3

Legendary motorcycle stuntman Evel Knievel blasts off in his custom rocket-powered Harley-Davidson Skycycle in an attempt to jump the Snake River Canyon in 1974. A parachute prematurely deployed and caused the craft to fall into the canyon, just short of the other side. Knievel survived.

VECTORS AND TWO-DIMENSIONAL MOTION

In our discussion of one-dimensional motion in Chapter 2, we used the concept of vectors only to a limited extent. In our further study of motion, manipulating vector quantities will become increasingly important, so much of this chapter is devoted to vector techniques. We'll then apply these mathematical tools to two-dimensional motion, especially that of projectiles, and to the understanding of relative motion.

3.1 VECTORS AND THEIR PROPERTIES

Each of the physical quantities we will encounter in this book can be categorized as either a *vector quantity* or a *scalar quantity*. As noted in Chapter 2, a vector has both direction and magnitude (size). A scalar can be completely specified by its magnitude with appropriate units; it has no direction. An example of each kind of quantity is shown in Figure 3.1.

As described in Chapter 2, displacement, velocity, and acceleration are vector quantities. Temperature is an example of a scalar quantity. If the temperature of an object is $-5°C$, that information completely specifies the temperature of the object; no direction is required. Masses, time intervals, and volumes are scalars as well. Scalar quantities can be manipulated with the rules of ordinary arithmetic. Vectors can also be added and subtracted from each other, and multiplied, but there are a number of important differences, as will be seen in the following sections.

When a vector quantity is handwritten, it is often represented with an arrow over the letter ($\vec{\mathbf{A}}$). As mentioned in Section 2.1, a vector quantity in this book will be represented by boldface type with an arrow on top (for example, $\vec{\mathbf{A}}$). The magnitude of the vector $\vec{\mathbf{A}}$ will be represented by italic type, as A. Italic type will also be used to represent scalars.

Equality of Two Vectors. Two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are equal if they have the same magnitude and the same direction. This property allows us to translate a vector parallel to itself in a diagram without affecting the vector. In fact, for most purposes, any vector can be moved parallel to itself without being affected. (See Fig. 3.2.)

Adding Vectors. When two or more vectors are added, they must all have the same units. For example, it doesn't make sense to add a velocity vector, carrying units of meters per second, to a displacement vector, carrying units of meters. Scalars obey the same rule: It would be similarly meaningless to add temperatures to volumes or masses to time intervals.

Vectors can be added geometrically or algebraically. (The latter is discussed at the end of the next section.) To add vector $\vec{\mathbf{B}}$ to vector $\vec{\mathbf{A}}$ geometrically, first draw $\vec{\mathbf{A}}$ on a piece of graph paper to some scale, such as 1 cm = 1 m, so that its direction is specified relative a coordinate system. Then draw vector $\vec{\mathbf{B}}$ to the same scale with the tail of $\vec{\mathbf{B}}$ starting at the tip of $\vec{\mathbf{A}}$, as in Active Figure 3.3a. Vector $\vec{\mathbf{B}}$ must be drawn along the direction that makes the proper angle relative vector $\vec{\mathbf{A}}$. The **resultant vector** $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ is the vector drawn from the tail of $\vec{\mathbf{A}}$ to the tip of $\vec{\mathbf{B}}$. This procedure is known as the **triangle method of addition**.

When two vectors are added, their sum is independent of the order of the addition: $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$. This relationship can be seen from the geometric construction in Active Figure 3.3b, and is called the **commutative law of addition**.

This same general approach can also be used to add more than two vectors, as is done in Figure 3.4 (page 56) for four vectors. The resultant vector sum $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}}$ is the vector drawn from the tail of the first vector to the tip of the last. Again, the order in which the vectors are added is unimportant.

Negative of a Vector. The negative of the vector $\vec{\mathbf{A}}$ is defined as the vector that gives zero when added to $\vec{\mathbf{A}}$. This means that $\vec{\mathbf{A}}$ and $-\vec{\mathbf{A}}$ have the same magnitude but opposite directions.

Subtracting Vectors. Vector subtraction makes use of the definition of the negative of a vector. We define the operation $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ as the vector $-\vec{\mathbf{B}}$ added to the vector $\vec{\mathbf{A}}$:

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + (-\vec{\mathbf{B}}) \qquad [3.1]$$

Vector subtraction is really a special case of vector addition. The geometric construction for subtracting two vectors is shown in Figure 3.5 (page 56).

Multiplying or Dividing a Vector by a Scalar. Multiplying or dividing a vector by a scalar gives a vector. For example, if vector $\vec{\mathbf{A}}$ is multiplied by the scalar number 3, the result, written $3\vec{\mathbf{A}}$, is a vector with a magnitude three times that of $\vec{\mathbf{A}}$ and pointing in the same direction. If we multiply vector $\vec{\mathbf{A}}$ by the scalar -3, the result

Mack Henley/Visuals Unlimited

(a)

© Thomson Learning/George Semple

(b)

FIGURE 3.1 (a) The number of grapes in this bunch ripe for picking is one example of a scalar quantity. Can you think of other examples? (b) This helpful person pointing in the right direction tells us to travel five blocks north to reach the courthouse. A vector is a physical quantity that must be specified by both magnitude and direction.

TIP 3.1 Vector Addition vs. Scalar Addition

$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{C}}$ is very different from $A + B = C$. The first is a vector sum, which must be handled graphically or with components, whereas the second is a simple arithmetic sum of numbers.

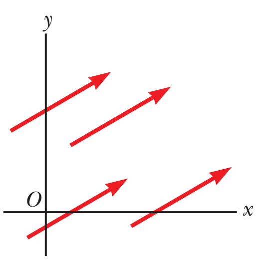
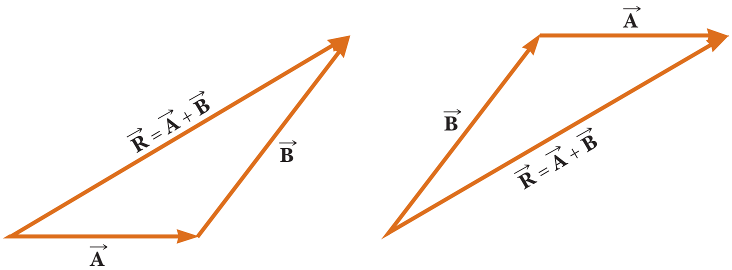


FIGURE 3.2 four vectors are equal because they have equal lengths and point in the same direction.



(a) (b)

ACTIVE FIGURE 3.3
(a) When vector $\vec{\mathbf{B}}$ is added to vector $\vec{\mathbf{A}}$, the vector sum $\vec{\mathbf{R}}$ is the vector that runs from the tail of $\vec{\mathbf{A}}$ to the tip of $\vec{\mathbf{B}}$. (b) Here the resultant runs from the tail of $\vec{\mathbf{B}}$ to the tip of $\vec{\mathbf{A}}$. These constructions prove that $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$.

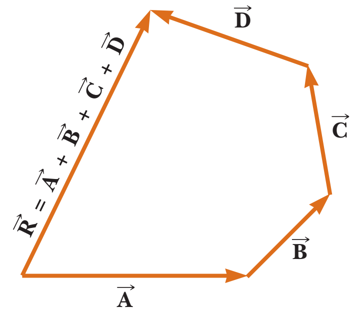


FIGURE 3.4 A geometric construction for summing four vectors. The resultant vector $\vec{\mathbf{R}}$ is the vector that completes the polygon.

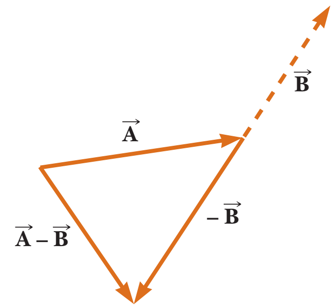


FIGURE 3.5 This construction shows how to subtract vector $\vec{\mathbf{B}}$ from vector $\vec{\mathbf{A}}$. The vector has the same magnitude as the vector $\vec{\mathbf{B}}$, but points in the opposite direction.

is $-3\vec{\mathbf{A}}$, a vector with a magnitude three times that of $\vec{\mathbf{A}}$ and pointing in the opposite direction (because of the negative sign).

QUICK QUIZ 3.1 The magnitudes of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are 12 units and 8 units, respectively. What are the largest and smallest possible values for the magnitude of the resultant vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$? (a) 14.4 and 4 (b) 12 and 8 (c) 20 and 4 (d) none of these.

EXAMPLE 3.1 Taking a Trip

Goal Find the sum of two vectors by using a graph.

Problem A car travels 20.0 km due north and then 35.0 km in a direction 60° west of north, as in Figure 3.6. Using a graph, find the magnitude and direction of a single vector that gives the net effect of the car's trip. This vector is called the car's *resultant displacement.*

Strategy Draw a graph and represent the displacement vectors as arrows. Graphically locate the vector resulting from the sum of the two displacement vectors. Measure its length and angle with respect to the vertical.

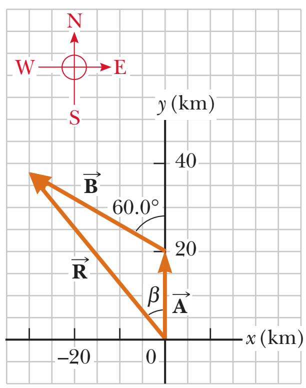


FIGURE 3.6 (Example 3.1) A graphical method for finding the resultant displacement vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$.

Solution

Let $\vec{\mathbf{A}}$ represent the first displacement vector, 20.0 km north, and $\vec{\mathbf{B}}$ the second displacement vector, extending west of north. Carefully graph the two vectors, drawing a resultant vector $\vec{\mathbf{R}}$ with its base touching the base of $\vec{\mathbf{A}}$ and extending to the tip of $\vec{\mathbf{B}}$. Measure the length of this vector, which turns out to be about 48 km. The angle β, measured with a protractor, is about 39° west of north.

Remarks Notice that ordinary arithmetic doesn't work here: the correct answer of 48 km is not equal to 20.0 km + 35.0 km = 55.0 km!

QUESTION 3.1

Suppose two vectors are added. Under what conditions would the sum of the magnitudes of the vectors equal the magnitude of the resultant vector?

EXERCISE 3.1

Graphically determine the magnitude and direction of the displacement if a man walks 30.0 km 45° north of east and then walks due east 20.0 km.

Answer 46 km, 27° north of east

3.2 COMPONENTS OF A VECTOR

One method of adding vectors makes use of the projections of a vector along the axes of a rectangular coordinate system. These projections are called **components.** Any vector can be completely described by its components.

Consider a vector $\vec{\mathbf{A}}$ in a rectangular coordinate system, as shown in Figure 3.7. $\vec{\mathbf{A}}$ can be expressed as the sum of two vectors: $\vec{\mathbf{A}}_x$, parallel to the x-axis; and $\vec{\mathbf{A}}_y$, parallel to the y-axis. Mathematically,

$$\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$$

where $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$ are the component vectors of $\vec{\mathbf{A}}$. The projection of $\vec{\mathbf{A}}$ along the x-axis, A_x, is called the x-component of $\vec{\mathbf{A}}$, and the projection of along the y-axis, A_y, is called the y-component of $\vec{\mathbf{A}}$. These components can be either positive or negative numbers with units. From the definitions of sine and cosine, we see that $\cos\theta = A_x/A$ and $\sin\theta = A_y/A$, so the components of $\vec{\mathbf{A}}$ are

$$A_x = A\cos\theta \qquad [3.2]$$

$$A_y = A\sin\theta$$

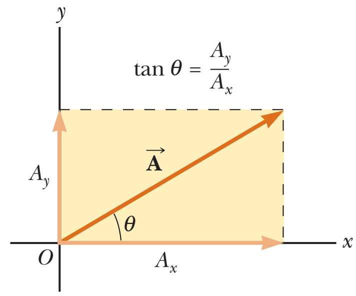


FIGURE 3.7 Any vector $\vec{\mathbf{A}}$ lying in the xy-plane can be represented by its rectangular components A_x and A_y.

These components form two sides of a right triangle having a hypotenuse with magnitude A. It follows that $\vec{\mathbf{A}}$'s magnitude and direction are related to its components through the Pythagorean theorem and the definition of the tangent:

$$A = \sqrt{A_x^2 + A_y^2} \qquad [3.3]$$

$$\tan\theta = \frac{A_y}{A_x} \qquad [3.4]$$

To solve for the angle θ, which is measured from the positive x-axis by convention, we can write Equation 3.4 in the form

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

This formula gives the right answer only half the time! The inverse tangent function returns values only from $-90°$ to $+90°$, so the answer in your calculator window will only be correct if the vector happens to lie in the first or fourth quadrant. If it lies in the second or third quadrant, adding 180° to the number in the calculator window will always give the right answer. The angle in Equations 3.2 and 3.4 must be measured from the positive x-axis. Other choices of reference line are possible, but certain adjustments must then be made. (See Tip 3.2 and Fig. 3.8.)

If a coordinate system other than the one shown in Figure 3.7 is chosen, the components of the vector must be modified accordingly. In many applications it's more convenient to express the components of a vector in a coordinate system having axes that are not horizontal and vertical, but are still perpendicular to each other. Suppose a vector $\vec{\mathbf{B}}$ makes an angle θ' with the x'-axis defined in Figure 3.9 (page 58).

TIP 3.2 x- and y-components

Equation 3.2 for the x- and y-components of a vector associates cosine with the x-component and sine with the y-component, as in Figure 3.8a. This association is due *solely* to the fact that we chose to measure the angle θ with respect to the positive x-axis. If the angle were measured with respect to the y-axis, as in Figure 3.8b, the components would be given by $A_x = A\sin\theta$ and $A_y = A\cos\theta$.

TIP 3.3 Inverse Tangents on Calculators: Right Half the Time

The inverse tangent function on calculators returns an angle between $-90°$ and $+90°$. If the vector lies in the second or third quadrant, the angle, as measured from the positive x-axis, will be the angle returned by your calculator plus 180°.

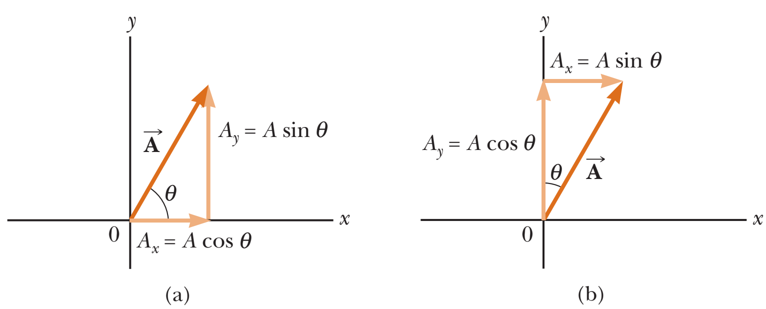


FIGURE 3.8 The angle θ need not always be defined from the positive x-axis.

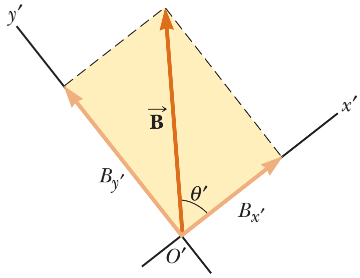


FIGURE 3.9 The components of vector $\vec{\mathbf{B}}$ in a tilted coordinate system.

The rectangular components of $\vec{\mathbf{B}}$ along the axes of the figure are given by $B_{x'} = B\cos\theta'$ and $B_{y'} = B\sin\theta'$, as in Equations 3.2. The magnitude and direction of $\vec{\mathbf{B}}$ are then obtained from expressions equivalent to Equations 3.3 and 3.4.

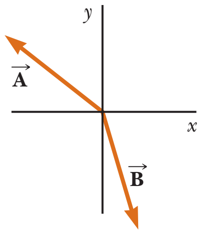


FIGURE 3.10 (Quick Quiz 3.2)

QUICK QUIZ 3.2 Figure 3.10 shows two vectors lying in the *xy*-plane. Determine the signs of the *x*- and *y*-components of $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{A}} + \vec{\mathbf{B}}$, and place your answers in the following table:

Vector	*x*-component	*y*-component
$\vec{\mathbf{A}}$		
$\vec{\mathbf{B}}$		
$\vec{\mathbf{A}} + \vec{\mathbf{B}}$		

EXAMPLE 3.2 Help Is on the Way!

Goal Find vector components, given a magnitude and direction, and vice versa.

Problem **(a)** Find the horizontal and vertical components of the 1.00×10^2 m displacement of a superhero who flies from the top of a tall building along the path shown in Figure 3.11a. **(b)** Suppose instead the superhero leaps in the other direction along a displacement vector $\vec{\mathbf{B}}$ to the top of a flagpole where the displacement components are given by $B_x = -25.0$ m and $B_y = 10.0$ m. Find the magnitude and direction of the displacement vector.

Strategy **(a)** The triangle formed by the displacement and its components is shown in Figure 3.11b. Simple trigonometry gives the components relative to the standard *x*-*y* coordinate system: $A_x = A\cos\theta$ and $A_y = A\sin\theta$ (Eqs. 3.2). Note that $\theta = -30.0°$, negative because it's measured clockwise from the positive *x*-axis. **(b)** Apply Equations 3.3 and 3.4 to find the magnitude and direction of the vector.

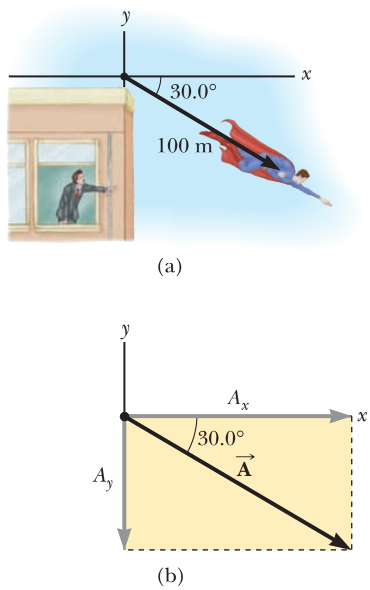


FIGURE 3.11 (Example 3.2)

Solution

(a) Find the vector components of $\vec{\mathbf{A}}$ from its magnitude and direction.

Use Equations 3.2 to find the components of the displacement vector $\vec{\mathbf{A}}$:

$$A_x = A\cos\theta = (1.00 \times 10^2 \text{ m})\cos(-30.0°) = \boxed{+86.6 \text{ m}}$$

$$A_y = A\sin\theta = (1.00 \times 10^2 \text{ m})\sin(-30.0°) = \boxed{-50.0 \text{ m}}$$

(b) Find the magnitude and direction of the displacement vector $\vec{\mathbf{B}}$ from its components.

Compute the magnitude of $\vec{\mathbf{B}}$ from the Pythagorean theorem:

$$B = \sqrt{B_x^{\,2} + B_y^{\,2}} = \sqrt{(-25.0\ \text{m})^2 + (10.0\ \text{m})^2} = 26.9\ \text{m}$$

Calculate the direction of $\vec{\mathbf{B}}$ using the inverse tangent, remembering to add 180° to the answer in your calculator window, because the vector lies in the second quadrant:

$$\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \tan^{-1}\left(\frac{10.0}{-25.0}\right) = -21.8°$$

$$\theta = 158°$$

Remarks In part (a), note that $\cos(-\theta) = \cos\theta$; however, $\sin(-\theta) = -\sin\theta$. The negative sign of A_y reflects the fact that displacement in the y-direction is *downward*.

QUESTION 3.2

What other functions, if any, can be used to find the angle in part (b)?

EXERCISE 3.2

(a) Suppose the superhero had flown 150 m at a 120° angle with respect to the positive x-axis. Find the components of the displacement vector. (b) Suppose instead the superhero had leaped with a displacement having an x-component of 32.5 m and a y-component of 24.3 m. Find the magnitude and direction of the displacement vector.

Answers (a) $A_x = -75$ m, $A_y = 130$ m (b) 40.6 m, 36.8°

Adding Vectors Algebraically

The graphical method of adding vectors is valuable in understanding how vectors can be manipulated, but most of the time vectors are added algebraically in terms of their components. Suppose $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. Then the components of the resultant vector $\vec{\mathbf{R}}$ are given by

$$R_x = A_x + B_x \qquad \textbf{[3.5a]}$$

$$R_y = A_y + B_y \qquad \textbf{[3.5b]}$$

So x-components are added only to x-components, and y-components only to y-components. The magnitude and direction of $\vec{\mathbf{R}}$ can subsequently be found with Equations 3.3 and 3.4.

Subtracting two vectors works the same way because it's a matter of adding the negative of one vector to another vector. You should make a rough sketch when adding or subtracting vectors, in order to get an approximate geometric solution as a check.

EXAMPLE 3.3 Take a Hike

Goal Add vectors algebraically and find the resultant vector.

Problem A hiker begins a trip by first walking 25.0 km 45.0° south of east from her base camp. On the second day she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower. **(a)** Determine the components of the hiker's displacements in the first and second days. **(b)** Determine the components of the hiker's total displacement for the trip. **(c)** Find the

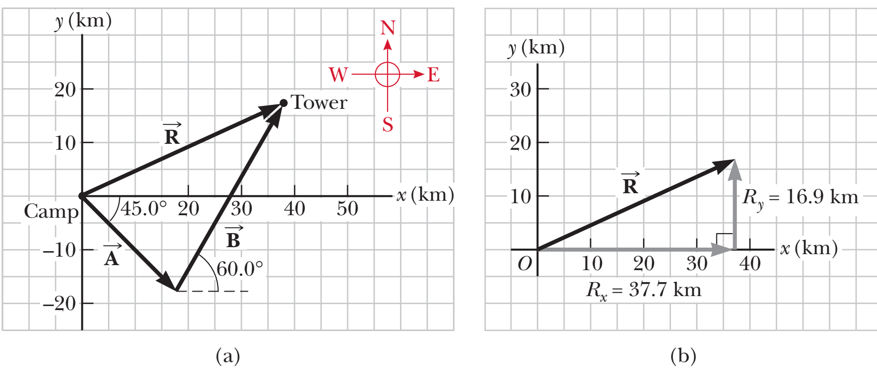


FIGURE 3.12 (Example 3.3) (a) Hiker's path and the resultant vector. (b) Components of the hiker's total displacement from camp.

magnitude and direction of the displacement from base camp.

Strategy This problem is just an application of vector addition using components, Equations 3.5. We denote the displacement vectors on the first and second days by $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, respectively. Using the camp as the origin of the coordinates, we get the vectors shown in Figure 3.12a. After finding x- and y-components for each vector, we add them "componentwise." Finally, we determine the magnitude and direction of the resultant vector $\vec{\mathbf{R}}$, using the Pythagorean theorem and the inverse tangent function.

Solution

(a) Find the components of $\vec{\mathbf{A}}$.

Use Equations 3.2 to find the components of $\vec{\mathbf{A}}$:

$$A_x = A\cos(-45.0°) = (25.0\text{ km})(0.707) = 17.7\text{ km}$$

$$A_y = A\sin(-45.0°) = -(25.0\text{ km})(0.707) = -17.7\text{ km}$$

Find the components of $\vec{\mathbf{B}}$:

$$B_x = B\cos 60.0° = (40.0\text{ km})(0.500) = 20.0\text{ km}$$

$$B_y = B\sin 60.0° = (40.0\text{ km})(0.866) = 34.6\text{ km}$$

(b) Find the components of the resultant vector, $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$.

To find R_x, add the x-components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$:

$$R_x = A_x + B_x = 17.7\text{ km} + 20.0\text{ km} = 37.7\text{ km}$$

To find R_y, add the y-components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$:

$$R_y = A_y + B_y = -17.7\text{ km} + 34.6\text{ km} = 16.9\text{ km}$$

(c) Find the magnitude and direction of $\vec{\mathbf{R}}$.

Use the Pythagorean theorem to get the magnitude:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(37.7\text{ km})^2 + (16.9\text{ km})^2} = 41.3\text{ km}$$

Calculate the direction of $\vec{\mathbf{R}}$ using the inverse tangent function:

$$\theta = \tan^{-1}\left(\frac{16.9\text{ km}}{37.7\text{ km}}\right) = 24.1°$$

Remarks Figure 3.12b shows a sketch of the components of $\vec{\mathbf{R}}$ and their directions in space. The magnitude and direction of the resultant can also be determined from such a sketch.

QUESTION 3.3

A second hiker follows the same path the first day, but then walks 15.0 km east on the second day before turning and reaching the ranger's tower. Is the second hiker's resultant displacement vector the same as the first hiker's, or different?

EXERCISE 3.3

A cruise ship leaving port travels 50.0 km 45.0° north of west and then 70.0 km at a heading 30.0° north of east. Find (a) the ship's displacement vector and (b) the displacement vector's magnitude and direction.

Answer (a) $R_x = 25.3$ km, $R_y = 70.4$ km (b) 74.8 km, 70.2° north of east

3.3 DISPLACEMENT, VELOCITY, AND ACCELERATION IN TWO DIMENSIONS

In one-dimensional motion, as discussed in Chapter 2, the direction of a vector quantity such as a velocity or acceleration can be taken into account by specifying whether the quantity is positive or negative. The velocity of a rocket, for example, is positive if the rocket is going up and negative if it's going down. This simple solution is no longer available in two or three dimensions. Instead, we must make full use of the vector concept.

Consider an object moving through space as shown in Figure 3.13. When the object is at some point Ⓟ at time t_i, its position is described by the position vector $\vec{\mathbf{r}}_i$, drawn from the origin to Ⓟ. When the object has moved to some other point Ⓠ at time t_f, its position vector is $\vec{\mathbf{r}}_f$. From the vector diagram in Figure 3.13, the final position vector is the sum of the initial position vector and the displacement $\Delta\vec{\mathbf{r}}$: $\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \Delta\vec{\mathbf{r}}$. From this relationship, we obtain the following one:

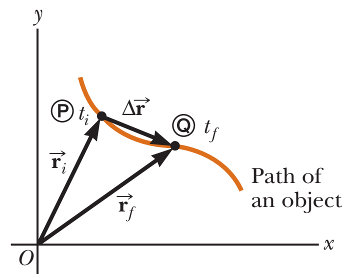


FIGURE 3.13 An object moving along some curved path between points Ⓟ and Ⓠ. The displacement vector $\Delta\vec{\mathbf{r}}$ is the difference in the position vectors: $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$.

An object's **displacement** is defined as the change in its position vector, or

$$\Delta\vec{\mathbf{r}} \equiv \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i \qquad [3.6]$$

SI unit: meter (m)

We now present several generalizations of the definitions of velocity and acceleration given in Chapter 2.

← Average velocity

An object's **average velocity** during a time interval Δt is its displacement divided by Δt:

$$\vec{\mathbf{v}}_{\text{av}} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \qquad [3.7]$$

SI unit: meter per second (m/s)

Because the displacement is a vector quantity and the time interval is a scalar quantity, we conclude that the average velocity is a *vector* quantity directed along $\Delta\vec{\mathbf{r}}$.

← Instantaneous velocity

An object's **instantaneous velocity** $\vec{\mathbf{v}}$ is the limit of its average velocity as Δt goes to zero:

$$\vec{\mathbf{v}} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \qquad [3.8]$$

SI unit: meter per second (m/s)

The direction of the instantaneous velocity vector is along a line that is tangent to the object's path and in the direction of its motion.

← Average acceleration

An object's **average acceleration** during a time interval Δt is the change in its velocity $\Delta\vec{\mathbf{v}}$ divided by Δt, or

$$\vec{\mathbf{a}}_{\text{av}} \equiv \frac{\Delta\vec{\mathbf{v}}}{\Delta t} \qquad [3.9]$$

SI unit: meter per second squared (m/s^2)

← Instantaneous acceleration

An object's **instantaneous acceleration** vector $\vec{\mathbf{a}}$ is the limit of its average acceleration vector as Δt goes to zero:

$$\vec{\mathbf{a}} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t} \qquad [3.10]$$

SI unit: meter per second squared (m/s^2)

It's important to recognize that an object can accelerate in several ways. First, the magnitude of the velocity vector (the speed) may change with time. Second, the direction of the velocity vector may change with time, even though the speed is constant, as can happen along a curved path. Third, both the magnitude and the direction of the velocity vector may change at the same time.

QUICK QUIZ 3.3 Which of the following objects can't be accelerating? (a) An object moving with a constant speed; (b) an object moving with a constant velocity; (c) an object moving along a curve.

QUICK QUIZ 3.4 Consider the following controls in an automobile: gas pedal, brake, steering wheel. The controls in this list that cause an acceleration of the car are (a) all three controls, (b) the gas pedal and the brake, (c) only the brake, or (d) only the gas pedal.

3.4 MOTION IN TWO DIMENSIONS

In Chapter 2 we studied objects moving along straight-line paths, such as the x-axis. In this chapter, we look at objects that move in both the x- and y-directions simultaneously under constant acceleration. An important special case of this two-dimensional motion is called **projectile motion**.

Projectile motion →

Anyone who has tossed any kind of object into the air has observed projectile motion. If the effects of air resistance and the rotation of Earth are neglected, the path of a projectile in Earth's gravity field is curved in the shape of a parabola, as shown in Active Figure 3.14.

The positive x-direction is horizontal and to the right, and the y-direction is vertical and positive upward. The most important experimental fact about projectile motion in two dimensions is that **the horizontal and vertical motions are completely independent of each other**. This means that motion in one direction has no effect on motion in the other direction. If a baseball is tossed in a parabolic path, as in Active Figure 3.14, the motion in the y-direction will look just like a ball tossed straight up under the influence of gravity. Active Figure 3.15 shows the effect of various initial angles; note that complementary angles give the same horizontal range.

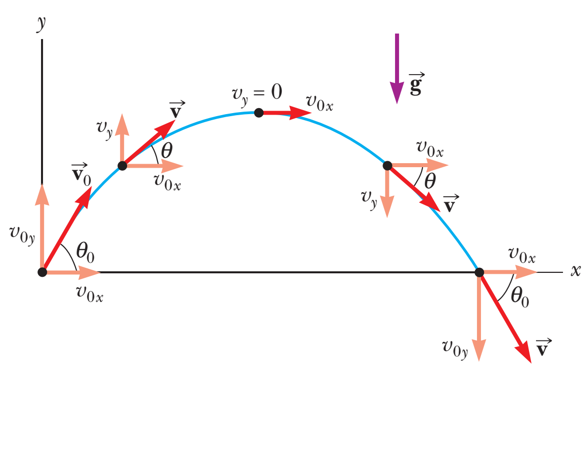
TIP 3.4 Acceleration at the Highest Point

The acceleration in the y-direction is *not* zero at the top of a projectile's trajectory. Only the y-component of the velocity is zero there. If the acceleration were zero, too, the projectile would never come down!

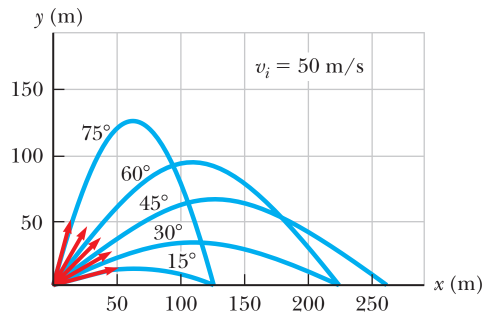
In general, the equations of constant acceleration developed in Chapter 2 follow separately for both the x-direction and the y-direction. An important difference is that the initial velocity now has two components, not just one as in that chapter. We assume that at $t = 0$ the projectile leaves the origin with an initial velocity $\vec{\mathbf{v}}_0$. If the velocity vector makes an angle θ_0 with the horizontal, where θ_0 is called the *projection angle*, then from the definitions of the cosine and sine functions and Active Figure 3.14 we have

$$v_{0x} = v_0 \cos \theta_0 \qquad \text{and} \qquad v_{0y} = v_0 \sin \theta_0$$

where v_{0x} is the initial velocity (at $t = 0$) in the x-direction and v_{0y} is the initial velocity in the y-direction.



ACTIVE FIGURE 3.14
The parabolic trajectory of a particle that leaves the origin with a velocity of $\vec{\mathbf{v}}_0$. Note that $\vec{\mathbf{v}}$ changes with time. However, the x-component of the velocity, v_x, remains constant in time. Also, $v_y = 0$ at the peak of the trajectory, but the acceleration is always equal to the free-fall acceleration and acts vertically downward.



ACTIVE FIGURE 3.15
A projectile launched from the origin with an initial speed of 50 m/s at various angles of projection. Note that complementary values of the initial angle θ result in the same value of R (the range of the projectile).

HIRB/Index Stock

The individual water streams of this fountain follow parabolic trajectories. The horizontal range and maximum height of a given stream of water depend on the elevation angle of that stream's initial velocity as well as its initial speed.

Now, Equations 2.6, 2.9, and 2.10 developed in Chapter 2 for motion with constant acceleration in one dimension carry over to the two-dimensional case; there is one set of three equations for each direction, with the initial velocities modified as just discussed. In the x-direction, with a_x constant, we have

$$v_x = v_{0x} + a_x t \qquad \textbf{[3.11a]}$$

$$\Delta x = v_{0x} t + \tfrac{1}{2} a_x t^2 \qquad \textbf{[3.11b]}$$

$$v_x^{\,2} = v_{0x}^{\,2} + 2a_x\,\Delta x \qquad \textbf{[3.11c]}$$

where $v_{0x} = v_0 \cos\theta_0$. In the y-direction, we have

$$v_y = v_{0y} + a_y t \qquad \textbf{[3.12a]}$$

$$\Delta y = v_{0y} t + \tfrac{1}{2} a_y t^2 \qquad \textbf{[3.12b]}$$

$$v_y^{\,2} = v_{0y}^{\,2} + 2a_y\,\Delta y \qquad \textbf{[3.12c]}$$

where $v_{0y} = v_0 \sin\theta_0$ and a_y is constant. The object's speed v can be calculated from the components of the velocity using the Pythagorean theorem:

$$v = \sqrt{v_x^{\,2} + v_y^{\,2}}$$

The angle that the velocity vector makes with the x-axis is given by

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

This formula for θ, as previously stated, must be used with care, because the inverse tangent function returns values only between $-90°$ and $+90°$. Adding 180° is necessary for vectors lying in the second or third quadrant.

The kinematic equations are easily adapted and simplified for projectiles close to the surface of the Earth. In that case, assuming air friction is negligible, the acceleration in the x-direction is 0 (because air resistance is neglected). **This means that $a_x = 0$, and the projectile's velocity component along the x-direction remains constant.** If the initial value of the velocity component in the x-direction is $v_{0x} = v_0 \cos\theta_0$, then this is also the value of v at any later time, so

$$v_x = v_{0x} = v_0 \cos\theta_0 = \text{constant} \qquad \textbf{[3.13a]}$$

whereas the horizontal displacement is simply

$$\Delta x = v_{0x} t = (v_0 \cos\theta_0) t \qquad \textbf{[3.13b]}$$

For the motion in the y-direction, we make the substitution $a_y = -g$ and $v_{0y} = v_0 \sin\theta_0$ in Equations 3.12, giving

$$v_y = v_0 \sin\theta_0 - gt \qquad \textbf{[3.14a]}$$

$$\Delta y = (v_0 \sin\theta_0) t - \tfrac{1}{2} g t^2 \qquad \textbf{[3.14b]}$$

$$v_y^{\,2} = (v_0 \sin\theta_0)^2 - 2g\,\Delta y \qquad \textbf{[3.14c]}$$

The important facts of projectile motion can be summarized as follows:

1. Provided air resistance is negligible, the horizontal component of the velocity v_x remains constant because there is no horizontal component of acceleration.
2. The vertical component of the acceleration is equal to the free-fall acceleration $-g$.
3. The vertical component of the velocity v_y and the displacement in the y-direction are identical to those of a freely falling body.
4. Projectile motion can be described as a superposition of two independent motions in the x- and y-directions.

EXAMPLE 3.4 Projectile Motion with Diagrams

Goal Approximate answers in projectile motion using a motion diagram.

Problem A ball is thrown so that its initial vertical and horizontal components of velocity are 40 m/s and 20 m/s, respectively. Use a motion diagram to estimate the ball's total time of flight and the distance it traverses before hitting the ground.

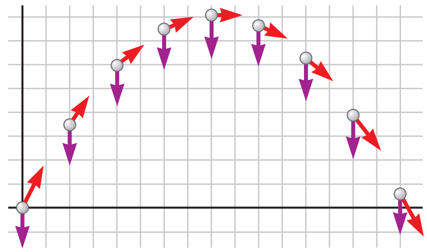


FIGURE 3.16 (Example 3.4) Motion diagram for a projectile.

Strategy Use the diagram, estimating the acceleration of gravity as $-10\ \text{m/s}^2$. By symmetry, the ball goes up and comes back down to the ground at the same y-velocity as when it left, except with opposite sign. With this fact and the fact that the acceleration of gravity decreases the velocity in the y-direction by 10 m/s every second, we can find the total time of flight and then the horizontal range.

Solution

In the motion diagram shown in Figure 3.16, the acceleration vectors are all the same, pointing downward with magnitude of nearly $10\ \text{m/s}^2$. By symmetry, we know that the ball will hit the ground at the same speed in the y-direction as when it was thrown, so the velocity in the y-direction goes from 40 m/s to -40 m/s in steps of -10 m/s every second; hence, approximately 8 seconds elapse during the motion.

The velocity vector constantly changes direction, but the horizontal velocity never changes because the acceleration in the horizontal direction is zero. Therefore, the displacement of the ball in the x-direction is given by Equation 3.13b, $\Delta x \approx v_{0x}t = (20\ \text{m/s})(8\ \text{s}) = 160\ \text{m}$.

Remarks This example emphasizes the independence of the x- and y-components in projectile motion problems.

QUESTION 3.4

Is the magnitude of the velocity vector at impact greater than, less than, or equal to the magnitude of the initial velocity vector? Why?

EXERCISE 3.4

Estimate the maximum height in this same problem.

Answer 80 m

QUICK QUIZ 3.5 Suppose you are carrying a ball and running at constant speed, and wish to throw the ball and catch it as it comes back down. Should you (a) throw the ball at an angle of about 45° above the horizontal and maintain the same speed, (b) throw the ball straight up in the air and slow down to catch it, or (c) throw the ball straight up in the air and maintain the same speed?

QUICK QUIZ 3.6 As a projectile moves in its parabolic path, the velocity and acceleration vectors are perpendicular to each other (a) everywhere along the projectile's path, (b) at the peak of its path, (c) nowhere along its path, or (d) not enough information is given.

PROBLEM-SOLVING STRATEGY

PROJECTILE MOTION

1. Select a coordinate system and sketch the path of the projectile, including initial and final positions, velocities, and accelerations.
2. Resolve the initial velocity vector into x- and y-components.
3. Treat the horizontal motion and the vertical motion independently.
4. Follow the techniques for solving problems with constant velocity to analyze the horizontal motion of the projectile.
5. Follow the techniques for solving problems with constant acceleration to analyze the vertical motion of the projectile.

EXAMPLE 3.5 Stranded Explorers

Goal Solve a two-dimensional projectile motion problem in which an object has an initial horizontal velocity.

Problem An Alaskan rescue plane drops a package of emergency rations to a stranded hiker, as shown in Figure 3.17. The plane is traveling horizontally at 40.0 m/s at a height of 1.00×10^2 m above the ground. **(a)** Where does the package strike the ground relative to the point at which it was released? **(b)** What are the horizontal and vertical components of the velocity of the package just before it hits the ground?

Strategy Here, we're just taking Equations 3.13 and 3.14, filling in known quantities, and solving for the remaining unknown quantities. Sketch the problem using a coordinate system as in Figure 3.17. In part (a), set the y-component of the displacement equations equal to -1.00×10^2 m—the ground level where the package lands—and solve for the time it takes the package to reach the ground. Substitute this time into the displacement equation for the x-component to find the range. In part (b), substitute the time found in part (a) into the velocity components. Notice that the initial velocity has only an x-component, which simplifies the math.

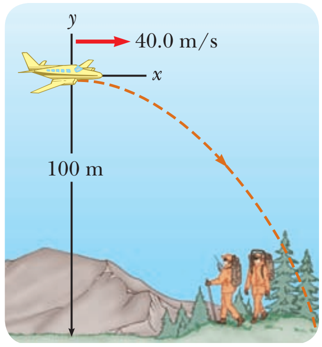


FIGURE 3.17 (Example 3.5) From the point of view of an observer on the ground, a package released from the rescue plane travels along the path shown.

Solution

(a) Find the range of the package.

Use Equation 3.14b to find the y-displacement:

$$\Delta y = y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2$$

Substitute $y_0 = 0$ and $v_{0y} = 0$, set $y = -1.00 \times 10^2$ m—the final vertical position of the package relative the airplane—and solve for time:

$$y = -(4.90\ \mathrm{m/s^2})t^2 = -1.00 \times 10^2\ \mathrm{m}$$

$$t = 4.52\ \mathrm{s}$$

Use Equation 3.13b to find the x-displacement:

$$\Delta x = x - x_0 = v_{0x}t$$

Substitute $x_0 = 0$, $v_{0x} = 40.0$ m/s, and the time:

$$x = (40.0\ \mathrm{m/s})(4.52\ \mathrm{s}) = \boxed{181\ \mathrm{m}}$$

(b) Find the components of the package's velocity at impact:

Find the x-component of the velocity at the time of impact:

$$v_x = v_0 \cos\theta = (40.0\ \mathrm{m/s})\cos 0° = \boxed{40.0\ \mathrm{m/s}}$$

Find the y-component of the velocity at the time of impact:

$$v_y = v_0 \sin\theta - gt = 0 - (9.80\ \mathrm{m/s^2})\ (4.52\ \mathrm{s}) = \boxed{-44.3\ \mathrm{m/s}}$$

Remarks Notice how motion in the *x*-direction and motion in the *y*-direction are handled separately.

QUESTION 3.5

Neglecting air friction effects, what path does the package travel as observed by the pilot? Why?

EXERCISE 3.5

A bartender slides a beer mug at 1.50 m/s toward a customer at the end of a frictionless bar that is 1.20 m tall. The customer makes a grab for the mug and misses, and the mug sails off the end of the bar. (a) How far away from the end of the bar does the mug hit the floor? (b) What are the speed and direction of the mug at impact?

Answers (a) 0.742 m (b) 5.08 m/s, $\theta = -72.8°$

EXAMPLE 3.6 The Long Jump

Goal Solve a two-dimensional projectile motion problem involving an object starting and ending at the same height.

Problem A long jumper (Fig. 3.18) leaves the ground at an angle of 20.0° to the horizontal and at a speed of 11.0 m/s. **(a)** How long does it take for him to reach maximum height? **(b)** What is the maximum height? **(c)** How far does he jump? (Assume his motion is equivalent to that of a particle, disregarding the motion of his arms and legs.) **(d)** Use Equation 3.14c to find the maximum height he reaches.

Strategy Again, we take the projectile equations, fill in the known quantities, and solve for the unknowns. At the maximum height, the velocity in the *y*-direction is zero, so setting Equation 3.14a equal to zero and solving gives the time it takes him to reach his maximum height. By symmetry, given that his trajectory starts and ends at the same height, doubling this time gives the total time of the jump.

Mike Powell/Allsport/Getty Images

FIGURE 3.18 (Example 3.6) Mike Powell, current holder of the world long-jump record of 8.95 m.

Solution

(a) Find the time t_{max} taken to reach maximum height.

Set $v_y = 0$ in Equation 3.14b and solve for t_{max}:

$$v_y = v_0 \sin\theta_0 - g t_{max} = 0$$

$$t_{max} = \frac{v_0 \sin\theta_0}{g} = \frac{(11.0 \text{ m/s})(\sin 20.0°)}{9.80 \text{ m/s}^2}$$

$$= 0.384 \text{ s}$$

(b) Find the maximum height he reaches.

Substitute the time t_{max} into the equation for the *y*-displacement:

$$y_{max} = (v_0 \sin\theta_0) t_{max} - \tfrac{1}{2} g (t_{max})^2$$

$$y_{max} = (11.0 \text{ m/s})(\sin 20.0°)(0.384 \text{ s}) - \tfrac{1}{2}(9.80 \text{ m/s}^2)(0.384 \text{ s})^2$$

$$y_{max} = 0.722 \text{ m}$$

(c) Find the horizontal distance he jumps.

First find the time for the jump, which is twice t_{max}:

$$t = 2t_{max} = 2(0.384 \text{ s}) = 0.768 \text{ s}$$

Substitute this result into the equation for the *x*-displacement:

$$\Delta x = (v_0 \cos\theta_0) t = (11.0 \text{ m/s})(\cos 20.0°)(0.768 \text{ s})$$

$$= 7.94 \text{ m}$$

(d) Use an alternate method to find the maximum height.

Use Equation 3.14c, solving for Δy:

$$v_y^2 - v_{0y}^2 = -2g\,\Delta y$$

$$\Delta y = \frac{v_y^2 - v_{0y}^2}{-2g}$$

Substitute $v_y = 0$ at maximum height, and the fact that $v_{0y} = (11.0 \text{ m/s}) \sin 20.0°$:

$$\Delta y = \frac{0 - [(11.0 \text{ m/s}) \sin 20.0°]^2}{-2(9.80 \text{ m/s}^2)} = 0.722 \text{ m}$$

Remarks Although modeling the long jumper's motion as that of a projectile is an oversimplification, the values obtained are reasonable.

QUESTION 3.6

True or False: Because the x-component of the displacement doesn't depend explicitly on g, the horizontal distance traveled doesn't depend on the acceleration of gravity.

EXERCISE 3.6

A grasshopper jumps a horizontal distance of 1.00 m from rest, with an initial velocity at a 45.0° angle with respect to the horizontal. Find (a) the initial speed of the grasshopper and (b) the maximum height reached.

Answers (a) 3.13 m/s (b) 0.250 m

EXAMPLE 3.7 The Range Equation

Goal Find an equation for the maximum horizontal displacement of a projectile fired from ground level.

Problem An athlete participates in a long-jump competition, leaping into the air with a velocity v_0 at an angle θ_0 with the horizontal. Obtain an expression for the length of the jump in terms of v_0, θ_0, and g.

Strategy Use the results of Example 3.6, eliminating the time t from Equations (1) and (2).

Solution

Use Equation (1) of Example 3.6 to find the time of flight, t:

$$t = 2t_{\text{max}} = \frac{2v_0 \sin \theta_0}{g}$$

Substitute this expression for t into Equation (2) of Example 3.6:

$$\Delta x = (v_0 \cos \theta_0)t = (v_0 \cos \theta_0)\left(\frac{2v_0 \sin \theta_0}{g}\right)$$

Simplify:

$$\Delta x = \frac{2{v_0}^2 \cos \theta_0 \sin \theta_0}{g}$$

Substitute the identity $2 \cos \theta_0 \sin \theta_0 = \sin 2\theta_0$ to reduce the foregoing expression to a single trigonometric function:

$$(1) \quad \Delta x = \frac{{v_0}^2 \sin 2\theta_0}{g}$$

Remarks The use of a trigonometric identity in the final step isn't necessary, but it makes Question 3.7 easier to answer.

QUESTION 3.7

What angle θ_0 produces the longest jump?

EXERCISE 3.7

Obtain an expression for the athlete's maximum displacement in the vertical direction, Δy_{max} in terms of v_0, θ_0, and g.

Answer $\Delta y_{\text{max}} = \dfrac{{v_0}^2 \sin^2 \theta_0}{2g}$

EXAMPLE 3.8 That's Quite an Arm

Goal Solve a two-dimensional kinematics problem with a nonhorizontal initial velocity, starting and ending at different heights.

Problem A stone is thrown upward from the top of a building at an angle of 30.0° to the horizontal and with an initial speed of 20.0 m/s, as in Figure 3.19. The point of release is 45.0 m above the ground. **(a)** How long does it take for the stone to hit the ground? **(b)** Find the stone's speed at impact. **(c)** Find the horizontal range of the stone. Neglect air resistance.

Strategy Choose coordinates as in the figure, with the origin at the point of release. **(a)** Fill in the constants of Equation 3.14b for the y-displacement and set the displacement equal to −45.0 m, the y-displacement when the stone hits the ground. Using the quadratic formula, solve for the time. To solve part **(b)**, substitute the time from part (a) into the components of the velocity, and substitute the same time into the equation for the x-displacement to solve part **(c)**.

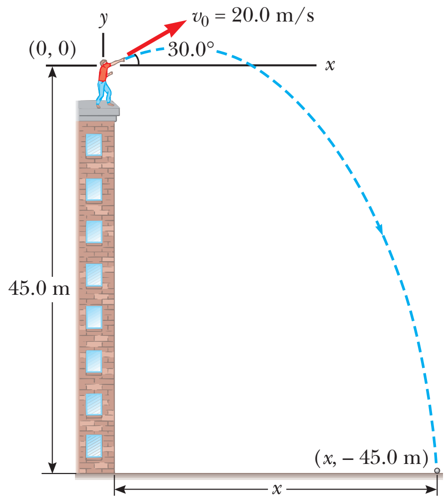


FIGURE 3.19 (Example 3.8)

Solution

(a) Find the time of flight.

Find the initial x- and y-components of the velocity:

$$v_{0x} = v_0 \cos\theta_0 = (20.0\text{ m/s})(\cos 30.0°) = +17.3\text{ m/s}$$

$$v_{0y} = v_0 \sin\theta_0 = (20.0\text{ m/s})(\sin 30.0°) = +10.0\text{ m/s}$$

Find the y-displacement, taking $y_0 = 0$, $y = -45.0$ m, and $v_{0y} = 10.0$ m/s:

$$\Delta y = y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2$$

$$-45.0\text{ m} = (10.0\text{ m/s})t - (4.90\text{ m/s}^2)t^2$$

Reorganize the equation into standard form and use the quadratic formula (see Appendix A) to find the positive root:

$$t = \boxed{4.22\text{ s}}$$

(b) Find the speed at impact.

Substitute the value of t found in part (a) into Equation 3.14a to find the y-component of the velocity at impact:

$$v_y = v_{0y} - gt = 10.0\text{ m/s} - (9.80\text{ m/s}^2)(4.22\text{ s})$$

$$= -31.4\text{ m/s}$$

Use this value of v_y, the Pythagorean theorem, and the fact that $v_x = v_{0x} = 17.3$ m/s to find the speed of the stone at impact:

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{(17.3\text{ m/s})^2 + (-31.4\text{ m/s})^2}$$

$$= \boxed{35.9\text{ m/s}}$$

(c) Find the horizontal range of the stone.

Substitute the time of flight into the range equation:

$$\Delta x = x - x_0 = (v_0 \cos\theta)t = (20.0\text{ m/s})(\cos 30.0°)(4.22\text{ s})$$

$$= \boxed{73.1\text{ m}}$$

Remarks The angle at which the ball is thrown affects the velocity vector throughout its subsequent motion, but doesn't affect the speed at a given height. This is a consequence of the conservation of energy, described in Chapter 5.

QUESTION 3.8

True or False: All other things being equal, if the ball is thrown at half the given speed it will travel half as far.

EXERCISE 3.8

Suppose the stone is thrown from the same height as in the example at an angle of 30.0° degrees below the horizontal. If it strikes the ground 57.0 m away, find (a) the time of flight, (b) the initial speed, and (c) the speed and the angle of the velocity vector with respect to the horizontal at impact. (*Hint:* For part (a), use the equation for the *x*-displacement to eliminate v_0t from the equation for the *y*-displacement.)

Answers (a) 1.57 s (b) 41.9 m/s (c) 51.3 m/s, −45.0°

So far we have studied only problems in which an object with an initial velocity follows a trajectory determined by the acceleration of gravity alone. In the more general case, other agents, such as air drag, surface friction, or engines, can cause accelerations. These accelerations, taken together, form a vector quantity with components a_x and a_y. When both components are constant, we can use Equations 3.11 and 3.12 to study the motion, as in the next example.

EXAMPLE 3.9 The Rocket

Goal Solve a problem involving accelerations in two directions.

Problem A jet plane traveling horizontally at 1.00×10^2 m/s drops a rocket from a considerable height. (See Fig. 3.20.) The rocket immediately fires its engines, accelerating at 20.0 m/s² in the *x*-direction while falling under the influence of gravity in the *y*-direction. When the rocket has fallen 1.00 km, find **(a)** its velocity in the *y*-direction, **(b)** its velocity in the *x*-direction, and **(c)** the magnitude and direction of its velocity. Neglect air drag and aerodynamic lift.

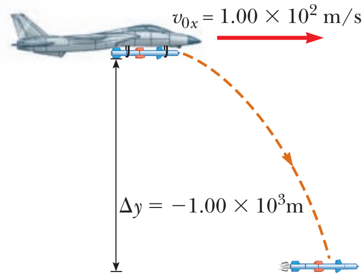


FIGURE 3.20 (Example 3.9)

Strategy Because the rocket maintains a horizontal orientation (say, through gyroscopes), the *x*- and *y*-components of acceleration are independent of each other. Use the time-independent equation for the velocity in the *y*-direction to find the *y*-component of the velocity after the rocket falls 1.00 km. Then calculate the time of the fall and use that time to find the velocity in the *x*-direction.

Solution

(a) Find the velocity in the *y*-direction.

Use Equation 3.14c:

$$v_y^2 = v_{0y}^2 - 2g\,\Delta y$$

Substitute $v_{0y} = 0$, $g = -9.80$ m/s², and $\Delta y = -1.00 \times 10^3$ m, and solve for v_y:

$$v_y^2 - 0 = 2(-9.8 \text{ m/s}^2)(-1.00 \times 10^3 \text{ m})$$

$$v_y = \boxed{-1.40 \times 10^2 \text{ m/s}}$$

(b) Find the velocity in the *x*-direction.

Find the time it takes the rocket to drop 1.00×10^3 m, using the *y*-component of the velocity:

$$v_y = v_{0y} + a_yt$$

$$-1.40 \times 10^2 \text{ m/s} = 0 - (9.80 \text{ m/s}^2)t \quad \rightarrow \quad t = 14.3 \text{ s}$$

Substitute t, v_{0x}, and a_x into Equation 3.11a to find the velocity in the *x*-direction:

$$v_x = v_{0x} + a_xt = 1.00 \times 10^2 \text{ m/s} + (20.0 \text{ m/s}^2)(14.3 \text{ s})$$

$$= \boxed{386 \text{ m/s}}$$

(c) Find the magnitude and direction of the velocity.

Find the magnitude using the Pythagorean theorem and the results of parts (a) and (b):

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-1.40 \times 10^2 \text{ m/s})^2 + (386 \text{ m/s})^2}$$

$$= \boxed{411 \text{ m/s}}$$

Use the inverse tangent function to find the angle: $$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-1.40 \times 10^2 \text{ m/s}}{386 \text{ m/s}}\right) = -19.9°$$

Remarks Notice the symmetry: The kinematic equations for the *x*- and *y*-directions are handled in exactly the same way. Having a nonzero acceleration in the *x*-direction doesn't greatly increase the difficulty of the problem.

QUESTION 3.9

True or False: Neglecting air friction, a projectile with a horizontal acceleration stays in the air longer than a projectile that is freely falling.

EXERCISE 3.9

Suppose a rocket-propelled motorcycle is fired from rest horizontally across a canyon 1.00 km wide. (a) What minimum constant acceleration in the *x*-direction must be provided by the engines so the cycle crosses safely if the opposite side is 0.750 km lower than the starting point? (b) At what speed does the motorcycle land if it maintains this constant horizontal component of acceleration? Neglect air drag, but remember that gravity is still acting in the negative *y*-direction.

Answers (a) 13.1 m/s^2 (b) 202 m/s

In a stunt similar to that described in Exercise 3.9, motorcycle daredevil Evel Knievel tried to vault across Hells Canyon, part of the Snake River system in Idaho, on his rocket-powered Harley-Davidson X-2 "Skycycle." (See the chapter-opening photo on page 54). He lost consciousness at takeoff and released a lever, prematurely deploying his parachute and falling short of the other side. He landed safely in the canyon.

3.5 RELATIVE VELOCITY

Relative velocity is all about relating the measurements of two different observers, one moving with respect to the other. The measured velocity of an object depends on the velocity of the observer with respect to the object. On highways, for example, cars moving in the same direction are often moving at high speed relative to Earth, but relative to each other they hardly move at all. To an observer at rest at the side of the road, a car might be traveling at 60 mi/h, but to an observer in a truck traveling in the same direction at 50 mi/h, the car would appear to be traveling only 10 mi/h.

So measurements of velocity depend on the **reference frame** of the observer. Reference frames are just coordinate systems. Most of the time, we use a **stationary frame of reference** relative to Earth, but occasionally we use a **moving frame of reference** associated with a bus, car, or plane moving with constant velocity relative to Earth.

In two dimensions relative velocity calculations can be confusing, so a systematic approach is important and useful. Let E be an observer, assumed stationary with respect to Earth. Let two cars be labeled A and B, and introduce the following notation (see Fig. 3.21):

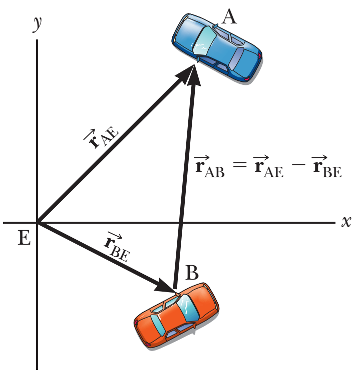


FIGURE 3.21 The position of Car A relative to Car B can be found by vector subtraction. The rate of change of the resultant vector with respect to time is the relative velocity equation.

$\vec{\mathbf{r}}_{AE}$ = the position of Car A as measured by E (in a coordinate system fixed with respect to Earth).

$\vec{\mathbf{r}}_{BE}$ = the position of Car B as measured by E.

$\vec{\mathbf{r}}_{AB}$ = the position of Car A as measured by an observer in Car B.

According to the preceding notation, the first letter tells us what the vector is pointing at and the second letter tells us where the position vector starts. The position vectors of Car A and Car B relative to E, $\vec{\mathbf{r}}_{AE}$ and $\vec{\mathbf{r}}_{BE}$, are given in the figure. How do we find $\vec{\mathbf{r}}_{AB}$, the position of Car A as measured by an observer in Car B? We simply draw an arrow pointing from Car B to Car A, which can be obtained by subtracting $\vec{\mathbf{r}}_{BE}$ from $\vec{\mathbf{r}}_{AE}$:

$$\vec{\mathbf{r}}_{AB} = \vec{\mathbf{r}}_{AE} - \vec{\mathbf{r}}_{BE} \quad [3.15]$$

Now, the rate of change of these quantities with time gives us the relationship between the associated velocities:

$$\vec{\mathbf{v}}_{AB} = \vec{\mathbf{v}}_{AE} - \vec{\mathbf{v}}_{BE} \quad [3.16]$$

The coordinate system of observer E need not be fixed to Earth, although it often is. Take careful note of the pattern of subscripts; rather than memorize Equation 3.16, it's better to study the short derivation based on Figure 3.21. Note also that the equation doesn't work for observers traveling a sizable fraction of the speed of light, when Einstein's theory of special relativity comes into play.

PROBLEM-SOLVING STRATEGY

RELATIVE VELOCITY

1. Label each object involved (usually three) with a letter that reminds you of what it is (for example, E for Earth).
2. Look through the problem for phrases such as "The velocity of A relative to B" and write the velocities as $\vec{\mathbf{v}}_{AB}$. When a velocity is mentioned but it isn't explicitly stated as relative to something, it's almost always relative to Earth.
3. Take the three velocities you've found and assemble them into an equation just like Equation 3.16, with subscripts in an analogous order.
4. There will be two unknown components. Solve for them with the x- and y-components of the equation developed in step 3.

EXAMPLE 3.10 Pitching Practice on the Train

Goal Solve a one-dimensional relative velocity problem.

Problem A train is traveling with a speed of 15.0 m/s relative to Earth. A passenger standing at the rear of the train pitches a baseball with a speed of 15.0 m/s relative to the train off the back end, in the direction opposite the motion of the train. What is the velocity of the baseball relative to Earth?

Strategy Solving these problems involves putting the proper subscripts on the velocities and arranging them as in Equation 3.16. In the first sentence of the problem statement, we are informed that the train travels at "15.0 m/s relative to Earth." This quantity is $\vec{\mathbf{v}}_{TE}$, with T for train and E for Earth. The passenger throws the baseball at "15 m/s relative to the train," so this quantity is $\vec{\mathbf{v}}_{BT}$, where B stands for baseball. The second sentence asks for the velocity of the baseball relative to Earth, $\vec{\mathbf{v}}_{BE}$. The rest of the problem can be solved by identifying the correct components of the known quantities and solving for the unknowns, using an analog of Equation 3.16.

Solution

Write the x-components of the known quantities:

$$(\vec{\mathbf{v}}_{TE})_x = +15 \text{ m/s}$$

$$(\vec{\mathbf{v}}_{BT})_x = -15 \text{ m/s}$$

Follow Equation 3.16:

$$(\vec{\mathbf{v}}_{BT})_x = (\vec{\mathbf{v}}_{BE})_x - (\vec{\mathbf{v}}_{TE})_x$$

Insert the given values and solve:

$$-15 \text{ m/s} = (\vec{\mathbf{v}}_{BE})_x - 15 \text{ m/s}$$

$$(\vec{\mathbf{v}}_{BE})_x = 0$$

QUESTION 3.10

Describe the motion of the ball as related by an observer on the ground.

EXERCISE 3.10

A train is traveling at 27 m/s relative to Earth, and a passenger standing in the train throws a ball at 15 m/s relative to the train in the same direction as the train's motion. Find the speed of the ball relative to Earth.

Answer 42 m/s

EXAMPLE 3.11 Crossing a River

Goal Solve a simple two-dimensional relative motion problem.

Problem The boat in Figure 3.22 is heading due north as it crosses a wide river with a velocity of 10.0 km/h relative to the water. The river has a uniform velocity of 5.00 km/h due east. Determine the velocity of the boat with respect to an observer on the riverbank.

Strategy Again, we look for key phrases. "The boat (has) . . . a velocity of 10.0 km/h relative to the water" gives $\vec{\mathbf{v}}_{BR}$. "The river has a uniform velocity of 5.00 km/h due east" gives $\vec{\mathbf{v}}_{RE}$, because this implies velocity with respect to Earth. The observer on the riverbank is in a reference frame at rest with respect to Earth. Because we're looking for the velocity of the boat with respect to that observer, this last velocity is designated $\vec{\mathbf{v}}_{BE}$. Take east to be the $+x$-direction, north the $+y$-direction.

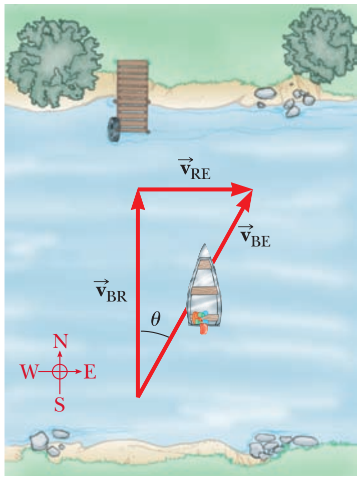


FIGURE 3.22 (Example 3.10)

Solution

Arrange the three quantities into the proper relative velocity equation:

$$\vec{\mathbf{v}}_{BR} = \vec{\mathbf{v}}_{BE} - \vec{\mathbf{v}}_{RE}$$

Write the velocity vectors in terms of their components. For convenience, these are organized in the following table:

Vector	x-Component (km/h)	y-Component (km/h)
$\vec{\mathbf{v}}_{BR}$	0	10.0
$\vec{\mathbf{v}}_{BE}$	v_x	v_y
$\vec{\mathbf{v}}_{RE}$	5.00	0

Find the x-component of velocity:

$$0 = v_x - 5.00 \text{ km/h} \quad \rightarrow \quad v_x = 5.00 \text{ km/h}$$

Find the y-component of velocity:

$$10.0 \text{ km/h} = v_y - 0 \quad \rightarrow \quad v_y = 10.0 \text{ km/h}$$

Find the magnitude of $\vec{\mathbf{v}}_{BE}$:

$$v_{BE} = \sqrt{v_x^{\,2} + v_y^{\,2}}$$

$$= \sqrt{(5.00 \text{ km/h})^2 + (10.0 \text{ km/h})^2} = 11.2 \text{ km/h}$$

Find the direction of $\vec{\mathbf{v}}_{BE}$:

$$\theta = \tan^{-1}\left(\frac{v_x}{v_y}\right) = \tan^{-1}\left(\frac{5.00 \text{ m/s}}{10.0 \text{ m/s}}\right) = 26.6°$$

Remark The boat travels at a speed of 11.2 km/h in the direction 26.6° east of north with respect to Earth.

QUESTION 3.11

If the speed of the boat relative to the water is increased, what happens to the angle?

EXERCISE 3.11

Suppose the river is flowing east at 3.00 m/s and the boat is traveling south at 4.00 m/s with respect to the river. Find the speed and direction of the boat relative to Earth.

Answer 5.00 m/s, 53.1° south of east

EXAMPLE 3.12 Bucking the Current

Goal Solve a complex two-dimensional relative motion problem.

Problem If the skipper of the boat of Example 3.11 moves with the same speed of 10.0 km/h relative to the water but now wants to travel due north, as in Figure 3.23, in what direction should he head? What is the speed of the boat, according to an observer on the shore? The river is flowing east at 5.00 km/h.

Strategy Proceed as in the previous example. In this situation, we must find the heading of the boat and its velocity with respect to the water, using the fact that the boat travels due north.

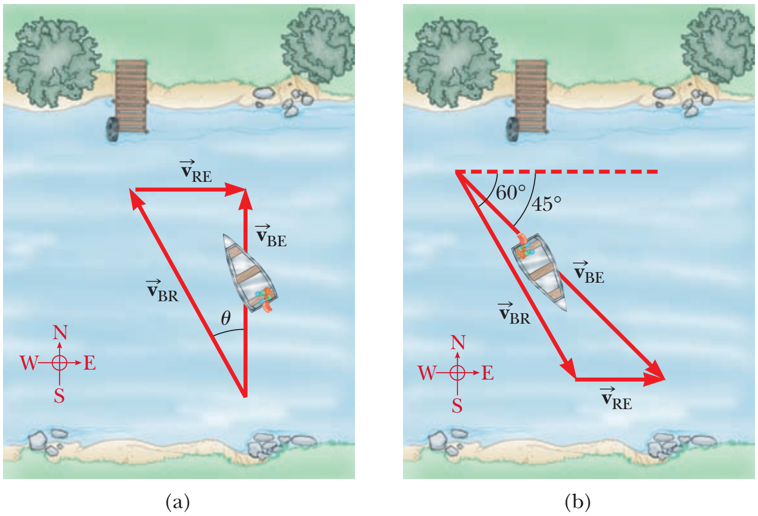


FIGURE 3.23 (a) (Example 3.12) (b) (Exercise 3.12)

Solution

Arrange the three quantities, as before:

$$\vec{v}_{BR} = \vec{v}_{BE} - \vec{v}_{RE}$$

Organize a table of velocity components:

Vector	x-Component (km/h)	y-Component (km/h)
$\vec{v}_{BR}$	$-(10.0 \text{ km/h}) \sin\theta$	$(10.0 \text{ km/h}) \cos\theta$
$\vec{v}_{BE}$	0	v
$\vec{v}_{RE}$	5.00 km/h	0

The *x*-component of the relative velocity equation can be used to find θ:

$$-(10.0 \text{ m/s}) \sin\theta = 0 - 5.00 \text{ km/h}$$

$$\sin\theta = \frac{5.00 \text{ km/h}}{10.0 \text{ km/h}} = \frac{1.00}{2.00}$$

Apply the inverse sine function and find θ, which is the boat's heading, east of north:

$$\theta = \sin^{-1}\left(\frac{1.00}{2.00}\right) = 30.0^\circ$$

The *y*-component of the relative velocity equation can be used to find v:

$$(10.0 \text{ km/h}) \cos\theta = v \quad \rightarrow \quad v = 8.66 \text{ km/h}$$

Remarks From Figure 3.23, we see that this problem can be solved with the Pythagorean theorem, because the problem involves a right triangle: the boat's *x*-component of velocity exactly cancels the river's velocity. When this is not the case, a more general technique is necessary, as shown in the following exercise. Notice that in the *x*-component of the relative velocity equation a minus sign had to be included in the term $-(10.0 \text{ km/h})\sin\theta$ because the *x*-component of the boat's velocity with respect to the river is negative.

QUESTION 3.12

The speeds in this example are the same as in Example 3.11. Why isn't the angle the same as before?

EXERCISE 3.12

Suppose the river is moving east at 5.00 km/h and the boat is traveling 45.0° south of east with respect to Earth. Find (a) the speed of the boat with respect to Earth and (b) the speed of the boat with respect to the river if the boat's heading in the water is 60.0° south of east. (See Fig. 3.23b.) You will have to solve two equations with two unknowns.

Answers (a) 16.7 km/h (b) 13.7 km/h

SUMMARY

3.1 Vectors and Their Properties

Two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ can be added geometrically with the **triangle method.** The two vectors are drawn to scale on graph paper, with the tail of the second vector located at the tip of the first. The **resultant** vector is the vector drawn from the tail of the first vector to the tip of the second.

The negative of a vector $\vec{\mathbf{A}}$ is a vector with the same magnitude as $\vec{\mathbf{A}}$, but pointing in the opposite direction. A vector can be multiplied by a scalar, changing its magnitude, and its direction if the scalar is negative.

3.2 Components of a Vector

A vector $\vec{\mathbf{A}}$ can be split into two components, one pointing in the x-direction and the other in the y-direction. These components form two sides of a right triangle having a hypotenuse with magnitude A and are given by

$$A_x = A\cos\theta \qquad A_y = A\sin\theta \qquad [3.2]$$

The magnitude and direction of $\vec{\mathbf{A}}$ are related to its components through the Pythagorean theorem and the definition of the tangent:

$$A = \sqrt{A_x^2 + A_y^2} \qquad [3.3]$$

$$\tan\theta = \frac{A_y}{A_x} \qquad [3.4]$$

If $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, then the components of the resultant vector $\vec{\mathbf{R}}$ are

$$R_x = A_x + B_x \qquad [3.5a]$$

$$R_y = A_y + B_y \qquad [3.5b]$$

3.3 Displacement, Velocity, and Acceleration in Two Dimensions

The displacement of an object in two dimensions is defined as the change in the object's position vector:

$$\Delta\vec{\mathbf{r}} \equiv \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i \qquad [3.6]$$

The average velocity of an object during the time interval Δt is

$$\vec{\mathbf{v}}_{\text{av}} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \qquad [3.7]$$

Taking the limit of this expression as Δt gets arbitrarily small gives the instantaneous velocity $\vec{\mathbf{v}}$:

$$\vec{\mathbf{v}} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \qquad [3.8]$$

The direction of the instantaneous velocity vector is along a line that is tangent to the path of the object and in the direction of its motion.

The average acceleration of an object with a velocity changing by $\Delta\vec{\mathbf{v}}$ in the time interval Δt is

$$\vec{\mathbf{a}}_{\text{av}} \equiv \frac{\Delta\vec{\mathbf{v}}}{\Delta t} \qquad [3.9]$$

Taking the limit of this expression as Δt gets arbitrarily small gives the instantaneous acceleration vector $\vec{\mathbf{a}}$:

$$\vec{\mathbf{a}} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t} \qquad [3.10]$$

3.4 Motion in Two Dimensions

The general kinematic equations in two dimensions for objects with constant acceleration are, for the x-direction,

$$v_x = v_{0x} + a_x t \qquad [3.11a]$$

$$\Delta x = v_{0x}t + \tfrac{1}{2}a_x t^2 \qquad [3.11b]$$

$$v_x^2 = v_{0x}^2 + 2a_x\,\Delta x \qquad [3.11c]$$

where $v_{0x} = v_0\cos\theta_0$, and, for the y-direction,

$$v_y = v_{0y} + a_y t \qquad [3.12a]$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}a_y t^2 \qquad [3.12b]$$

$$v_y^2 = v_{0y}^2 + 2a_y\,\Delta y \qquad [3.12c]$$

where $v_{0y} = v_0\sin\theta_0$. The speed v of the object at any instant can be calculated from the components of velocity at that instant using the Pythagorean theorem:

$$v = \sqrt{v_x^2 + v_y^2}$$

The angle that the velocity vector makes with the x-axis is given by

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

The kinematic equations are easily adapted and simplified for projectiles close to the surface of the Earth. The equations for the motion in the horizontal or x-direction are

$$v_x = v_{0x} = v_0\cos\theta_0 = \text{constant} \qquad [3.13a]$$

$$\Delta x = v_{0x}t = (v_0\cos\theta_0)t \qquad [3.13b]$$

while the equations for the motion in the vertical or y-direction are

$$v_y = v_0\sin\theta_0 - gt \qquad [3.14a]$$

$$\Delta y = (v_0\sin\theta_0)t - \tfrac{1}{2}gt^2 \qquad [3.14b]$$

$$v_y^2 = (v_0\sin\theta_0)^2 - 2g\,\Delta y \qquad [3.14c]$$

Problems are solved by algebraically manipulating one or more of these equations, which often reduces the system to two equations and two unknowns.

3.5 Relative Velocity

Let E be an observer, and B a second observer traveling with velocity $\vec{\mathbf{v}}_{\text{BE}}$ as measured by E. If E measures the velocity of an object A as $\vec{\mathbf{v}}_{\text{AE}}$, then B will measure A's velocity as

$$\vec{\mathbf{v}}_{\text{AB}} = \vec{\mathbf{v}}_{\text{AE}} - \vec{\mathbf{v}}_{\text{BE}} \qquad [3.16]$$

Solving relative velocity problems involves identifying the velocities properly and labeling them correctly, substituting into Equation 3.16, and then solving for unknown quantities.

MULTIPLE-CHOICE QUESTIONS

1. A catapult launches a large stone at a speed of 45.0 m/s at an angle of 55.0° with the horizontal. What maximum height does the stone reach? (Neglect air friction.) (a) 45.7 m (b) 32.7 m (c) 69.3 m (d) 83.2 m (e) 102 m

2. A skier leaves the end of a horizontal ski jump at 22.0 m/s and falls 3.20 m before landing. Neglecting friction, how far horizontally does the skier travel in the air before landing? (a) 9.8 m (b) 12.2 m (c) 14.3 m (d) 17.8 m (e) 21.6 m

3. A cruise ship sails due north at 4.50 m/s while a coast guard patrol boat heads 45.0° north of west at 5.20 m/s. What is the velocity of the cruise ship relative to the patrol boat? (a) v_x = 3.68 m/s; v_y = 0.823 m/s (b) v_x = −3.68 m/s; v_y = 8.18 m/s (c) v_x = 3.68 m/s; v_y = 8.18 m/s (d) v_x = −3.68 m/s; v_y = −0.823 m/s (e) v_x = 3.68 m/s; v_y = 1.82 m/s

4. A vector lying in the *xy*-plane has components of opposite sign. The vector must lie in which quadrant? (a) the first quadrant (b) the second quadrant (c) the third quadrant (d) the fourth quadrant (e) either the second or the fourth quadrant

5. An athlete runs three-fourths of the way around a circular track. Which of the following statements is true? (a) His average speed is greater than the magnitude of his average velocity. (b) The magnitude of his average velocity is greater than his average speed. (c) His average speed is equal to the magnitude of his average velocity. (d) His average speed is the same as the magnitude of his average velocity if his instantaneous speed is constant. (e) None of statements (a) through (d) is true.

6. A car moving around a circular track with constant speed (a) has zero acceleration, (b) has an acceleration component in the direction of its velocity, (c) has an acceleration directed away from the center of its path, (d) has an acceleration directed toward the center of its path, or (e) has an acceleration with a direction that cannot be determined from the information given.

7. A NASA astronaut hits a golf ball on the Moon. Which of the following quantities, if any, remain constant as the ball travels through the lunar vacuum? (a) speed (b) acceleration (c) velocity (d) horizontal component of velocity (e) vertical component of velocity

8. A projectile is launched from Earth's surface at a certain initial velocity at an angle above the horizontal, reaching maximum height after time t_{max}. Another projectile is launched with the same initial velocity and angle from the surface of the Moon, where the acceleration of gravity is one-sixth that of Earth. Neglecting air resistance (on Earth) and variations in the acceleration of gravity with height, how long does it take the projectile on the Moon to reach its maximum height? (a) t_{max} (b) $t_{max}/6$ (c) $\sqrt{6}t_{max}$ (d) $36t_{max}$ (e) $6t_{max}$

9. A sailor drops a wrench from the top of a sailboat's vertical mast while the boat is moving rapidly and steadily straight forward. Where will the wrench hit the deck? (a) ahead of the base of the mast (b) at the base of the mast (c) behind the base of the mast (d) on the windward side of the base of the mast (e) None of choices (a) through (d) is correct.

10. A baseball is thrown from the outfield toward the catcher. When the ball reaches its highest point, which statement is true? (a) Its velocity and its acceleration are both zero. (b) Its velocity is not zero, but its acceleration is zero. (c) Its velocity is perpendicular to its acceleration. (d) Its acceleration depends on the angle at which the ball was thrown. (e) None of statements (a) through (d) is true.

11. A student throws a heavy red ball horizontally from a balcony of a tall building with an initial speed v_0. At the same time, a second student drops a lighter blue ball from the same balcony. Neglecting air resistance, which statement is true? (a) The blue ball reaches the ground first. (b) The balls reach the ground at the same instant. (c) The red ball reaches the ground first. (d) Both balls hit the ground with the same speed. (e) None of statements (a) through (d) is true.

12. As an apple tree is transported by a truck moving to the right with a constant velocity, one of its apples shakes loose and falls toward the bed of the truck. Of the curves shown in Figure MCQ3.12, (i) which best describes the path followed by the apple as seen by a stationary observer on the ground, who observes the truck moving from his left to his right? (ii) Which best describes the path as seen by an observer sitting in the truck?

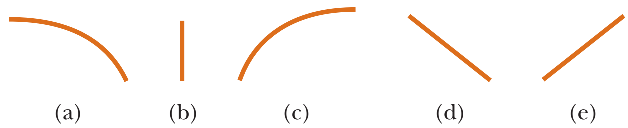


FIGURE MCQ3.12

13. Which of the following quantities are vectors? (a) the velocity of a sports car (b) temperature (c) the volume of water in a can (d) the displacement of a tennis player from the backline of the court to the net (e) the height of a building

CONCEPTUAL QUESTIONS

1. If $\vec{\mathbf{B}}$ is added to $\vec{\mathbf{A}}$, under what conditions does the resultant vector have a magnitude equal to $A + B$? Under what conditions is the resultant vector equal to zero?

2. Under what circumstances would a vector have components that are equal in magnitude?

3. As a projectile moves in its path, is there any point along the path where the velocity and acceleration vectors are (a) perpendicular to each other? (b) Parallel to each other?

4. Two vectors have unequal magnitudes. Can their sum be zero? Explain.

5. Explain whether the following particles do or do not have an acceleration: (a) a particle moving in a straight line with constant speed and (b) a particle moving around a curve with constant speed.

6. A ball is projected horizontally from the top of a building. One second later, another ball is projected horizontally from the same point with the same velocity. At what point in the motion will the balls be closest to each other? Will the first ball always be traveling faster than the second? What will be the time difference between them when the balls hit the ground? Can the horizontal projection velocity of the second ball be changed so that the balls arrive at the ground at the same time?

7. A spacecraft drifts through space at a constant velocity. Suddenly, a gas leak in the side of the spacecraft causes it to constantly accelerate in a direction perpendicular to the initial velocity. The orientation of the spacecraft does not change, so the acceleration remains perpendicular to the original direction of the velocity. What is the shape of the path followed by the spacecraft?

8. Determine which of the following moving objects obey the equations of projectile motion developed in this chapter. (a) A ball is thrown in an arbitrary direction. (b) A jet airplane crosses the sky with its engines thrusting the plane forward. (c) A rocket leaves the launch pad. (d) A rocket moves through the sky after its engines have failed. (e) A stone is thrown under water.

9. Two projectiles are thrown with the same initial speed, one at an angle θ with respect to the level ground and the other at angle $90° - \theta$. Both projectiles strike the ground at the same distance from the projection point. Are both projectiles in the air for the same length of time?

10. A ball is thrown upward in the air by a passenger on a train that is moving with constant velocity. (a) Describe the path of the ball as seen by the passenger. Describe the path as seen by a stationary observer outside the train. (b) How would these observations change if the train were accelerating along the track?

PROBLEMS

ENHANCED WebAssign The Problems for this chapter may be assigned online at WebAssign.

1, 2, 3 = straightforward, intermediate, challenging

GP = denotes guided problem

ecp = denotes enhanced content problem

= biomedical application

☐ = denotes full solution available in *Student Solutions Manual/Study Guide*

SECTION 3.1 VECTORS AND THEIR PROPERTIES

1. Vector $\vec{\mathbf{A}}$ has a magnitude of 29 units and points in the positive y-direction. When vector $\vec{\mathbf{B}}$ is added to $\vec{\mathbf{A}}$, the resultant vector $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ points in the negative y-direction with a magnitude of 14 units. Find the magnitude and direction of $\vec{\mathbf{B}}$.

2. Vector $\vec{\mathbf{A}}$ has a magnitude of 8.00 units and makes an angle of 45.0° with the positive x-axis. Vector $\vec{\mathbf{B}}$ also has a magnitude of 8.00 units and is directed along the negative x-axis. Using graphical methods, find (a) the vector sum $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ and (b) the vector difference $\vec{\mathbf{A}} - \vec{\mathbf{B}}$.

3. Vector $\vec{\mathbf{A}}$ is 3.00 units in length and points along the positive x-axis. Vector $\vec{\mathbf{B}}$ is 4.00 units in length and points along the negative y-axis. Use graphical methods to find the magnitude and direction of the vectors (a) $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ and (b) $\vec{\mathbf{A}} - \vec{\mathbf{B}}$.

4. Each of the displacement vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ shown in Figure P3.4 has a magnitude of 3.00 m. Graphically find (a) $\vec{\mathbf{A}} + \vec{\mathbf{B}}$, (b) $\vec{\mathbf{A}} - \vec{\mathbf{B}}$, (c) $\vec{\mathbf{B}} - \vec{\mathbf{A}}$, and (d) $\vec{\mathbf{A}} - 2\vec{\mathbf{B}}$.

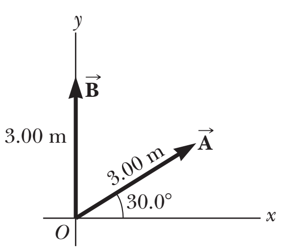


FIGURE P3.4

5. A roller coaster moves 200 ft horizontally and then rises 135 ft at an angle of 30.0° above the horizontal. Next, it travels 135 ft at an angle of 40.0° below the horizontal. Use graphical techniques to find the roller coaster's displacement from its starting point to the end of this movement.

6. ecp An airplane flies 200 km due west from city A to city B and then 300 km in the direction of 30.0° north of west from city B to city C. (a) In straight-line distance, how far is city C from city A? (b) Relative to city A, in what direction is city C? (c) Why is the answer only approximately correct?

7. A plane flies from base camp to lake A, a distance of 280 km at a direction of 20.0° north of east. After dropping off supplies, the plane flies to lake B, which is 190 km and 30.0° west of north from lake A. Graphically determine the distance and direction from lake B to the base camp.

8. A jogger runs 100 m due west, then changes direction for the second leg of the run. At the end of the run, she is 175 m away from the starting point at an angle of 15.0° north of west. What were the direction and length of her second displacement? Use graphical techniques.

9. A man lost in a maze makes three consecutive displacements so that at the end of his travel he is right back where he started. The first displacement is 8.00 m westward, and the second is 13.0 m northward. Use the graphical method to find the magnitude and direction of the third displacement.

SECTION 3.2 COMPONENTS OF A VECTOR

10. The magnitude of vector $\vec{\mathbf{A}}$ is 35.0 units and points in the direction 325° counterclockwise from the positive x-axis. Calculate the x- and y-components of this vector.

11. A golfer takes two putts to get his ball into the hole once he is on the green. The first putt displaces the ball 6.00 m east, the second 5.40 m south. What displacement would have been needed to get the ball into the hole on the first putt?

12. A figure skater glides along a circular path of radius 5.00 m. If she coasts around one half of the circle, find (a) the magnitude of the displacement vector and (b) what distance she skated. (c) What is the magnitude of the displacement if she skates all the way around the circle?

13. A girl delivering newspapers covers her route by traveling 3.00 blocks west, 4.00 blocks north, and then 6.00 blocks east. (a) What is her resultant displacement? (b) What is the total distance she travels?

14. ecp A hiker starts at his camp and moves the following distances while exploring his surroundings: 75.0 m north, 2.50×10^2 m east, 125 m at an angle 30.0° north of east, and 1.50×10^2 m south. (a) Find his resultant displacement from camp. (Take east as the positive *x*-direction and north as the positive *y*-direction.) (b) Would changes in the order in which the hiker makes the given displacements alter his final position? Explain.

15. A vector has an *x*-component of −25.0 units and a *y*-component of 40.0 units. Find the magnitude and direction of the vector.

16. A quarterback takes the ball from the line of scrimmage, runs backwards for 10.0 yards, then runs sideways parallel to the line of scrimmage for 15.0 yards. At this point, he throws a 50.0-yard forward pass straight downfield, perpendicular to the line of scrimmage. What is the magnitude of the football's resultant displacement?

17. The eye of a hurricane passes over Grand Bahama Island in a direction 60.0° north of west with a speed of 41.0 km/h. Three hours later the course of the hurricane suddenly shifts due north, and its speed slows to 25.0 km/h. How far from Grand Bahama is the hurricane 4.50 h after it passes over the island?

18. A small map shows Atlanta to be 730 miles in a direction 5° north of east from Dallas. The same map shows that Chicago is 560 miles in a direction 21° west of north from Atlanta. Assume a flat Earth and use the given information to find the displacement from Dallas to Chicago.

19. A commuter airplane starts from an airport and takes the route shown in Figure P3.19. The plane first flies to city *A*, located 175 km away in a direction 30.0° north of east. Next, it flies for 150 km 20.0° west of north, to city *B*. Finally, the plane flies 190 km due west, to city *C*. Find the location of city *C* relative to the location of the starting point.

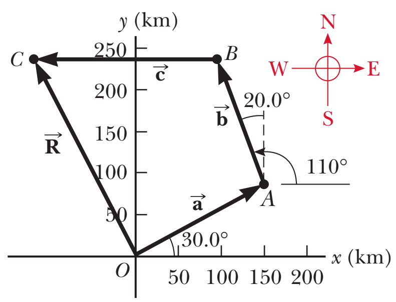


FIGURE P3.19

20. The helicopter view in Figure P3.20 shows two people pulling on a stubborn mule. Find (a) the single force that is equivalent to the two forces shown and (b) the force a third person would have to exert on the mule to make the net force equal to zero. The forces are measured in units of newtons (N).

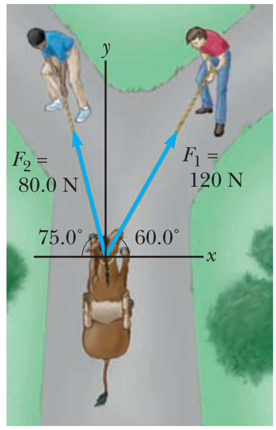


FIGURE P3.20

21. A novice golfer on the green takes three strokes to sink the ball. The successive displacements of the ball are 4.00 m to the north, 2.00 m 45.0° north of east, and 1.00 m at 30.0° west of south. Starting at the same initial point, an expert golfer could make the hole in what single displacement?

SECTION 3.3 DISPLACEMENT, VELOCITY, AND ACCELERATION IN TWO DIMENSIONS

SECTION 3.4 MOTION IN TWO DIMENSIONS

22. One of the fastest recorded pitches in major-league baseball, thrown by Joel Zumaya in 2006, was clocked at 101.0 mi/h (Fig. P3.22). If a pitch were thrown horizontally with this velocity, how far would the ball fall vertically by the time it reached home plate, 60.5 ft away?

Mark Cunningham/MLB Photos via Getty Images

FIGURE P3.22 Joel Zumaya throws a baseball.

23. GP A student stands at the edge of a cliff and throws a stone horizontally over the edge with a speed of 18.0 m/s. The cliff is 50.0 m above a flat, horizontal beach as shown in Figure P3.23 (page 78). (a) What are the coordinates of the initial position of the stone? (b) What are the components of the initial velocity? (c) Write the equations for the *x*- and *y*-components of the velocity of the stone with time. (d) Write the equations for the position of the stone with time, using the coordinates in Figure P3.23. (e) How long after being released does the stone strike the beach below the cliff? (f) With what speed and angle of impact does the stone land?

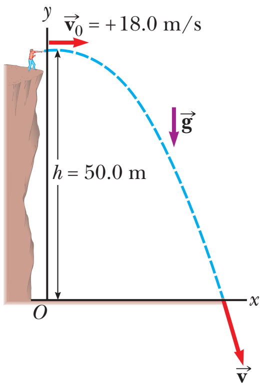


FIGURE P3.23

24. A peregrine falcon (Fig. P3.24) is the fastest bird, flying at a speed of 200 mi/h. Nature has adapted the bird to reach such a speed by placing baffles in its nose to prevent air from rushing in and slowing it down. Also, the bird's eyes adjust their focus faster than the eyes of any other creature, so the falcon can focus quickly on its prey. Assume a peregrine falcon is moving horizontally at its top speed at a height of 100 m above the ground when it brings its wings into its sides and begins to drop in free fall. How far will the bird fall vertically while traveling horizontally a distance of 100 m?

FIGURE P3.24 Notice the structure within the peregrine falcon's nostrils.

25. The best leaper in the animal kingdom is the puma, which can jump to a height of 12 ft when leaving the ground at an angle of 45°. With what speed, in SI units, must the animal leave the ground to reach that height?

26. ecp The record distance in the sport of throwing cowpats is 81.1 m. This record toss was set by Steve Urner of the United States in 1981. Assuming the initial launch angle was 45° and neglecting air resistance, determine (a) the initial speed of the projectile and (b) the total time the projectile was in flight. (c) Qualitatively, how would the answers change if the launch angle were greater than 45°? Explain.

27. A tennis player standing 12.6 m from the net hits the ball at 3.00° above the horizontal. To clear the net, the ball must rise at least 0.330 m. If the ball just clears the net at the apex of its trajectory, how fast was the ball moving when it left the racket?

28. GP From the window of a building, a ball is tossed from a height y_0 above the ground with an initial velocity of 8.00 m/s and angle of 20.0° below the horizontal. It strikes the ground 3.00 s later. (a) If the base of the building is taken to be the origin of the coordinates, with upward the positive y-direction, what are the initial coordinates of the ball? (b) With the positive x-direction chosen to be out the window, find the x- and y-components of the initial velocity. (c) Find the equations for the x- and y-components of the position as functions of time. (d) How far horizontally from the base of the building does the ball strike the ground? (e) Find the height from which the ball was thrown. (f) How long does it take the ball to reach a point 10.0 m below the level of launching?

29. A brick is thrown upward from the top of a building at an angle of 25° to the horizontal and with an initial speed of 15 m/s. If the brick is in flight for 3.0 s, how tall is the building?

30. An artillery shell is fired with an initial velocity of 300 m/s at 55.0° above the horizontal. To clear an avalanche, it explodes on a mountainside 42.0 s after firing. What are the x- and y-coordinates of the shell where it explodes, relative to its firing point?

31. A car is parked on a cliff overlooking the ocean on an incline that makes an angle of 24.0° below the horizontal. The negligent driver leaves the car in neutral, and the emergency brakes are defective. The car rolls from rest down the incline with a constant acceleration of 4.00 m/s^2 for a distance of 50.0 m to the edge of the cliff, which is 30.0 m above the ocean. Find (a) the car's position relative to the base of the cliff when the car lands in the ocean and (b) the length of time the car is in the air.

32. A fireman 50.0 m away from a burning building directs a stream of water from a ground-level fire hose at an angle of 30.0° above the horizontal. If the speed of the stream as it leaves the hose is 40.0 m/s, at what height will the stream of water strike the building?

33. A projectile is launched with an initial speed of 60.0 m/s at an angle of 30.0° above the horizontal. The projectile lands on a hillside 4.00 s later. Neglect air friction. (a) What is the projectile's velocity at the highest point of its trajectory? (b) What is the straight-line distance from where the projectile was launched to where it hits its target?

34. A soccer player kicks a rock horizontally off a 40.0-m-high cliff into a pool of water. If the player hears the sound of the splash 3.00 s later, what was the initial speed given to the rock? Assume the speed of sound in air to be 343 m/s.

SECTION 3.5 RELATIVE VELOCITY

35. GP A jet airliner moving initially at 3.00×10^2 mi/h due east enters a region where the wind is blowing 1.00×10^2 mi/h in a direction 30.0° north of east. (a) Find the components of the velocity of the jet airliner relative to the air, $\vec{v}_{JA}$. (b) Find the components of the velocity of the air relative to Earth, $\vec{v}_{AE}$. (c) Write an equation analogous to Equation 3.16 for the velocities $\vec{\mathbf{v}}_{JA}$, $\vec{\mathbf{v}}_{AE}$, and $\vec{\mathbf{v}}_{JE}$. (d) What is the speed and direction of the aircraft relative to the ground?

36. A boat moves through the water of a river at 10 m/s relative to the water, regardless of the boat's direction. If the water in the river is flowing at 1.5 m/s, how long does it take the boat to make a round trip consisting of a 300-m displacement downstream followed by a 300-m displacement upstream?

37. A chinook (king) salmon (genus *Oncorhynchus*) can jump out of water with a speed of 6.26 m/s. (See Problem 4.9, page 111 for an investigation of how the fish can leave the water at a higher speed than it can swim underwater.) If the salmon is in a stream with water speed equal to 1.50 m/s, how high in the air can the fish jump if it leaves the water traveling vertically upwards relative to the Earth?

38. A river flows due east at 1.50 m/s. A boat crosses the river from the south shore to the north shore by maintaining a constant velocity of 10.0 m/s due north relative to the water. (a) What is the velocity of the boat relative to the shore? (b) If the river is 300 m wide, how far downstream has the boat moved by the time it reaches the north shore?

39. A rowboat crosses a river with a velocity of 3.30 mi/h at an angle 62.5° north of west relative to the water. The river is 0.505 mi wide and carries an eastward current of 1.25 mi/h. How far upstream is the boat when it reaches the opposite shore?

40. Suppose a chinook salmon needs to jump a waterfall that is 1.50 m high. If the fish starts from a distance 1.00 m from the base of the ledge over which the waterfall flows, find the x- and y-components of the initial velocity the salmon would need to just reach the ledge at the top of its trajectory. Can the fish make this jump? (Remember that a chinook salmon can jump out of the water with a speed of 6.26 m/s.)

41. ecp A river has a steady speed of 0.500 m/s. A student swims upstream a distance of 1.00 km and swims back to the starting point. (a) If the student can swim at a speed of 1.20 m/s in still water, how long does the trip take? (b) How much time is required in still water for the same length swim? (c) Intuitively, why does the swim take longer when there is a current?

42. ecp A river has a steady speed of v_s. A student swims upstream a distance d and back to the starting point. (a) If the student can swim at a speed of v in still water, how much time t_{up} does it take the student to swim upstream a distance d? Express the answer in terms of d, v, and v_s. (b) Using the same variables, how much time t_{down} does it take to swim back downstream to the starting point? (c) Sum the answers found in parts (a) and (b) and show that the time t_a required for the whole trip can be written as

$$t_a = \frac{2d/v}{1 - v_s^{\,2}/v^2}$$

(d) How much time t_b does the trip take in still water? (e) Which is larger, t_a or t_b? Is it always larger?

43. A bomber is flying horizontally over level terrain at a speed of 275 m/s relative to the ground and at an altitude of 3.00 km. (a) The bombardier releases one bomb. How far does the bomb travel horizontally between its release and its impact on the ground? Ignore the effects of air resistance. (b) Firing from the people on the ground suddenly incapacitates the bombardier before he can call, "Bombs away!" Consequently, the pilot maintains the plane's original course, altitude, and speed through a storm of flak. Where is the plane relative to the bomb's point of impact when the bomb hits the ground? (c) The plane has a telescopic bombsight set so that the bomb hits the target seen in the sight at the moment of release. At what angle from the vertical was the bombsight set?

ADDITIONAL PROBLEMS

44. ecp A moving walkway at an airport has a speed v_1 and a length L. A woman stands on the walkway as it moves from one end to the other, while a man in a hurry to reach his flight walks on the walkway with a speed of v_2 relative to the moving walkway. (a) How long does it take the woman to travel the distance L? (b) How long does it take the man to travel this distance?

45. How long does it take an automobile traveling in the left lane of a highway at 60.0 km/h to overtake (become even with) another car that is traveling in the right lane at 40.0 km/h when the cars' front bumpers are initially 100 m apart?

46. You can use any coordinate system you like to solve a projectile motion problem. To demonstrate the truth of this statement, consider a ball thrown off the top of a building with a velocity $\vec{\mathbf{v}}$ at an angle θ with respect to the horizontal. Let the building be 50.0 m tall, the initial horizontal velocity be 9.00 m/s, and the initial vertical velocity be 12.0 m/s. Choose your coordinates such that the positive y-axis is upward, the x-axis is to the right, and the origin is at the point where the ball is released. (a) With these choices, find the ball's maximum height above the ground and the time it takes to reach the maximum height. (b) Repeat your calculations choosing the origin at the base of the building.

47. ecp A Nordic jumper goes off a ski jump at an angle of 10.0° below the horizontal, traveling 108 m horizontally and 55.0 m vertically before landing. (a) Ignoring friction and aerodynamic effects, calculate the speed needed by the skier on leaving the ramp. (b) Olympic Nordic jumpers can make such jumps with a jump speed of 23.0 m/s, which is considerably less than the answer found in part (a). Explain how that is possible.

48. ecp In a local diner, a customer slides an empty coffee cup down the counter for a refill. The cup slides off the counter and strikes the floor at distance d from the base of the counter. If the height of the counter is h, (a) find an expression for the time t it takes the cup to fall to the floor in terms of the variables h and g. (b) With what speed does the mug leave the counter? Answer in terms of the variables d, g, and h. (c) In the same terms, what is the speed of the cup immediately before it hits the floor? (d) In terms of h and d, what is the direction of the cup's velocity immediately before it hits the floor?

49. Towns A and B in Figure P3.49 are 80.0 km apart. A couple arranges to drive from town A and meet a couple driving from town B at the lake, L. The two couples leave simultaneously and drive for 2.50 h in the directions shown. Car 1 has a speed of 90.0 km/h. If the cars arrive simultaneously at the lake, what is the speed of car 2?

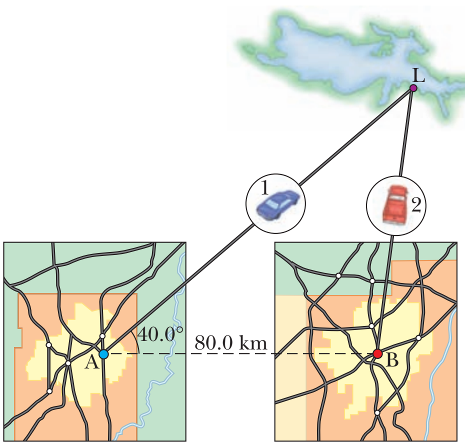


FIGURE P3.49

50. A chinook salmon has a maximum underwater speed of 3.58 m/s, but it can jump out of water with a speed of 6.26 m/s. To move upstream past a waterfall, the salmon does not need to jump to the top of the fall, but only to a point in the fall where the water speed is less than 3.58 m/s; it can then swim up the fall for the remaining distance. Because the salmon must make forward progress in the water, let's assume it can swim to the top if the water speed is 3.00 m/s. If water has a speed of 1.50 m/s as it passes over a ledge, how far below the ledge will the water be moving with a speed of 3.00 m/s? (Note that water undergoes projectile motion once it leaves the ledge.) If the salmon is able to jump vertically upward from the base of the fall, what is the maximum height of waterfall that the salmon can clear?

51. A rocket is launched at an angle of 53.0° above the horizontal with an initial speed of 100 m/s. The rocket moves for 3.00 s along its initial line of motion with an acceleration of 30.0 m/s^2. At this time, its engines fail and the rocket proceeds to move as a projectile. Find (a) the maximum altitude reached by the rocket, (b) its total time of flight, and (c) its horizontal range.

52. Two canoeists in identical canoes exert the same effort paddling and hence maintain the same speed relative to the water. One paddles directly upstream (and moves upstream), whereas the other paddles directly downstream. With downstream as the positive direction, an observer on shore determines the velocities of the two canoes to be −1.2 m/s and +2.9 m/s, respectively. (a) What is the speed of the water relative to the shore? (b) What is the speed of each canoe relative to the water?

53. If a person can jump a maximum horizontal distance (by using a 45° projection angle) of 3.0 m on Earth, what would be his maximum range on the Moon, where the free-fall acceleration is $g/6$ and $g = 9.80$ m/s^2? Repeat for Mars, where the acceleration due to gravity is $0.38g$.

54. A daredevil decides to jump a canyon. Its walls are equally high and 10 m apart. He takes off by driving a motorcycle up a short ramp sloped at an angle of 15°. What minimum speed must he have in order to clear the canyon?

55. A home run is hit in such a way that the baseball just clears a wall 21 m high, located 130 m from home plate. The ball is hit at an angle of 35° to the horizontal, and air resistance is negligible. Find (a) the initial speed of the ball, (b) the time it takes the ball to reach the wall, and (c) the velocity components and the speed of the ball when it reaches the wall. (Assume the ball is hit at a height of 1.0 m above the ground.)

56. A ball is thrown straight upward and returns to the thrower's hand after 3.00 s in the air. A second ball is thrown at an angle of 30.0° with the horizontal. At what speed must the second ball be thrown so that it reaches the same height as the one thrown vertically?

57. A quarterback throws a football toward a receiver with an initial speed of 20 m/s at an angle of 30° above the horizontal. At that instant the receiver is 20 m from the quarterback. In what direction and with what constant speed should the receiver run in order to catch the football at the level at which it was thrown?

58. A 2.00-m-tall basketball player is standing on the floor 10.0 m from the basket, as in Figure P3.58. If he shoots the ball at a 40.0° angle with the horizontal, at what initial speed must he throw the basketball so that it goes through the hoop without striking the backboard? The height of the basket is 3.05 m.

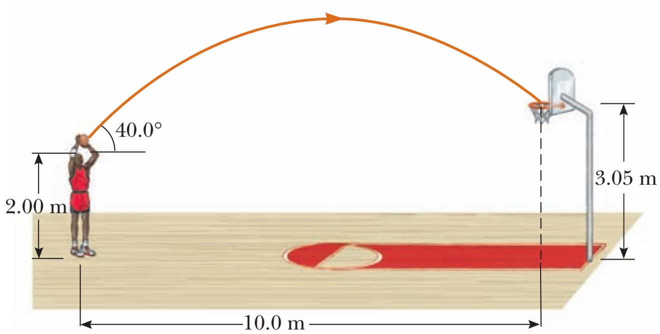


FIGURE P3.58

59. ecp In a very popular lecture demonstration, a projectile is fired at a falling target as in Figure P3.59. The projectile leaves the gun at the same instant the target is

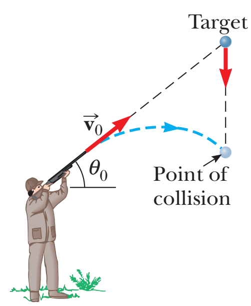


FIGURE P3.59

dropped from rest. Assuming the gun is initially aimed at the target, show that the projectile will hit the target. (One restriction of this experiment is that the projectile must reach the target before the target strikes the floor.)

60. Figure P3.60 illustrates the difference in proportions between the male (m) and female (f) anatomies. The displacements $\vec{\mathbf{d}}_{1m}$ and $\vec{\mathbf{d}}_{1f}$ from the bottom of the feet to the navel have magnitudes of 104 cm and 84.0 cm, respectively. The displacements $\vec{\mathbf{d}}_{2m}$ and $\vec{\mathbf{d}}_{2f}$ have magnitudes of 50.0 cm and 43.0 cm, respectively. (a) Find the vector sum of the displacements $\vec{\mathbf{d}}_{d1}$ and $\vec{\mathbf{d}}_{d2}$ in each case. (b) The male figure is 180 cm tall, the female 168 cm. Normalize the displacements of each figure to a common height of 200 cm and re-form the vector sums as in part (a). Then find the vector difference between the two sums.

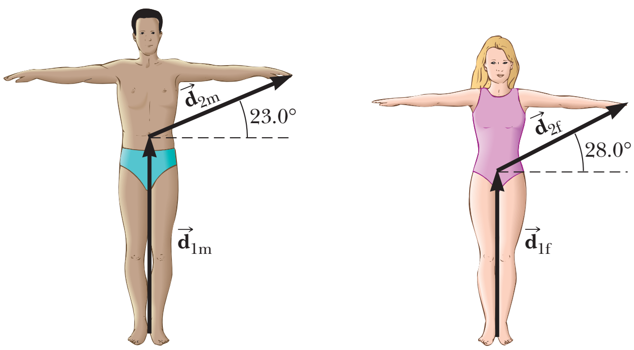


FIGURE P3.60

61. ecp By throwing a ball at an angle of 45°, a girl can throw the ball a maximum horizontal distance R on a level field. How far can she throw the same ball vertically upward? Assume her muscles give the ball the same speed in each case. (Is this assumption valid?)

62. ecp The equation of a parabola is $y = ax^2 + bx + c$, where a, b, and c are constants. The x- and y-coordinates of a projectile launched from the origin as a function of time are given by $x = v_{0x}t$ and $y = v_{0y}t - \frac{1}{2}gt^2$, where v_{0x} and v_{0y} are the components of the initial velocity. (a) Eliminate t from these two equations and show that the path of a projectile is a parabola and has the form $y = ax + bx^2$. (b) What are the values of a, b, and c for the projectile?

63. A hunter wishes to cross a river that is 1.5 km wide and flows with a speed of 5.0 km/h parallel to its banks. The hunter uses a small powerboat that moves at a maximum speed of 12 km/h with respect to the water. What is the minimum time necessary for crossing?

64. ecp When baseball outfielders throw the ball, they usually allow it to take one bounce, on the theory that the ball arrives at its target sooner that way. Suppose that, after the bounce, the ball rebounds at the same angle θ that it had when it was released (as in Fig. P3.64), but loses half its speed. (a) Assuming that the ball is always thrown with the same initial speed, at what angle θ should the ball be thrown in order to go the same distance D with one bounce as a ball thrown upward at 45.0° with no bounce? (b) Determine the ratio of the times for the one-bounce and no-bounce throws.

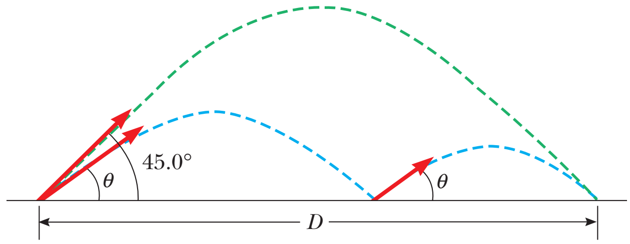


FIGURE P3.64

65. A daredevil is shot out of a cannon at 45.0° to the horizontal with an initial speed of 25.0 m/s. A net is positioned a horizontal distance of 50.0 m from the cannon. At what height above the cannon should the net be placed in order to catch the daredevil?

66. Chinook salmon are able to move upstream faster by jumping out of the water periodically; this behavior is called *porpoising*. Suppose a salmon swimming in still water jumps out of the water with a speed of 6.26 m/s at an angle of 45°, sails through the air a distance L before returning to the water, and then swims a distance L underwater at a speed of 3.58 m/s before beginning another porpoising maneuver. Determine the average speed of the fish.

67. ecp A student decides to measure the muzzle velocity of a pellet shot from his gun. He points the gun horizontally. He places a target on a vertical wall a distance x away from the gun. The pellet hits the target a vertical distance y below the gun. (a) Show that the position of the pellet when traveling through the air is given by $y = Ax^2$, where A is a constant. (b) Express the constant A in terms of the initial (muzzle) velocity and the free-fall acceleration. (c) If $x = 3.00$ m and $y = 0.210$ m, what is the initial speed of the pellet?

68. A sailboat is heading directly north at a speed of 20 knots (1 knot = 0.514 m/s). The wind is blowing towards the east with a speed of 17 knots. Determine the magnitude and direction of the wind velocity as measured on the boat. What is the component of the wind velocity in the direction parallel to the motion of the boat? (See Problem 4.58 for an explanation of how a sailboat can move "into the wind.")

69. A golf ball with an initial speed of 50.0 m/s lands exactly 240 m downrange on a level course. (a) Neglecting air friction, what *two* projection angles would achieve this result? (b) What is the maximum height reached by the ball, using the two angles determined in part (a)?

70. ecp A landscape architect is planning an artificial waterfall in a city park. Water flowing at 0.750 m/s leaves the end of a horizontal channel at the top of a vertical wall 2.35 m high and falls into a pool. (a) How far from the wall will the water land? Will the space behind the waterfall be wide enough for a pedestrian walkway? (b) To sell her plan to the city council, the architect wants to build a model to standard scale, one-twelfth actual size. How fast should the water flow in the channel in the model?

71. One strategy in a snowball fight is to throw a snowball at a high angle over level ground. Then, while your opponent is watching that snowball, you throw a second one at a low angle timed to arrive before or at the same time as the first one. Assume both snowballs are thrown with a speed of 25.0 m/s. The first is thrown at an angle of 70.0° with respect to the horizontal. (a) At what angle should the second snowball be thrown to arrive at the same point as the first? (b) How many seconds later should the second snowball be thrown after the first in order for both to arrive at the same time?

72. A dart gun is fired while being held horizontally at a height of 1.00 m above ground level and while it is at rest relative to the ground. The dart from the gun travels a horizontal distance of 5.00 m. A college student holds the same gun in a horizontal position while sliding down a 45.0° incline at a constant speed of 2.00 m/s. How far will the dart travel if the student fires the gun when it is 1.00 m above the ground?

73. The determined Wile E. Coyote is out once more to try to capture the elusive roadrunner. The coyote wears a new pair of Acme power roller skates, which provide a constant horizontal acceleration of 15 m/s^2, as shown in Figure P3.73. The coyote starts off at rest 70 m from the edge of a cliff at the instant the roadrunner zips by in the direction of the cliff. (a) If the roadrunner moves with constant speed, find the minimum speed the roadrunner must have to reach the cliff before the coyote. (b) If the cliff is 100 m above the base of a canyon, find where the coyote lands in the canyon. (Assume his skates are still in operation when he is in "flight" and that his horizontal component of acceleration remains constant at 15 m/s^2.)

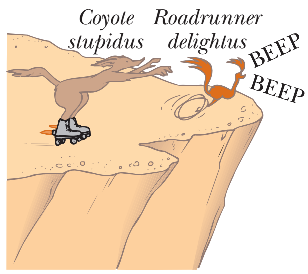


FIGURE P3.73