Sec 6.1:
Slope Fields, Differential Equations & Euler's Method

A differential equation is an equation involving a function and one or more of its derivatives. The goal of solving a differential equation is to find a function, y, in terms of x, that satisfies the equation.

Determine whether the given function is a solution of the differential equation below.

$$y'' - y = 0$$

$y = \sin x$

$y = 4e^{-x}$

The order of a differential equation is determined by the highest-order derivative in the equation. For example,

$y' = 4y$ → ______________ - order DE

$y'' - y = 0$ → ______________ - order DE

The general solution of a differential equation contains arbitrary constants (*C*). In fact, a differential equation of order *n* has a general solution with *n* arbitrary constants.

To find the specific solution of a differential equation, we must be given an initial conditions must be equal to the number of arbitrary constants in the general solution.

There are three ways to solve a differential equation:

1) Algebraically by separation and integration

2) Graphically by slope fields

3) Numerically by Euler's method

Solve the following DE algebraically, finding the general solution.

$$\frac{dy}{dx} = \frac{2x}{y}$$

$$\frac{dy}{dx} = y^2$$

Solve the following DE algebraically, finding the general solution.

$$\frac{dy}{dx} = \frac{x + \sin x}{3y^2}$$

$$\frac{du}{dt} = e^{u+2t}$$

Solve the following DE algebraically, using the initial condition to find the specific solution.

$$\frac{dy}{dx} = y^2 + 1, \quad y(1) = 0$$

$$xe^{-t}\frac{dx}{dt} = 1, \quad x(0) = 1$$

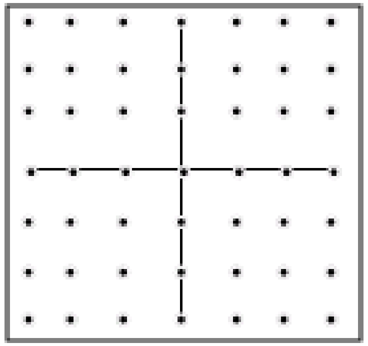
Graph a solution to the given DE given the initial conditions.

$$\frac{dy}{dx} = x$$

Initial conditions:

(1, 1) (-2, 0) (2, -2)

(x, y)	-3	-2	-1	0	1	2	3
3							
2							
1							
0							
-1							
-2							
-3							

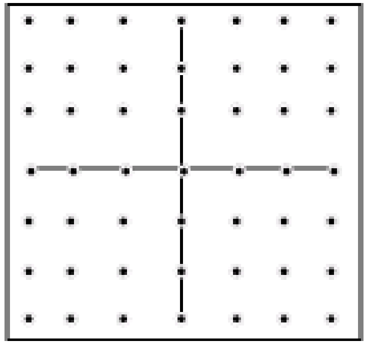


$$\frac{dy}{dx} = y$$

Initial conditions:

(1, 1) (-2, -1)

(x, y)	-3	-2	-1	0	1	2	3
3							
2							
1							
0							
-1							
-2							
-3							



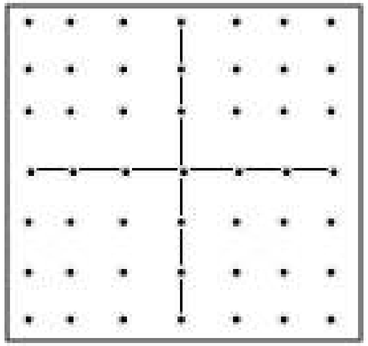
Graph a solution to the given DE given the initial conditions.

$$\frac{dy}{dx} = \frac{-x}{y}$$

Initial conditions:

(1, 1)   (-2, 0)

(x, y)	-3	-2	-1	0	1	2	3
3							
2							
1							
0							
-1							
-2							
-3							

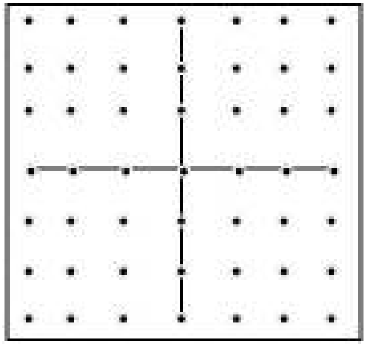


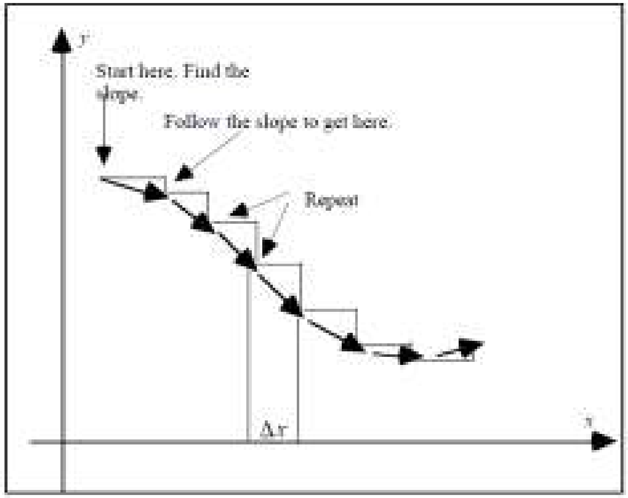
$$\frac{dy}{dx} = x + y$$

Initial conditions:

(1, 1)   (-2, -1)   (0, -1)

(x, y)	-3	-2	-1	0	1	2	3
3							
2							
1							
0							
-1							
-2							
-3							





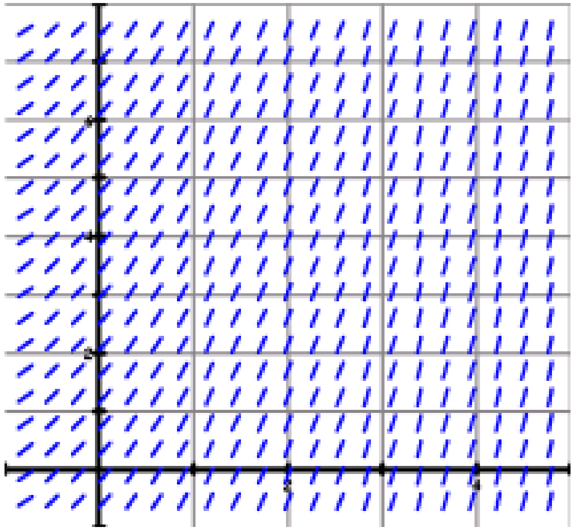
Many differential equations are too difficult to solve algebraically, and a slope field is not accurate enough to approximate a value. Euler's Method is a numeric approach to approximating a particular solution for a given DE.

Here is the procedure for Euler's Method

- Start at a given point (x, y) on the graph.
- Calculate the slope at this point by using the differential equation.
- For a given value of Δx, calculate the value of dy, using the fact that $dy \approx \frac{dy}{dx}\Delta x$ which will be the change in y along the graph.
- Add Δx to x and add dy to y to get a new point (x, y).
- Repeat the process

Use Euler's Method to approximate $y(2.5)$ given $y'=x+2$ and initial point (0, 0) using increments of 0.5.

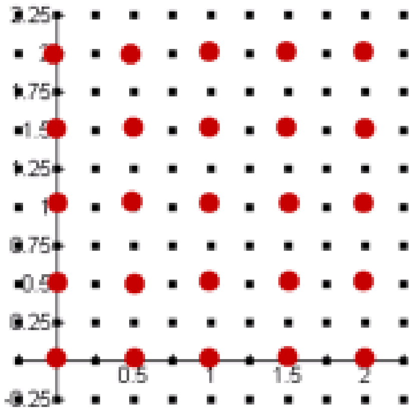
(x, y)	$\frac{dy}{dx} = x + 2$	Δx	$\Delta y = \frac{dy}{dx} \cdot \Delta x$	$(x + \Delta x, y + \Delta y)$



Consider the differential equation and initial condition given below.

$$\frac{dy}{dx} = 2x \qquad y(0) = 0$$

Draw a slope field at the indicated points.



Carry out Euler's Method with 4 steps to estimate $y(1)$

Sovle the DE algebraically to find y and then find the actual value of $y(1)$