Sec 6.5:
Logistic Growth

In exponential growth, we assume that the rate of increase (or decrease) of a quantity is directly proportional to that quantity.

However, in many situations a quantities growth levels off and approaches a limiting number L (the carrying capacity) because of limited resources. In these situations the rate of increase/decrease is directly proportional to both P and L-P. This type of growth is called logistic growth and is represented by the differential equation:

$$\frac{dP}{dt} = kP(L-P)$$

Let's solve this differential equation.

$$\frac{dP}{dt} = kP(L-P)$$

Let's look at one other thing. It is often asked when the growth rate is the fastest. We can find this by finding the max of the rate of change (i.e. finding where the second derivative changes from +/-).

$$\frac{dP}{dt} = kP(L-P)$$

So, the DE for a logistic growth function

can be solved for P to yield

and it's increasing at its fastest rate at

The population P(t) of fish in a lake satisfies the logistic differential equation given below, where t is measured in years.

$$\frac{dP}{dt} = 3P - \frac{P^2}{6000}$$

If P(0)=4000, what is $\lim_{t \to \infty} P(t)$? Is the solution curve increasing or decreasing? Sketch the solution curve. Does it have an inflection point? What is the range for P(t)?

The population P(t) of fish in a lake satisfies the logistic differential equation given below, where t is measured in years.

$$\frac{dP}{dt} = 3P - \frac{P^2}{6000}$$

If P(0)=10,000, what is $\lim_{t \to \infty} P(t)$? Is the solution curve increasing or decreasing? Sketch the solution curve. Does it have an inflection point? What is the range for P(t)?

The population P(t) of fish in a lake satisfies the logistic differential equation given below, where t is measured in years.

$$\frac{dP}{dt} = 3P - \frac{P^2}{6000}$$

If P(0)=20,000, what is $\lim_{t \to \infty} P(t)$? Is the solution curve increasing or decreasing? Sketch the solution curve. Does it have an inflection point? What is the range for P(t)?

The rate at which the flu spreads through a community is modeled by the logistic differential equation below, where t is measured in days.

$$\frac{dP}{dt} = 0.001P(3000 - P)$$

a) If P(0)=50, solve for P as a function of t.

b) Use your solution to find the size of the population after 2 days.

c) Use your solution to find the number of days that have occured when the flu is spreading the fastest.