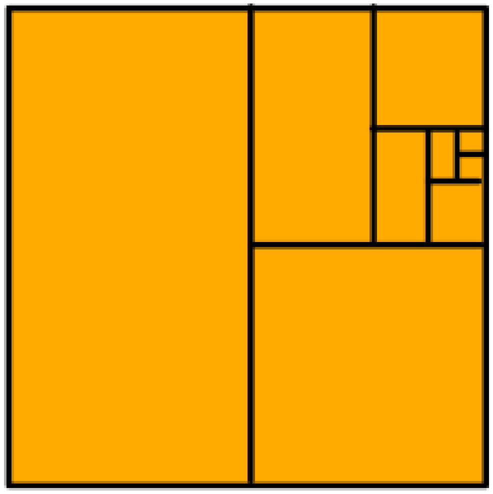
Sec 9.1: Intro to Series & Power Series

We looked at sequences earlier and now we're going to look at series. Do you remember the difference between a sequence and a series?



How would you write the area of each piece of the rectangle below? What does it all sum to?

Can you write $0.\overline{3}$ as a series? What is it's sum?

So an inifinite series can have a sum. Let's look at some others and see if they have a sum as well.

$$\sum_{n=1}^{\infty} \frac{3}{2^n} =$$

Find the first 10 partial sums.

$S_1=$

$S_2=$

$S_3=$

$S_4=$

$S_5=$

$S_6=$

$S_7=$

$S_8=$

$S_9=$

$S_{10}=$

$$\lim_{n\to\infty} S_n =$$

$$\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n =$$

Find the first 5 partial sums.

$S_1=$

$S_2=$

$S_3=$

$S_4=$

$S_5=$

$$\lim_{n\to\infty} S_n =$$

Why does the first series converge and the second one not converge to a sum?

Can anyone remember what the sum of an infinite geometric series is?

Geometric Series Test

A geometric series is in the form $\sum_{n=0}^{\infty} a_1(r)^n$ or $\sum_{n=1}^{\infty} a_1(r)^{n-1}$ $\quad a \neq 0$

- The geometric series *diverges* if $|r| \geq 1$
- The geometric series *converges* to the sum $S = \frac{a_1}{1-r}$ if $|r| < 1$

Determine if the following series converge or diverge. If they converge, what is the sum?

$$\sum_{n=0}^{\infty} \cos^n\left(\frac{\pi}{3} + n\pi\right)$$

$$\sum_{n=0}^{\infty} \frac{e^n}{3^{n+1}}$$

So now we are going to work it backwards. What if I give you a sum and ask you what the series was? Let's try it!

$Sum = \frac{1}{1-x}$ $a_1 =$ $r =$

Does this series make sense if we plug in 1/2?

What about if we plug in 2?

What x values would this series have a sum (converge)?

So let's compare the function $f(x)=\dfrac{1}{1-x}$ with the series we wrote on the last slide.

For what x-values does this series approximate the function well?

The series that involve both n and powers of x or (x-a) are called power series.

Power Series centered at x = 0:

$$\sum_{n=0}^{\infty} a_n (x)^n = a_0 + a_1 x + a_2 x^2 + \dots$$

Express each of the following as a power series and determine the interval of convergence.

$$f(x) = \frac{1}{1+x^2}$$ centered at x = 0

What is the center of the interval of convergence?

Express each of the following as a power series and determine the interval of convergence.

$$f(x) = \frac{1}{x+2}$$ centered at x = 0

What is the center of the interval of convergence?

Not all functions need to be approximated at x=0. We can shift the center to allow for other approximations.

Power Series centered at x = a:

$$\sum_{n=0}^{\infty} a_n (x-a)^n = a_0 + a_1(x-a) + a_2(x-a)^2 + \ldots$$

Express each of the following as a power series and determine the interval of convergence.

$$f(x) = \frac{1}{x+2}$$ centered at x = -1

Express the following as a power series and determine the interval of convergence.

$$f(x) = \frac{15}{2x-1} \quad \text{centered at } x = 2$$