Sec 6.4:
Exponential Growth & Decay

Consider the statement:

The rate of change of some quantity y is directly proportional to y.

Translation. . .

quantity:

rate of change of quantity:

directly proportional:

Which leads us to the differential equation:

Let's solve this DE and see what we get.

$$\frac{dy}{dt} = ky$$

So, the statement

The rate of change of some quantity y is directly proportional to y.

can be translated into: ___________________

which can be solved for y yielding: ________________________

MEMORIZE THIS TO SAVE TIME!

How about this. . .

Bacteria in a lab culture grows in such a way that the instantaneous rate of change of bacteria is directly proportional to the number of bacteria present.

Write a DE that expresses this relationship. Solve the DE for the number of bacteria as a function of time.

Suppose that there are 5 million bacteria present and 3 hours later, that number has grown to 7 million. Write the particular equation that epxresses the number of millions of bacteria as a function of hours.

What will the bacteria population be one full day after the first measurement?

When will the population reach 1 billion? Show work.

This only works if the rate of change is directly proportional to y. What if it isn't? Let's look at these.

The rate of change of y is proportional to 4y.

The rate of change of y is proportional to 4 - y.

The rate of change of y is inversely proportional to y.

The rate of change of y is proportional to the square root of y.

Let's try an AP problem. . .

Suppose you fill a tall tin can with water and then punch out a hole near the bottom. The water leaks out quickly at first, then more slowly as the depth of the water decreases. The rate at which the water's height, h, changes is directly proportional to the square root of its height.

a) Write a DE which states this relationship.

b) Suppose that at time t=0 min, the height is 12 cm and dh/dt = -3 cm/sec. Find the value of k which satisfies this relationship.

c) Solve this DE to find h as a function of time. Use the given info to find the particular solution.

d) Solve agebraically for the time when the can becomes empty.

Newton's Law of Cooling states that the rate of cooling an object is proportional to the temperature between the object and the outside air. Suppose that a roast turkey is taken from the oven when its temperature has reached 185 degrees and is placed on a table where the temp is 75 degrees. If R is the temp of the turkey after t minutes, then

$$\frac{dR}{dt} = k(R - 75)$$

Solve the DE equation for R.

If the temp of the turkey is 150 degrees after 30 min, what is the temp after 45 min?