

FREGE

An Introduction to the Founder of
Modern Analytic Philosophy

Anthony Kenny

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CONTENTS

Preface	vii
Acknowledgements	ix
Abbreviations in references to works of Frege	x
1 Biographical Introduction to Frege's Philosophy	1
2 <i>Concept Script</i> , I	12
3 <i>Concept Script</i> , II	38
4 <i>The Foundations of Arithmetic</i> , I	50
5 <i>The Foundations of Arithmetic</i> , II	78
6 Function, Concept and Object	100
7 Sense and Reference	126
8 <i>Grundgesetze der Arithmetik</i> , I	142
9 <i>Grundgesetze der Arithmetik</i> , II	160
10 <i>Logical Investigations</i> , I	178
11 <i>Logical Investigations</i> , II	195
12 Frege's Achievement	207
Appendices	
I Frege's Symbolic Notation	214
II A Note on Translation	216
Index	219

and if every sentence of the form ' $a = b$ ' really signified a relationship between symbols, it would not express any knowledge about the extra-linguistic world. 'The evening star = the morning star' would record a lexical fact rather than an astronomical fact. This was a point which Frege had already realized at the time of writing *Concept Script*, where he says that the need for having a symbol for equality of content rests on the fact that the same content can be fully determined in different ways (as he illustrates with a geometrical example); and that a synthetic judgement of identity states that the same content is given by two ways of determining it (see p. 33).

In 'Sense and Reference', Frege says that a statement of identity can be informative only if the difference between the signs corresponds to a difference in the mode of presentation of what is designated.

Let a , b , c be the lines connecting the vertices of a triangle with the midpoints of the opposite sides. The point of intersection of a and b is then the same as the point of intersection of b and c . So we have different designations for the same point, and these names ('point of intersection of a and b ', 'point of intersection of b and c ') likewise indicate the mode of presentation; and hence the statement contains actual knowledge. (CP, p. 158.)

It is this mode of presentation which constitutes the *sense* of the sign. The reference of the two expressions quoted in the passage above is the same, but the sense of each is different.

We can say, then, that an identity statement will be true and informative if the equals sign is flanked by two names with the same reference but different senses. The word 'name' is, as the examples show, being used in a broad sense to include complex designations of objects. Frege is prepared to call all such designations 'proper names'.

In Frege's mature account of meaning there are items at three levels: signs, their senses and their references. Signs, we may say, express their senses and stand for, or denote, their references. By using signs we express a sense and denote a reference (CP, p. 161).

CHAPTER 7

SENSE AND REFERENCE

The distinction between sense and reference first presented in 'Function and Concept' in 1891 was developed in another paper which was written in conjunction with it, although it was not published until 1892. This second paper, 'Sense and Reference', presents arguments for, and answers objections to, some of the more puzzling theses which were baldly stated in the earlier paper.

The paper starts by asking a question about identity. Is identity a relation? If it is a relation, is it a relation between objects, or between signs for objects? The second answer suggests itself, because – to take the example used in 'Function and Concept' – 'the morning star = the morning star' is a statement very different in cognitive value from 'the morning star = the evening star.' The former is analytically true, while the second records an astronomical discovery. If we were to regard identity as a relation between what the signs stand for it would seem that if ' $a = b$ ' is true, then ' $a = a$ ' could not differ from ' $a = b$ '. 'A relation would thereby be expressed of a thing to itself, and indeed one in which each thing stands to itself but to no other thing.' (CP, p. 157.)

It seems natural to conclude, then, that equations express relations between signs, and that ' $a = b$ ' says that the signs ' a ' and ' b ' designate the same thing. This was the view Frege had adopted in Section 8 of *Concept Script*, where he introduced a sign for identity of content, and said that this signified the circumstance of two names having the same content.

But there is an objection to this view. Names are arbitrary,

In a well-regulated language, every sign would have a sense, and only one sense. In natural languages, a sign may have more than one sense: a word like 'staff' is ambiguous; the expression 'the dog', if it is to have a precise sense, needs different supplementation in different households; different scholars would give different explanations of the name 'Aristotle'. Frege says that we have to be content if the same word has the same sense in the same context. While in an ideal language there would be only one sense for every sign, there is no requirement, even in an ideal language, that every sense should have only one sign. In the real world the same sense may be expressed by different signs in different languages or even in the same language.

In a good translation, the sense of the original text will be preserved. What is lost in translation is not a matter of sense, but, Frege says, 'the colouring and shading which poetic eloquence seeks to give to the sense'. This is something which is not objective in the way that sense is objective.

Not every sense has a reference corresponding to it. The expressions 'The celestial body most distant from the Earth' and 'The least rapidly convergent series' both have senses, but the former probably, and the latter certainly, lacks a reference and stands for no object. Conversely, one and the same object may be referred to by expressions of quite different senses: this was the starting point for the drawing of the distinction in the first place. If one knew all that there was to know about an object one would know which senses would serve to pick it out and which would not. But of course we humans never attain such knowledge.

The sense of a word is what we grasp when we understand a word. It is quite different from a mental image, even though, if a sign refers to a tangible object, I may well have a mental image associated with it. Images are subjective and vary from person to person, and within the same person from time to time; but the sense of a sign, Frege says, 'may be the common property of many and therefore is not a part of a mode of the individual mind'.

Frege offers an analogy for the relationship between reference, sense and mental image:

Somebody observes the Moon through a telescope. I compare the Moon itself to the reference; it is the object of the observation, mediated by the real image projected by the object glass in the interior of the telescope, and by the retinal image of the observer. The former I compare to the sense, the latter is like the image or experience. The optical image in the telescope is indeed one-sided and dependent upon the standpoint of observation; but it is still objective, in as much as it can be used by several observers. It could be arranged for several people to use it simultaneously. But each one would have his own retinal image. (CP, p. 160.)

An image is *my* image or *your* image; but a sense does not, in the same way, have an owner. It is because senses are public and common that thoughts can be passed on from one generation to another.

We know, from 'Function and Concept', that it is not only proper names – simple or complex – that have senses and references. What of entire sentences, which express thoughts? Is the thought, the content of the sentence, the sense or the reference of a sentence?

Let us assume for the time being that the sentence has reference. If we now replace one word of the sentence by another having the same reference, but a different sense, this can have no bearing upon the reference of the sentence. Yet we can see that in such a case the thought changes; since, e.g., the thought in the sentence 'The morning star is a body illuminated by the Sun' differs from that in the sentence 'The evening star is a body illuminated by the Sun'. Anybody who did not know that the evening star is the morning star might hold the one thought to be true, the other false. The thought, accordingly, cannot be the reference of the sentence, but must rather be considered as the sense. (CP, p. 162.)

Frege is clearly operating with a tacit criterion for the identity of thoughts: two thoughts are identical if and only if it is not possible to hold either thought to be true without holding the other one to be true. This suggests a general criterion for the

identity of senses: that two senses are the same if and only if it would be impossible to grasp them both without realizing that they determined the same reference. For the truth-value of a proposition is its reference, as we know from 'Function and Concept'.

Here, in 'Sense and Reference', an argument is provided for that earlier assertion.

Is it possible that a sentence as a whole has only a sense, but no reference? At any rate, one might expect that such sentences occur, just as there are parts of sentences having sense but no reference. And sentences which contain proper names without reference will be of this kind. The sentence 'Odysseus was set ashore at Ithaca while sound asleep' obviously has a sense. But since it is doubtful whether the name 'Odysseus', occurring therein, has reference, it is also doubtful whether the whole sentence has one. (CP, p. 162.)

The argument seems to go like this. We must expect that the reference of a sentence is determined by the reference of the parts of a sentence. Let us inquire, therefore, what is missing from a sentence if one of its parts lacks a reference. If we were interested only in the thought, it would not matter to us whether 'Odysseus' has a reference or not, for the thought remains the same in either case, being determined by the senses, not the references, of the constituent parts of the proposition.

It is only if you want seriously to take the sentence as true or as false that you need to ascribe a reference to 'Odysseus'; for if you do not, there will be nothing for the predicate to be true or false of.

Why do we want every proper name to have not only a sense, but also a reference? Why is the thought not enough for us? Because, and to the extent that, we are concerned with its truth value. (CP, p. 163.)

The sense, and the associated images, are sufficient to make us enjoy an epic poem: what pushes us to advance from sense to reference is the attitude of scientific investigation, the striving for truth. We are, therefore, Frege says, driven into accepting as

the reference of a sentence its truth-value, the True or, as the case may be, the False. Every seriously propounded indicative sentence is a name of one or other of these objects. All true sentences have the same reference as each other, and so do all false sentences.

Frege's thesis that the relation between a sentence and its truth-value is the same as that between a name and its reference is connected with his general distinction between predication and assertion. It is no doubt more natural to think of the relation between a thought and its truth as being that of subject to predicate: we say, after all, 'The thought that 5 is a prime number is true.' But, Frege says, the thought expressed by that whole sentence is only the same thought as is contained by the simpler sentence '5 is a prime number.' (This is in accord with the tacit criterion for the identity of thoughts mentioned earlier.) These two sentences have the same sense, and both of them are names of the True; but only if they are asserted do they *say that* they are names of the True.

Subject and predicate (understood in the logical sense) are indeed elements of thought; they stand on the same level as items for comprehension. By combining subject and predicate one reaches only a thought, never passes from sense to reference, never from a thought to its truth-value. (CP, p. 164.)

It is by making a judgement, which is not the mere comprehension of a thought, but an acceptance of its truth, that we take the step from the level of thought to the level of reference.

The difference between assertion and predication, in ordinary discourse, is made out by grammatical mood, by context and by convention. In Frege's formal system, as we saw when we were considering *Concept Script*, this difference is supposed to be marked by a special assertion sign.

In the course of Frege's writing, at least two different functions are attributed to the assertion sign. Sometimes the assertion sign is considered as a sign that what follows is seriously meant; that is to say, that it is meant to be taken seriously and not as part of

a charade or fiction. At other times, it is taken as being the mark of the assertoric mood, as when it is said that its function is to distinguish an assertion from a supposition or a question.

Frege treats the assertion sign as a sign that what follows is seriously meant when he says that in fiction we are only interested in the sense of sentences and not in their truth-values. Actors on the stage, he says, utter words which are signs having only a sense; the assertion sign, which is used to assert that the reference of what follows it is the True, cannot be used in conjunction with signs that have no reference, and therefore cannot be used by an actor on the stage. The assertion sign, in Frege's thought, seems to be like the sign of the treble clef. The clef shows how each note which follows it is to be read, and so the assertion sign shows that each word following it is to be taken seriously.

But it seems obvious that there is something futile in introducing a sign to denote that a sentence is to be taken seriously. If it was in question whether an ordinary sentence was to be taken seriously, it could equally be questioned whether a sentence preceded by an assertion sign was to be taken seriously. According to Frege, the actor on the stage who says 'the thought that 5 is a prime number is true' no more makes a serious assertion than if he said '5 is a prime number' (CP, p. 164). But, equally, an actor who in the course of a play wrote the sentence preceded by an assertion sign on a blackboard would not have made a serious assertion either.

Whether an utterance is meant seriously is a different question from whether the utterance is in the assertoric mood. The question 'Did you mean that seriously?' can be asked of commands no less than it can be asked of statements. Even in the case of suppositions, which are Frege's favourite examples of sentences which are *not* assertions, there is a difference between an idle fantasy and a hypothesis put forward for exploration.¹

1. I have treated this issue at greater length in my book *Will, Freedom and Power*, pp. 36-8.

There is indeed scope for a sign to mark the difference between the assertoric mood and other moods, and natural languages have such signs in verb inflexions and auxiliary verbs. However, it is not necessary to make use of such a sign in order to expound Frege's system, since in his treatment he explicitly ignores sentences in other than assertoric moods since his overriding aim is to describe, and in part construct, a language which is appropriate for the purposes of theoretical science.

Acceptance of the distinction between assertion and predication, and of the distinction between the assertoric and other moods, does not force us, as Frege seems to think it does, to regard truth-values as the references of sentences. It is true that if a truth-value is an object then it cannot be represented by a predicate (which has to correspond to something unsaturated, like a concept). But the converse is not true.

Frege may be right that '...is true' is not a genuine predicate, and does not express any function. But his argument seems to involve two further steps. (1) The only entities in the world that correspond to language are objects and functions, so if truth is not a function it must be an object. (2) The only relation an object can have to language is being the reference of a name; hence truth is something which is named by true, sentences. But why cannot there be linguistic items which are neither names nor linguistic functions? Is not Frege's assertion sign itself such an item? And if so, why is '...is true' not also such an item?

Some philosophers have thought that for a sentence, or a thought, to be true was for it to correspond to something extralinguistic and extra-mental: a fact, perhaps, or a state of affairs, or what Frege in *Concept Script* called a circumstance. There are grave difficulties in identifying the facts, or states of affairs, or circumstances, to which true propositions are alleged to correspond: but it is surprising that Frege does not here consider this possibility, even if only to reject it.

Frege talks, not of facts, but of parts of truth-values. But he says:

I have here used 'part' in a special sense. That is, I have transferred to the reference of a sentence the relation between the whole and the parts of a sentence itself, by calling the reference of the word a part of the reference of the sentence, if the word itself is part of the sentence. (CP, p. 165.)

Frege admits that there is something objectionable about this, because 'the whole reference and one part of it do not suffice to determine the reference of the remainder'. Frege continued to worry about this problem, but he did not ever satisfactorily resolve, even in his own mind, the unease here expressed. For the present he contents himself with saying that perhaps a word other than 'part' should be invented.

At the three levels of Frege's system, corresponding to a sentence containing a name, there are three different, but parallel, part-whole relationships, which are of increasing obscurity.

First, at the level of signs, there is the distinction between function and argument within sentences, according to the linguistic function/argument analysis with which the *Concept Script* replaced the traditional distinction between grammatical subject and grammatical predicate. Thus, in the sentence 'Nero fiddled', the parts are the name 'Nero' and the predicate '... fiddled'.

Secondly, at the level of sense, there are two elements: the sense of the name involved, and the sense expressed by the predicates: in the case in point, *the fifth Roman Emperor* (or the like), and whatever it is that one grasps when one understands the predicate '... fiddled'. These parts constitute the whole, which is a thought, the thought *that Nero fiddled*.

Thirdly, at the level of reference, there is the whole, which is a truth-value, and there is at least one part which Frege identifies, the reference of the name, which in this case is the Emperor Nero himself. In 'Sense and Reference' Frege is very unforthcoming about the other part, 'the remainder'; but we know from 'Concept and Object' that it must be the concept ξ *fiddled*.²

2. Frege gives a cross-reference (CP, p. 158) to the passage of 'Concept and Object' discussed above at p. 123.

One might have expected that at the level of reference the two parts would be the reference of the name, Nero, and the extension of the concept ξ *fiddled*. But, no doubt for good reason, Frege hesitates to say this. None the less, he proceeds resolutely to make use of the 'part/whole' terminology at the level of reference, and he uses it to support the thesis that the reference of a sentence is its truth-value. If a part of a sentence is replaced by another with the same reference, then the reference of the whole must remain constant. And in fact the truth-value (and probably nothing but the truth-value) of a sentence does remain constant in such a case (CP, p. 164). If, say, in the example above, 'Nero' is replaced by 'The son of the younger Agrippina' the sentence remains true.

The same is true if the expression to be substituted is not a name but a sentence or clause within a proposition. If the reference of a sentence is, as the thesis claims, a truth-value, then the truth-value of a sentence containing another as a part must remain unchanged when the part is replaced by another sentence having the same truth-value.

This is the case, of course, when sentences are joined to each other to make larger sentences by means of conjunctions such as 'and', 'or' and 'if... then', if these conjunctions are understood in the manner explained in the *Concept Script* (see p. 27 above). These connectives, so understood, are truth-functional; that is to say, the truth-values of propositions compounded by these connectives depend only on the truth-values, and not on the content, of the propositions so compounded. It will follow that if we take any such compound proposition and replace one of its components with another proposition of different sense but the same truth-value the truth-value of the whole will not alter.

Frege takes here as an example the sentence

If the Sun has already risen, the sky is very cloudy,

and he says of this sentence:

Here it can be said that a relation between the truth-values of

conditional and dependent clauses has been asserted, viz. Such that the case does not occur in which the antecedent stands for the True and the consequent for the False. Accordingly, our sentence is true if the Sun has not yet risen, whether the sky is very cloudy or not, and also if the Sun has risen and the sky is very cloudy. Since only truth-values are here in question, each component clause can be replaced by another of the same truth-value without changing the truth-value of the whole. (CP, p. 173.)

The sense of the sentence is equivalent to 'Either the Sun has not risen yet or the sky is very cloudy'. In accordance with all this, 'If the Sun has already risen, 5 is a prime number' will come out true if uttered before sunrise on a cloudy day. Frege is happy to accept this; the sentence, he agrees, will appear in a strange light, 'as if a sad melody were to be sung in a lively fashion' – but this is nothing to do with the truth-value. He suggests also that 'although' and 'but' can be treated as truth-functional, no less than 'and' and 'or' can.

Partly as a result of the work of Frege, philosophers now tend to think of propositions joined by truth-functional connectives as coordinated to each other on an equal level. Frege, however, was writing at a time when it was more natural to think of one of the component propositions as in some sense subordinate to the other. He says that whenever the reference of a subordinate clause is a truth-value, then it may be replaced, without harm to the truth-value of the whole, by a sentence having the same truth-value.

However, Frege recognizes that in the case of many kinds of subordinate clause this is simply not the case, and most of the concluding part of 'Sense and Reference' is devoted to showing that in such cases, contrary to the general rules, the reference of the clause is not a truth-value. He recognizes three main categories: direct quotation, indirect quotation and definite descriptions.

(1) If a sentence occurs within quotation marks, then the reference of the sentence is not a truth-value, but the words quoted, as in 'Smith said "The world will end in the year

2000"'. In such a case, Frege says, 'One's own words first designate words of the other speaker, and only the latter have their usual reference' (CP, p. 159).

(2) If a sentence occurs in reported speech, as in 'Smith said that the world will end in the year 2000', then the reference of the sentence, according to Frege, is what would normally be its sense. This, he says, is indirect reference as opposed to customary reference; and we can say that the indirect reference of a word is its customary sense (CP, pp. 159, 165). Does this mean that in this case the distinction between sense and reference collapses? No: Frege says that the clause has as sense (indirect sense, presumably) 'not a thought, but the sense of the words "the thought, that..." which is only a part of the thought in the entire complex sentence' (CP, p. 166). Reports of thoughts, beliefs and indifference, and also hopes, regrets and similar feelings, are in this respect similar to indirect quotations: in all such cases the 'that' clauses refer to their sense and not to a truth-value (CP, p. 167).

It is easy to show that in such cases the clause does not occur truth-functionally. 'Copernicus believed that the planetary orbits are circular' and 'Copernicus believed that the Earth goes round the Sun' are both true, though the content of the first 'that' clause is false, and that of the second is true; and on the other hand, though the proposition 'Uranus goes round the Sun' is true no less than the proposition 'Venus goes round the Sun', the proposition 'Copernicus believed that Venus goes round the Sun' is true, while the proposition 'Copernicus believed that Uranus goes round the Sun' is false, since Uranus had not been discovered in Copernicus's day. Frege sums up

The main clause and the subordinate clause together have as their sense only a single thought, and the truth of the whole includes neither the truth nor the untruth of the subordinate clause. In such cases it is not permissible to replace one expression in the subordinate clause by another having the same customary reference, but only by one having the same indirect reference, i.e. the same customary sense. (CP, p. 166.)

This exception applies not only to the complete clauses, but also to names within them. In the sentence 'Duncan believed that Macbeth was trustworthy' we cannot substitute 'his murderer' for 'Macbeth'. The reference of the two expressions is the same, since Macbeth was Duncan's murderer, but Duncan never had the thought 'my murderer is trustworthy'.

Frege assimilates to the case of reported speech and belief a number of other constructions: indirect reports of commands and questions, and final clauses expressing a purpose. Consider the two sentences 'Napoleon believed that his right flank was to advance' and 'Napoleon gave orders that his right flank was to advance'. It is clear that a similar structure of argument and function can be discerned in the clause of the second sentence 'that his right flank was to advance' as in the clause of the first sentence 'that his right flank was to advance'. There is clearly something in common between the senses of the two clauses. The difference between the two is that if the two sentences were rewritten in direct speech, the first would contain a quotation in the assertoric mood, and the second would contain a quotation in the imperative mood. Frege sums this up by saying 'A command, a request, are indeed not thoughts, yet they stand on the same level as thoughts.' In subordinate clauses reporting commands, he says, words have their indirect reference.³ In these clauses, as in those reporting statements and beliefs, the reference is not a truth-value; it is, he says, a command or request. From which we conclude that for Frege a command is the sense of an imperative sentence.

Frege deals very briefly with final clauses. 'Obviously', he says, 'a purpose is a thought; therefore: indirect reference for the words, subjunctive mood.' This seems questionable: Napoleon's purposes were surely what he wanted to bring about in the world, and it was not thoughts that he wanted to bring about. 'Napoleon took the necessary steps in order that his right flank should advance' seems to be closer to 'Napoleon gave orders

3. It is not clear to me what Frege would count as the direct reference of a sentence in the imperative.

that his right flank was to advance' than to 'Napoleon believed that his right flank was to advance', and so, even if one accepts Frege's case for the reference being indirect, it would seem that he should say that it is not a thought, but to something analogous to a command or request.

Given Frege's account of reference, it is clear that in the cases where the reference of a clause is not a truth-value but a thought, a command or a purpose, the clause can be regarded as a proper name of the respective thought, command or purpose.

(3) Frege turns to a quite different group of cases, which would now be called (after Bertrand Russell) sentences containing definite descriptions. Consider the following sentence, adapted from Frege's own.

The man who discovered oxygen was guillotined.

Russell called expressions such as 'the man who discovered oxygen' definite descriptions, and he gave an elaborate analysis of the sentence in formal symbolism which was roughly equivalent in everyday language to the sentence 'one and only one man discovered oxygen, and that man was guillotined'.

Frege takes a different approach. He asks how we are to construe the relative clause

'who discovered oxygen'.

He says that such a clause does not have a thought occurring as its sense and a truth-value as its reference. It is not a complete thought; the grammatical subject 'who' has no independent sense and serves only to mediate a relationship with the main clause. The reference of the definite description is not a truth-value but an object, namely, the chemist Lavoisier.

One might think that the sense of the original sentence does include a thought as its part, namely, the thought that there was somebody who was the first to discover oxygen. At all events, nobody could take the sentence to be true who denied the existence of such a person. If 'the man who discovered oxygen' had no reference, the sentence could not be true.

Frege accepts this: but he says that the sentence does not state, but only presupposes, that there was somebody who was the first to discover oxygen. The sentence certainly presupposes that there was such a person, but that is quite different from containing, as part of the thought that it expresses, the thought that there was such a person. 'Lavoisier discovered oxygen' likewise presupposes that the name 'Lavoisier' has a reference; but that is not part of the thought that it expresses. If it were, the negation of 'Lavoisier was guillotined' would not be 'Lavoisier was not guillotined' but

Either Lavoisier was not guillotined, or 'Lavoisier' has no reference.

That 'Lavoisier' has a reference is presupposed equally both by 'Lavoisier was guillotined' and by 'Lavoisier was not guillotined'. This shows the difference between the case where one sentence presupposes the truth of another and the case where one sentence has another as part of its sense.

So 'the man who discovered oxygen was guillotined' does not state, but only presupposes, that there was one and only one man who discovered oxygen. The interpretation of sentences of this kind which is here scorned by Frege was later explored, in a fertile manner, by Russell and his followers. But to this day there are philosophers who think that Frege's approach was the more appropriate one.

Frege believed that the possibility of a definite description lacking reference was to be averted not by particular provisions in individual sentences, but rather by general rules about the construction of scientific languages.

A logically perfect concept-script should satisfy the conditions, that every expression grammatically well constructed as a proper name out of signs already introduced shall in fact designate an object, and that no new sign shall be introduced as a proper name without being secured a reference. (CP, p. 169.)

What Frege has in mind here is illustrated by the example he

gave in 'Function and Concept', where he says that we must see to it that we never perform calculations with empty signs in the belief that we are dealing with objects.

It is thus necessary to lay down rules from which it follows, e.g. what '☉ + 1' is to stand for, if '☉' is to stand for the Sun. What rule we lay down is a matter of comparative indifference; but it is essential that we should do so. (CP, p. 148.)