# Section 9.5: More Tests for Convergence

Integral Test

P-Series Test

Alternating Series Test

## Integral Test

If $f$ is positive, continuous and decreasing for $x \geq 1$ and $a_n = f(x)$ then

$$\sum_{n=1}^{\infty} a_n \text{ and } \int_1^{\infty} f(x)dx \quad \text{either both converge or both diverge.}$$

Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$$

## *p*-Series

A series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots$ is called a $p$-series, where $p$ is a positive constant.

The harmonic series (where $p=1$) is the series $\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots$

Let's use the integral test to figure out for which values of $p$ would cause the series above to converge or diverge.

| $p=1$ | $p=1.1$ | $p=.9$ |
|---|---|---|

## *p*-Series Test:

The $p$-series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots$

- converges if $p > 1$
- diverges if $p < 1$ or $p = 1$

The harmonic series (where $p = 1$) diverges.

Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$$

An alternating series is a series whose terms are alternately positive and negative.

## Alternating Series Test

If $a_n > 0$, then an alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ or $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$

converges if both of the following conditions are satisfied:

1) $\lim_{n\to\infty} a_n = 0$

2) $\{a_n\}$ is a decreasing sequence; that is $a_{n+1} < a_n$ for all $n$.

*You can't use this test to show that a series diverges! Use the nth term test for that!!!*

Determine if the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{2n-1}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{\ln(2n)}$$

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$$

Now let's try to figure out what an alternating series will converge to (if it converges). We can't find it exactly, so we will approximate it by finding a partial sum and looking at how big the error is.

**Alternating Series Remainder**

If the series has a sum $S$, then $|R_n| = |S - S_n| < a_{n+1}$

In other words, if an alternating series satisfies the conditions of the Alternating Series Test, you can approximate the sum of the series by using the nth partial sum and your error will have an absolute value no greater than the first term left off.

Approximate the sum of the series below by using its first six terms and find the error. Use your results to find an interval in which S must lie. Justify.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

Approximate the sum of the series below with an error less than 0.001. Justify.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$$

Now let's go back to our radius and interval of convergence problem. Last class we looked at series that were primarily Geometric, but what if your series is not a geometric series? We still want to figure out what x-values will make the series converge, but we'll have to use a test other than the geometric series test. Let's try the Ratio Test first and then some other test for the endpoints.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n \bullet 2^n}$$

We know what the radius of convergence is now, but what happens at the endpoints? Let's check:

at $x = 7$:

at $x = 3$:

Find the radius and interval of convergence for the following series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x)^{2n+1}}{(2n+1)!}$$

Find the radius and interval of convergence for the following series.

$$\sum_{n=0}^{\infty} n!(x-3)^n$$