11.1 Understanding Area

The standard units of AREA are square units.

Definition - 95 - The area of a closed region is the number of square units of space within the boundary of the region.

Go to pg. 511 and estimate the area of the kidney shape. Area about______units.

Postulate - 16 - The area of a rectangle is equal to the product of the base and the height for that base. \( A = bh \) Where \( b \) = the length of the base \( h \) = the height.

Theorem 99 - The area of a square is equal to the square of a side. \( A = s^2 \) Where the \( s \) is the length of a side.

3 Basic Properties of Area:

Postulate 17 - Every closed region has an area
Postulate 18 - If two closed figures are congruent then their areas are equal.
Postulate 19 - If two closed regions intersect only along a common boundary, then the area of their union is equal to the sum of their individual areas.

Find the area of the rectangle.

\[
\begin{array}{c}
\text{Rectangle } \hspace{2cm} 13 \text{ cm } \hspace{2cm} 5 \text{ cm}
\end{array}
\]

Given that the area of a rectangle is 20 sq. dm. and the altitude is 5 dm, find the base.

\[
\begin{array}{c}
\text{Rectangle } \hspace{2cm} A = 20 \hspace{2cm} 5
\end{array}
\]
Find the area of the shaded region.

11.2 Areas of Parallelograms and Triangles

Theorem 100 - The area of a parallelogram is equal to the product of the base and the height. Where $b$ is the base and $h$ is the height.

$$A = bh$$

*GET OUT YOUR 3X5 CARD AND SCISSORS. Pick any point on one of the long sides of the card. Draw a line from that point you selected to a vertex on the opposite side. Cut along the line you just drew.*

The area of a Triangle;
The area of any triangle can be shown to be one half of the area of a parallelogram with the same base and height.

Now create a parallelogram by using the triangle.

What do you have to do to the triangle to do it.

Theorem 101 - The area of a triangle is equal to half of the product of the base and the height (altitude) for that base. Where $b$ is the length of the base and $h$ is the altitude.

$$A_{\Delta} = \frac{1}{2}bh$$

Find the area of each triangle.
Find the base of a triangle with altitude 15 and area 60. Let $x$ be the base. Label the drawing.

Find the area of a parallelogram whose sides are 14 and 6 and whose acute angle is 60°. Draw and label.

Find the area of the trapezoid WXYZ.

11.3 The Area of a Trapezoid:
The area of a trapezoid can be found by dividing it into similar shapes such as triangle, rectangles and parallelograms (“divide and conquer”)

Theorem 102 - The area of a trapezoid equals one-half the product of the height and the sum of the bases. Where $b_1$ is the length of one base and $b_2$ is the length of the other base, and $h$ is the height.

$$A_{trap} = \frac{1}{2}h(b_1 + b_2)$$
The Median of a Trapezoid:
We can use the Midline Theorem to find out what happens when the midpoints of
the nonparallel sides of a trapezoid are joined.

Definition 96 - The line segment joining the midpoints of the nonparallel
sides of a trapezoid is called the median of the trapezoid.

Now let’s list all the things we know about this trapezoid.
P,____ and _____ are midpoints of the side of ΔWXZ and Δ_____.
P,Q and ____ are collinear, because PQ and ____ share Q and each segment is
parallel to ______ and _______.
PR is the _______ of trapezoid WXYZ.

By the Midline Theorem: PQ = \( \frac{1}{2} (____) \) and QR = \( \frac{1}{2} (__) \).
Thus PR = PQ +____= \( \frac{1}{2} \) of(____) + \( \frac{1}{2} \) of(_____) = \( \frac{1}{2} \) (_______+_______)

Theorem 103 - The measure of the median of a trapezoid equals the
average of the measures of the bases, where \( b_1 \) is the length of one base and
\( b_2 \) is the length of the other base.
\[
M = \frac{1}{2} (b_1 + b_2)
\]

Now you can easily prove a shorter form of Theorem 102.
Theorem 104 - The area of a trapezoid is the product of the median and
the height, where M is the length of the median, and h is the height.

\[
A_{\text{trap}} = Mh
\]

Given: Trapezoid WXYZ, with height 7,
lower base 18 and upper base 12 find the
area of the trapezoid,
Find the shorter base of a trapezoid if the trapezoid's area is 52, its altitude is 8 and its longer base is 10. Draw and label the trapezoid first, then solve.

The height of a trapezoid is 12. The bases are 6 and 14.
Find the median.

Find the area.

11.4 Areas of Kites and Related Figures:
Remember that in a kite the diagonals are ____________. Also the kite can be divided into two ____________ triangles with a common base, so that its area will equal the sum of the areas of these triangles.
Draw and label a kite to fit the description above.

\[ \text{Area of kite} = \text{Area of } \triangle ABD + \text{Area of } \triangle DBC \]
\[ \quad = \]
\[ \quad = \]
\[ \quad = \]
Notice that BD and AC are the diagonals of the kite.
We have just proven the following formula.
Theorem 106 - The area of a kite is equal to half the product of its diagonals. Where $d_1$ is the length of one diagonal and $d_2$ is the length of the other diagonal.

$$A_{kite} = \frac{1}{2} d_1 d_2$$

Find the area of a kite with diagonals 9 and 14.

Find the area of a rhombus whose perimeter is 20 and whose longer diagonal is 8.

11.5 Areas of Regular Polygons:

Area of an Equilateral Triangle - Give the angle measure of each angle.

Draw an altitude. Then put angle measures in each of the angles of one of the triangles you created. What type of triangle did you create?

If $WY = s$, then $ZY = \text{______}$ and $WZ = \text{______}$

If the area of triangle $= \frac{1}{2}bh$

= 

= 

Theorem 106 - The area of an equilateral triangle equals the product of $\frac{1}{4}$ the square of a side and the $\sqrt{3}$.

$$A_{equa} = \frac{s^2}{4} \sqrt{3} \quad \text{or} \quad A_{equa} = \frac{1}{4} s^2 \sqrt{3}$$
The Area of a Regular Polygon: Remember all interior angles and all sides are____________. PENTA is a regular polygon, O is the center, \( OA \) is the radius to a vertex and \( OM \) is an apothem.

Definition 95 - A radius of a regular polygon is an segment joining the center to any vertex.
Definition 96 - An _________ of a regular polygon is a segment joining the center to the _________ of any side.

(\textit{Draw and label a radius to each vertex of the polygon, and one apothem to the bottom edge of the polygon.})

List all the important observations about apothems and radii.

•
•
•
•
•
•

When all the radii of a regular polygon are drawn, the polygon is divided into __________ (What type of? ______________)triangles.

What is an altitude of each triangle called?_________

Can you write an expression for the sum of the areas of those isosceles triangles
Theorem 107 - The area of a regular polygon equals one-half the product of the apothem and the perimeter. Where \( a \) is the length of the apothem and \( p \) is the perimeter.

\[
A_{\text{reg. polygon}} = \frac{1}{2} ap
\]

a) A regular polygon has a perimeter of 40 and an apothem of 5. Find the area of the polygon.

b) An equilateral triangle has a side 10 cm long. Find the triangle’s area.

A circle with a radius of 6 is inscribed in an equilateral triangle. Find the area of the triangle. Center of the circle is \( O \).

\[\text{OP is ______ to AB, and is a _________}
\]

AO and OP and AP create what kind of triangle?___________

How can this help you solve the problem using the formula \( A = 0.5ap \)?

Find the area of a regular hexagon with sides 18 units long. \( AB=_______
\]

\( AP=_______
\]

\( AO=_______
\]
11.6 Areas of Circles, Sectors, and Segments:

Area of a circle-
Postulate 20 - The area of a circle is equal to the product of $\pi$ and the square of the radius.

$$A_{\text{circle}} = \pi r^2$$

Area of a Sector-
Definition 99 - A **sector of a circle** is a region bounded by two radii and an arc of the circle.

( Draw 2 radii and an intercepted arc. Label the center $O$ and the arc $HP$.)

The length of an arc is a fractional part of the __________.
The sector is a fractional part of the __________.

Theorem 108 - The area of a sector of a circle is equal to the area of the circle times the fractional part of the circle determined by the sector's arc. (Where $r$ is the radius and the arc is measured in degrees.)

$$A_{\text{sector - HOP}} = \left(\frac{mHP}{360}\right)\pi r^2$$

Area of a Segment-
Another way of dividing the interior of a circle produces a segment.
Definition 100 - A segment of a circle is a region bounded by a chord of the circle and its corresponding arc.

( Shade in the small segment formed by the chord.)

- Find the area of a circle whose diameter is 10.
- Find the circumference of a circle whose area is $49\pi$ sq. units.

- Find the area of a sector with a radius of 12 and a $45^\circ$ arc.
The measure of the arc of the segment (\(\hat{AB}\)) is 90. The radius of the circle is 10. Find the area of the segment. (Shade in the small segment formed by the chord.)

11.7 Ratios of Areas:

Computing the areas-
One way to compare areas of 2 figures is to write a ratio of their areas. Find the ratio of the area of these two figures.

\[
\frac{A_1}{A_2} = \frac{\frac{1}{2}bh_1}{\frac{1}{2}bh_2}
\]

In the diagram, \(AB = 5\) and \(BC = 2\). Find the ratio of the area of \(\triangle ABD\) to that of \(\triangle CBD\). (What do you notice about the height of each triangle?)

Similar Figures-
If you know that two triangle are similar, the ratio of any pairs of their corresponding altitudes, medians or angle bisectors equals the ratio of their corresponding sides.

Given that \(\triangle PQR \sim \triangle WXY\), find the ratio of their areas.
The previous example shows the key steps that can be used to prove a theorem about the areas of similar triangles. Because “convex polygons” can be divided into triangles, you may suspect that the areas of similar polygons have the same relationship.

Theorem 109 - If two figures are similar, then the ratio of their areas equals the square of the ratio of corresponding segments

(Similar-Figure Theorem) \( \frac{A_1}{A_2} = \left( \frac{s_1}{s_2} \right)^2 \), where the \( A_1, A_2 \) are areas and \( s_1, s_2 \) are measure of corresponding sides.

CORRESPONDING SEGMENTS can be any segments, associated with the figures, such as sides, altitudes, medians, diagonals or radii.

a) Given the similar pentagons shown find the ratio of their areas.

\[
\begin{array}{c}
12 \\
\text{1}
\end{array} \quad \begin{array}{c}
9 \\
\text{2}
\end{array}
\]

b) If \( \triangle ABC \sim \triangle DEF \), find the ratio of the areas of the two triangles.

\[
\begin{array}{c}
A \\
\text{12}
\end{array} \quad \begin{array}{c}
12 \\
\text{B}
\end{array} \quad \begin{array}{c}
C \\
\text{B}
\end{array} \quad \begin{array}{c}
D \\
\text{8}
\end{array} \quad \begin{array}{c}
E \\
\text{E}
\end{array}
\]

c) If the ratio of the areas of two similar parallelograms is 49:121, find the ratio of their bases.
Theorem 110 - A median of a triangle divides the triangle into two triangles with equal areas.

\[ \Rightarrow A_{\triangle QPR} = A_{\triangle PRS} \]

11.8 Hero's and Brahmagupta's:
A useful formula for finding the area of a triangle was developed nearly 2000 years ago by mathematician **Hero of Alexandria**.

Theorem 111 -

\[ A_{\triangle} = s(s-a)(s-b)(s-c) \]

where \(a, b,\) and \(c\) are the lengths of the sides of the triangle and

\[ s = \frac{a+b+c}{2} = \text{semiperimeter} \]

(Hero's formula)

About 628 A.D., a Hindu mathematician, **Brahmagupta**, recorded a formula for the area of an inscribed quadrilateral. This formula applies only to quadrilaterals that can be inscribed in circles (known as cyclic quadrilaterals).

Theorem 112 -

\[ A_{\text{cyclic-quad}} = \sqrt{(s-a)(s-b)(s-c)(s-d)} \]

where \(a, b, c,\) and \(d\) are the sides of the quadrilateral

and \( s = \text{semiperimeter} = \frac{a+b+c+d}{2} \).

• Find the area of a triangle with sides 3, 6, and 7.

Find the area of the inscribed quadrilateral with side, 2, 7, 6, and 9.