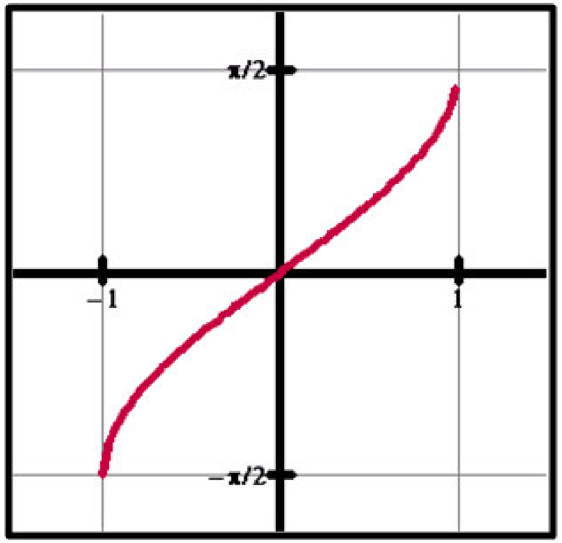
Section 3.8:

Derivatives of Inverse Trig Functions

First let's look at the inverse trig functions and the types of problems from PreCal.

$y = \sin^{-1}(x) = \arcsin(x)$



Domain: ____________

Range: ____________

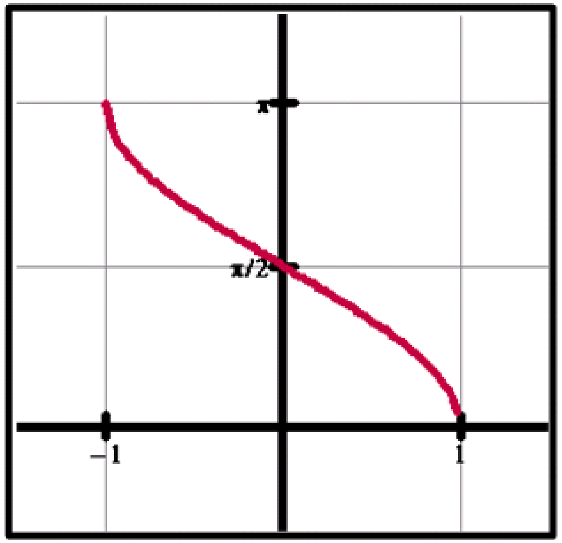
Try these:

$$\arcsin\left(-\frac{1}{2}\right) = \underline{\hspace{3cm}}$$

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \underline{\hspace{3cm}}$$

First let's look at the inverse trig functions and the types of problems from PreCal.

$y = \cos^{-1}(x) = \arccos(x)$



Domain: __________

Range: ___________

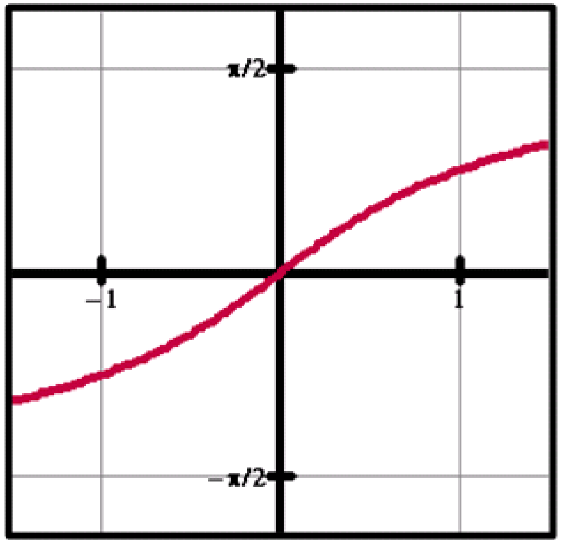
Try these:

$\arccos(1) =$ __________

$\cos^{-1}(0) =$ __________

First let's look at the inverse trig functions and the types of problems from PreCal.

$$y = \tan^{-1}(x) = \arctan(x)$$



Domain: ___________

Range: ___________

Try these:

$$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \_\_\_\_\_\_\_\_\_\_$$

$$\arctan(1) = \_\_\_\_\_\_\_\_\_\_$$

OK, how about some harder problems. Remember these? You will need to draw a triangle on the last two to describe the situation and answer the question.

$$\sin^{-1}\left(\cos\left(\frac{\pi}{4}\right)\right) = \_\_\_\_\_\_\_\_\_\_$$

$$\sec(\arctan(1)) = \_\_\_\_\_\_\_\_\_\_$$

$$\cos\left(\sin^{-1}(x)\right) = \_\_\_\_\_\_\_\_\_\_$$

$$\cot\left(\arccos\left(\frac{1}{\sqrt{x^2+1}}\right)\right) = \_\_\_\_\_\_\_\_\_\_$$

Now let's do some Calculus. We need to take the derivative of these three functions with respect to x.

$$y = \sin^{-1}(x)$$

(1) Take the sine of both sides.

(2) Draw a picture of the problem.

(3) Take the derivative of each side using implicit differentiation and solve for dy/dx. Use the picture to eliminate any y's.

$y = \cos^{-1}(x)$

$y = \tan^{-1}(x)$

Your turn!

$y = \csc^{-1}(x)$

$y = \sec^{-1}(x)$

$y = \cot^{-1}(x)$

Derivatives of Inverse Trig Functions:

$$\frac{d}{dx}\left[\sin^{-1}(x)\right] = \_\_\_\_\_\_\_\_\_\_$$

$$\frac{d}{dx}\left[\cos^{-1}(x)\right] = \_\_\_\_\_\_\_\_\_\_$$

$$\frac{d}{dx}\left[\tan^{-1}(x)\right] = \_\_\_\_\_\_\_\_\_\_$$

$$\frac{d}{dx}\left[\cot^{-1}(x)\right] = \_\_\_\_\_\_\_\_\_\_$$

$$\frac{d}{dx}\left[\sec^{-1}(x)\right] = \_\_\_\_\_\_\_\_\_\_$$

$$\frac{d}{dx}\left[\csc^{-1}(x)\right] = \_\_\_\_\_\_\_\_\_\_$$

Find the derivative of the following. Don't forget to look for the power, product, quotient and chain rule!

$$\frac{d}{dx}[\arcsin(2x)] = \_\_\_\_\_\_\_\_\_\_$$

$$\frac{d}{dx}[\arctan(3x)] = \_\_\_\_\_\_\_\_\_\_$$

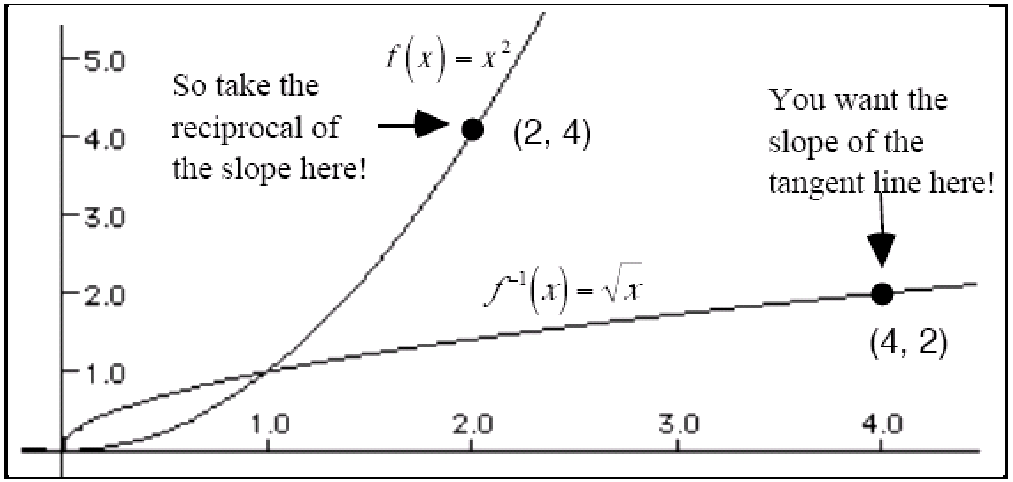
$$\frac{d}{dx}\left[\operatorname{arccot}\left(\sqrt{x}\right)\right] = \_\_\_\_\_\_\_\_\_\_$$

$$\frac{d}{dx}\left[\operatorname{arcsec}\left(x^2\right)\right] = \_\_\_\_\_\_\_\_\_\_$$

Find dy/dx.

$$y = \arcsin(x) + x\sqrt{1 - x^2}$$

Let's look at inverse functions in general. Take the two inverse functions $y = x^2$ and $y = \sqrt{x}$. What is the slope of the tangent line at the indicated points on the graph below?



So. . . the derivative of an inverse function is the ___________________ of the derivative of the original function at the "matching" point.

Try these:

Find $\left(f^{-1}\right)'(3)$ if $f(x)=x^3+x+1$

Find $\left(f^{-1}\right)'(2)$ if $f(x)=x^2+2x+3$