Section 3.6:
The Chain Rule

Let's do some investigating. . .

Find the derivative of $f(x)$ by first multiplying out the binomial.

$$f(x) = (3x - 5)^2$$

Now, decompose the function and let $y = u^2$ and $u = 3x - 5$.

Find the following. $\frac{dy}{dx}, \frac{dy}{du}, \frac{du}{dx}$

The Chain Rule:

(Derivative of the outside)(Derivative of the inside)

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\left(f \circ g \right)(x) = f\,'(g(x)) \cdot g\,'(x)$$

Find each derivative.

(1) $g(x)=\sqrt{x^2-4}$

(2) $f(x)=\left(8x-2x^2\right)^4$

(3) $f(x)=\dfrac{-3}{\left(3x-4\right)^2}$

(4) $y=3\sin(8x)$

Find each derivative.

(1) $y = 5\cos\left(3x - \frac{\pi}{2}\right)$

(2) $f(x) = 3\cos^5(x)$

(3) $f(x) = \sin^3(2x^5)$

(4) $g(\theta) = \sec^3(4\theta^2)$

Given the following information, find each derivative at $x = 2$.

$f(2) = -3$ $f'(2) = 6$ $g(2) = 3$ $g'(2) = -2$ $f'(3) = 4$

$f(x) \cdot g(x)$

$\dfrac{f(x)}{g(x)}$

$[f(x)]^3$

$f(g(x))$

Let's look at some parametric curves.

$$x = 2t^2 \qquad y = \sin t$$

With these equations, we can find $\frac{dx}{dt}$ and $\frac{dy}{dt}$. How can we use those to find $\frac{dy}{dx}$?

So. . . if your curve is defined parametrically, then

$$\frac{dy}{dx} = \underline{\qquad\qquad}$$

Find the line tangent to the right-hand hyperbola branch defined parametrically by

$$x = \sec t \qquad y = \tan t \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

at the point $\left(\sqrt{2}, 1\right)$ where $t = \dfrac{\pi}{4}$.