqrt{2x-x^2}$	$y=x^3$	$y=\ln(x+1)$	$y=1-\cos x$	$y=\sqrt{\frac{x}{x+1}}$

$y=0$	$x=0$	$y=x$

Local linearity means that a tangent line exists to the curve at that point. The slope of that tangent line is called the ***derivative***.

For all functions in the category $y = 0$, the slope of the tangent line is ______. Thus, the derivative of those functions at $x = 0$ is ______.

For all functions in the category $y = x$, the slope of the tangent line is ______. Thus, the derivative of those functions at $x = 0$ is ______.

For all functions in the category x $= 0$, the slope of the tangent line is _________. Thus, the derivative of those functions at $x = 0$ _________.

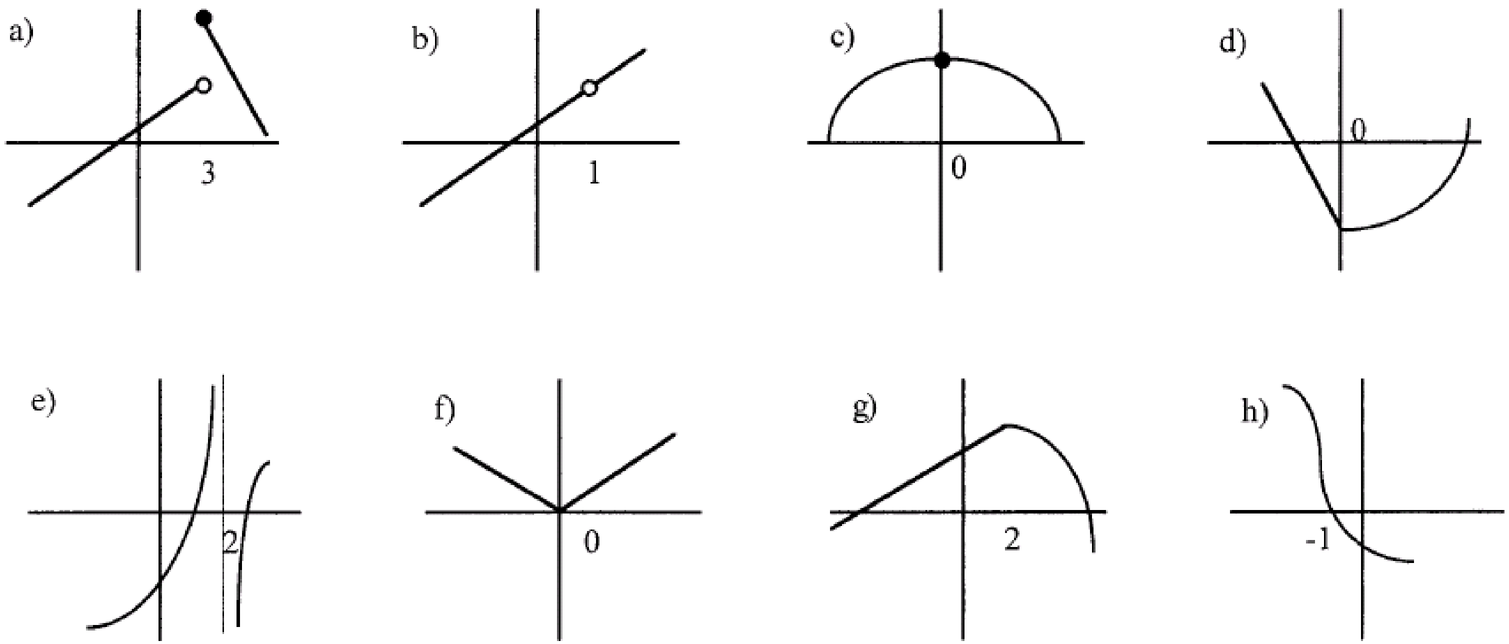
Let's look at some other functions and their local linearity at $x = 0$.

$y = \sin|x| + 1$ $y = |x|$

What do you notice as you zoom in?

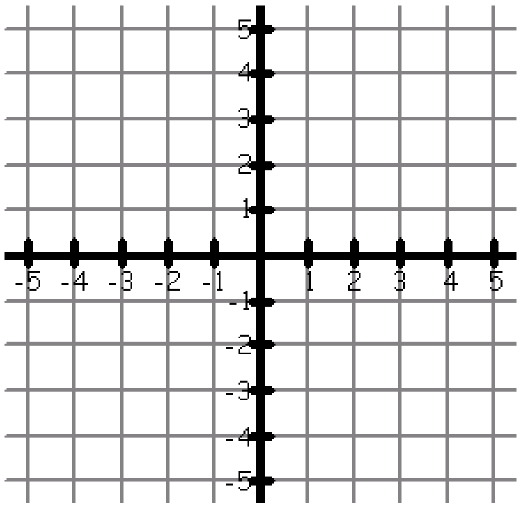
When does the derivative fail to exist? Remember, the derivative is the slope of the tangent line at a specified point.

Determine whether the functions are cointinuous, *differentiable*, neither or both at the indicated point.

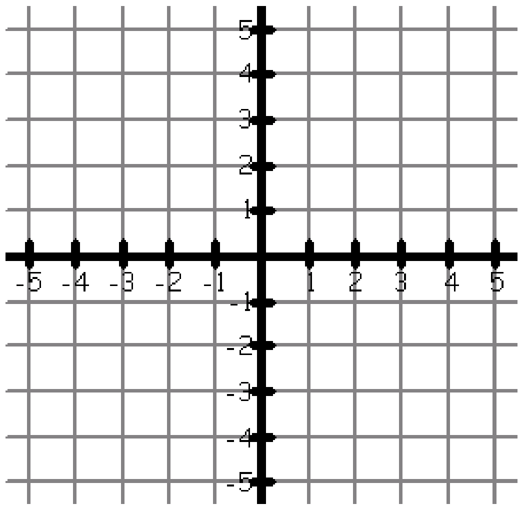


Graph each peicewise function and determine if each is continuous, differentiable, neither or both.

$$f(x)=\begin{cases} x^2 & x \geq 0 \\ x & x < 0 \end{cases}$$

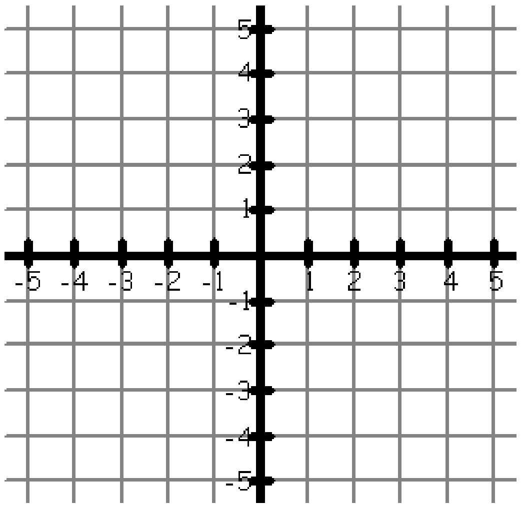


$$f(x)=\begin{cases} x^2+1 & x \geq 0 \\ x^3+1 & x < 0 \end{cases}$$



Graph each peicewise function and determine if each is continuous, differentiable, neither or both.

$$f(x) = \begin{cases} 4 - x^2 & x < 1 \\ 2x + 2 & x \geq 1 \end{cases}$$



$$f(x) = \begin{cases} e^x & x \geq 0 \\ \sin x + 1 & x < 0 \end{cases}$$