More Taylor Series
and
Radius of Convergence

Let's look at and review a Taylor polynomial from yesterday first.

Suppose that g is a function which has continuous derivatives, and that

$$g(2)=3 \qquad g'(2)=-4 \qquad g''(2)=7 \qquad g'''(2)=-5$$

Write the Taylor polynomial of degree 3 for g centered at 2.

Write the Taylor polynomial of degree 2 for $h(x)=g'(x)$ centered at 2.

Write the Taylor polynomial of degree 2 for $f(x)=\int_2^x g(t)dt$ centered at 2.

When you know the function that was used to create a Taylor series, you can often look at the graphs of some of the polynomials and estimte the interval of covergence. However, when you are simply given the series and you don't know the function, that method will not work. We are now going to investigate how to find the radius of convergence in these situations.

Consider the series below. Write the first four non-zero terms of the series and then graph the first four partial sums. Estimate the interval of covergence based on your graphs. Set your window to [-6, 4] by [-2, 6].

$$\sum_{n=1}^{\infty} \frac{x^{n-1}}{n \bullet 3^n}$$

Let's check our interval for specific values of x. What happens if $x = 2$?

What if we put other positive values in for x? What do you think the upper bound for x can be so that it will still converge?

Let's look at negative x values now. What do you think the lower bound for x can be so that it will still converge?

Now we want to find a more exact way to determine the interval of covergence for non-goemtric series. In a geometric series the terms have a constant ratio, no matter which two terms you use. To converge, that ratio must be less than 1. In non-geometric series this ratio is not constant. However, since the series is infinite it is reasonable to look at the limit of this ratio to discover the behavior of the terms in the long run.

Below are two lists of series. For each determine $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$

Converging Series

$$\sum_{n=0}^{\infty} \frac{2}{n^2+1} \qquad \sum_{n=0}^{\infty} \left(\frac{6}{7}\right)^n$$

$$\sum_{n=0}^{\infty} \frac{2^n}{n!} \qquad \sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$$

Diverging Series

$$\sum_{n=0}^{\infty} \frac{1}{n}\left(\frac{3}{2}\right)^n \qquad \sum_{n=0}^{\infty} \frac{n}{n+1}$$

$$\sum_{n=0}^{\infty} \frac{n!}{n^2} \qquad \sum_{n=0}^{\infty} \frac{2^n}{n^2}$$

Ratio Test

Let $\sum_{n=1}^{\infty} a_n$ be a series of nonzero terms.

1. $\sum_{n=1}^{\infty} a_n$ converges if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$

2. $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$

3. The Ratio Test is inconclusive if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$

*Series with factorials and exponential functions work especiall well with the Ratio Test.

Let's try the Ratio Test. Find the radius of convergence.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n \cdot 2^n}$$

Find the radius of convergence for the following series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n+1}}{(2n+1)!}$$

Find the radius of convergence for the following series.

$$\sum_{n=0}^{\infty} n!(x-3)^n$$