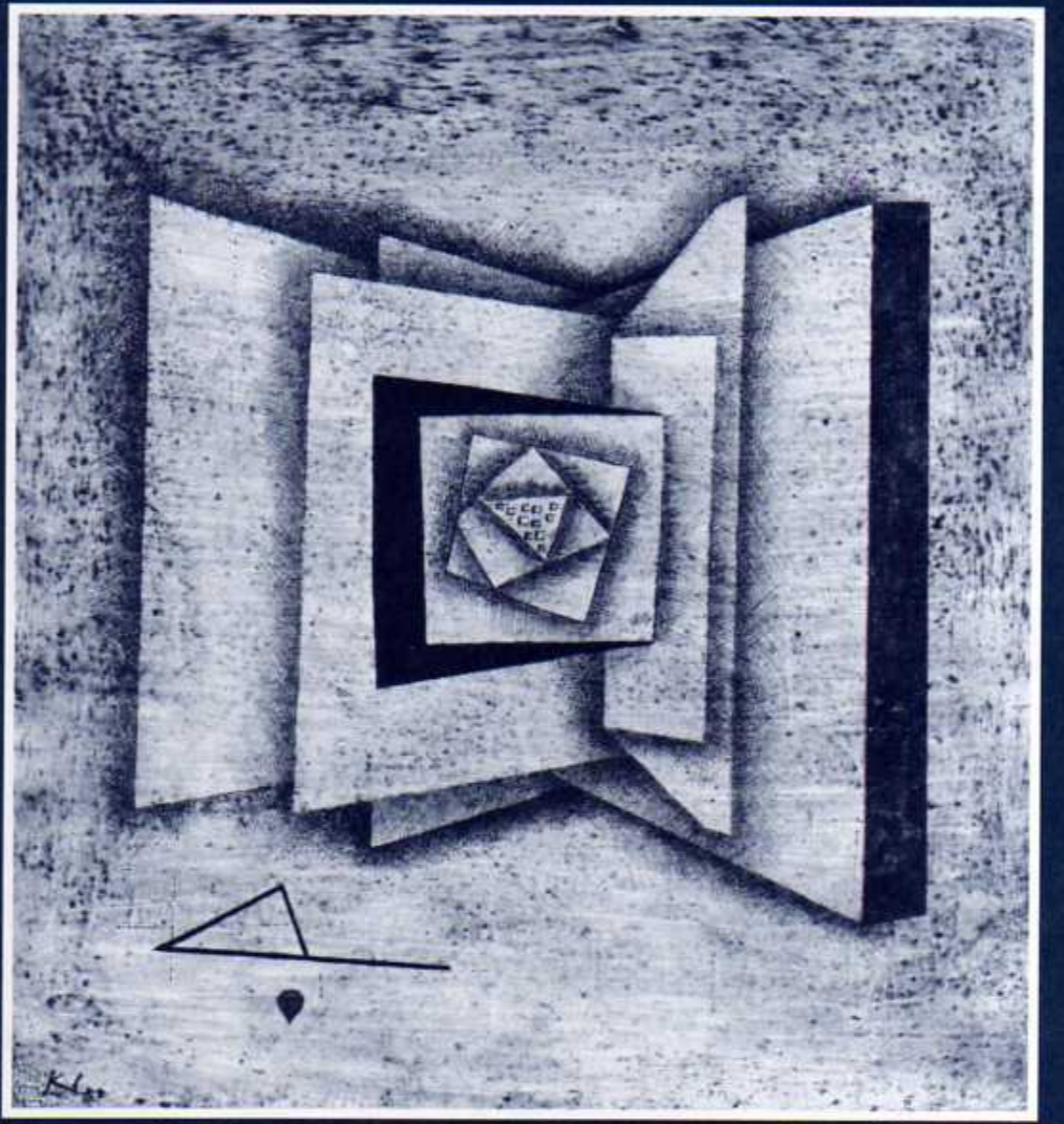
CHAPTER

1 INTRODUCTION TO GEOMETRY



This painting, *Open Book* by Paul Klee, incorporates geometric shapes and relationships.

1.1 GETTING STARTED

Objectives

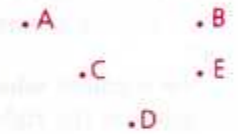
After studying this section, you will be able to

- Recognize points
- Recognize lines
- Recognize line segments
- Recognize rays
- Recognize angles
- Recognize triangles

Part One: Introduction

Points

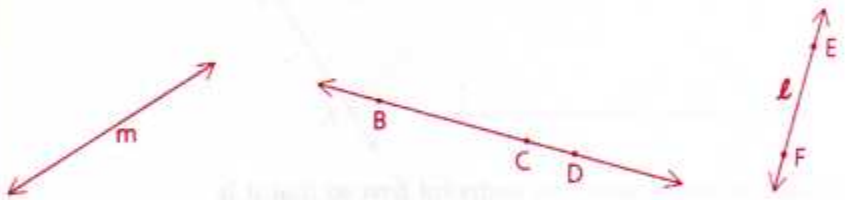
In the diagram at the right, five ***points*** are represented by five dots. The names of the points are A, B, C, D, and E. (We use capital letters to name points.)



Lines

The diagram below represents three ***lines.*** Lines are made up of points and are straight. The arrows on the ends of the figures show that the lines extend infinitely far in both directions.

All lines are straight and extend infinitely far in both directions.



- The line on the left is called line m.
- Since we can name a line in terms of any two points on it, the line in the middle can be called by a variety of names.

$\overleftrightarrow{BD}$ $\overleftrightarrow{BC}$ $\overleftrightarrow{CD}$ $\overleftrightarrow{CB}$ $\overleftrightarrow{DB}$ $\overleftrightarrow{DC}$

- The line on the right can be called by any of three names.

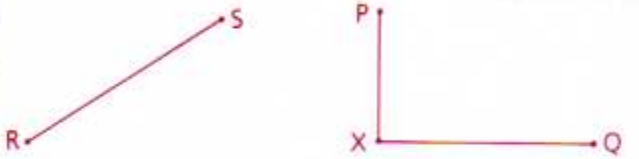
line ℓ $\overleftrightarrow{EF}$ $\overleftrightarrow{FE}$

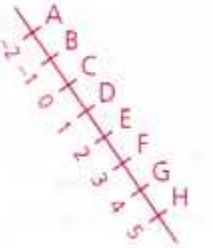
In algebra you learned that a ***number line*** is formed when a numerical value is assigned to each point on a line.

The coordinate of A is -2. The coordinate of B is $1\frac{1}{2}$.

Line Segments

The following diagram represents several ***line segments***, or simply ***segments***. Like lines, segments are made up of points and are straight. A segment, however, has a definite beginning and end.





A segment is named in terms of its two ***endpoints***

- The segment on the left can be called either $\overline{RS}$ or $\overline{SR}$.
- In the middle figure there are two segments. The vertical (up-and-down) segment can be called either $\overline{PX}$ or $\overline{XP}$. The horizontal (crosswise) segment can also be named in two ways. Can you name these two ways?
- How might we name the segment whose endpoints have coordinates 3 and 0 in the figure on the right?

Rays

In the diagram below, three ***rays*** are represented. Rays, like lines and segments, are made up of points and are straight. A ray differs from a line or a segment in that it begins at an endpoint and then extends infinitely far in *only one* direction.

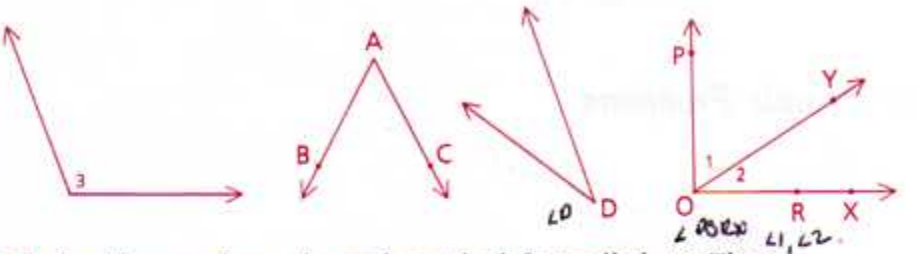
When we name a ray, we must name the endpoint first so that it is clear where the ray begins.

- The ray on the left is called $\overrightarrow{AB}$.
- The ray in the middle can be called $\overrightarrow{CD}$ or $\overrightarrow{CE}$. (As long as the endpoint is given first, any other point on the ray can be used in its name.)
- The ray on the right can be named in only one way. Do you know what its name is?

Angles

Two rays that have the same endpoint form an ***angle***.

Definition An ***angle*** is made up of two rays with a common endpoint. This point is called the ***vertex*** of the angle. The rays are called ***sides*** of the angle.



- In the diagram above, the angle on the left is called ∠3. The 3 placed inside the angle near the vertex names it.
- The second angle in the diagram can be called by any of three names.

  ∠BAC ∠CAB ∠A

  (Notice that when we use three letters, the vertex must be named in the middle.)
- The third angle is called ∠D.
- In the last figure above, there are three angles. Can you tell which angle is ∠O? Because names might refer to more than one angle in a diagram, we *never name an angle in a way that could result in confusion.*

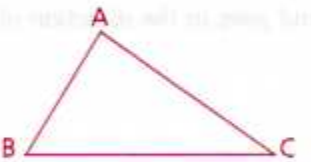
  ∠1 can also be called ∠POY or ∠YOP.

  ∠2 can also be called ∠YOR, ∠YOX, ∠ROY, or ∠XOY.

  The other angle in this figure can be named ∠POR. See if you can find three other names for this angle.

Triangles

We shall call the following figure ***triangle*** ABC (△ABC).



A triangle has three segments as its sides. You may wonder whether we can talk about an ∠B in the triangle, since there are no arrows in the diagram. The answer is yes. We shall often talk about rays, lines, and angles in a diagram of a triangle. So a triangle not only

has three sides but has three angles as well. Can you name the angle at the top of the triangle shown on the preceding page in three ways?

The triangle is the ***union*** (∪) of three segments.

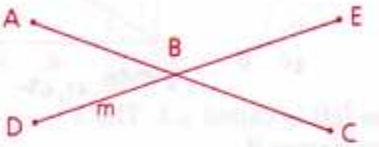
$$\triangle ABC = \overline{AB} \cup \overline{BC} \cup \overline{AC}$$

The ***intersection*** (∩) of any two sides is a ***vertex*** of the triangle.

$$\overline{AB} \cap \overline{BC} = B$$

Part Two: Sample Problems

Problem 1



a *How many lines are shown? (Imagine that there are arrows in the diagram.)*

b *Name these lines.*

c *Where do* $\overleftrightarrow{AC}$ *and* $\overleftrightarrow{DE}$ *intersect?*

d *Where does* $\overrightarrow{AC}$ *intersect* $\overleftrightarrow{BC}$? ($\overrightarrow{AC} \cap \overleftrightarrow{BC} =$ ___?___)

e *What is the union of* $\overrightarrow{BA}$ *and* $\overrightarrow{BD}$? ($\overrightarrow{BA} \cup \overrightarrow{BD} =$ ___?___)

Answers

a 2

b Line m, $\overleftrightarrow{DB}$, $\overleftrightarrow{DE}$, $\overleftrightarrow{BD}$, $\overleftrightarrow{BE}$, $\overleftrightarrow{EB}$, or $\overleftrightarrow{ED}$; $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{BA}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CA}$, or $\overleftrightarrow{CB}$

c B

d $\overrightarrow{AC}$ (Remember sets? If P and Q are two sets of points, then $P \cap Q$ = {all points in P *and* in Q}.)

e $\angle ABD$ ($P \cup Q$ = {all points in P *or* in Q or in both}.)

Problem 2

a *Name the ray that has endpoint A and goes in the direction of C.*

b *Name the segment joining A and B.*

Answers

a $\overrightarrow{AB}$ or $\overrightarrow{AC}$

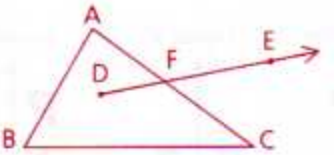
b $\overline{AB}$ or $\overline{BA}$

Problem 3 *Draw a diagram in which the intersection of* $\overrightarrow{AB}$ *with* $\overrightarrow{CA}$ *is* $\overline{AC}$ ($\overrightarrow{AB} \cap \overrightarrow{CA} = \overline{AC}$).

Solution

Problem 4 *Draw a diagram in which* $\triangle ABC \cap \overrightarrow{DE} = F$.

Solution



There are other correct answers, and a lot of wrong ones.

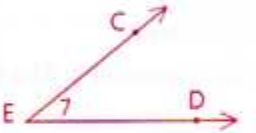
Part Three: Problem Sets

Problem Set A

In the back of the book, you will find answers to many of the problems. It will help you learn to check your answer in the back after you solve a problem. Then rethink your work if necessary.

1 What are three possible names for the line shown?

2 What are four possible names for the angle shown?



3 Can the ray shown be called $\overrightarrow{XY}$?

4 Name the sides of $\triangle RST$.

5 **a** $\overline{AB} \cap \overline{BC} =$?

b $\overrightarrow{EC} \cup \overrightarrow{EA} =$?

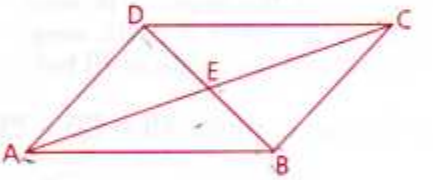
c $\overleftrightarrow{AC} \cap \overleftrightarrow{DB} =$?

d $\overline{DC} \cap \overline{AB} =$?

e $\overrightarrow{AC} \cap \overrightarrow{EC} =$?

f $\overrightarrow{BA} \cup \overrightarrow{BC} =$?

g $\overline{EC} \cup \overline{CB} \cup \overline{BE} =$?



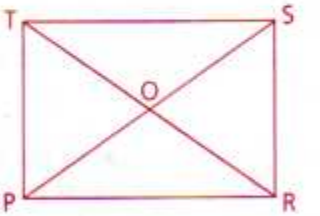
6 **a** Name $\angle OPR$ in all other possible ways.

b What is the vertex of $\angle TOS$?

c How many angles have vertex R?

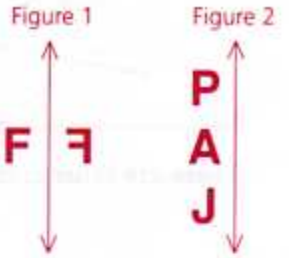
d Name $\angle TSP$ in all other possible ways.

e How many triangles are there in the figure?



Problem Set A, *continued*

7 Figure 1 shows the reflection of the letter *F* over a line. Copy Figure 2 and draw the reflections of the letters *P*, *A*, and *J* over the given line.



8 **a** A line is made up of ___?___.

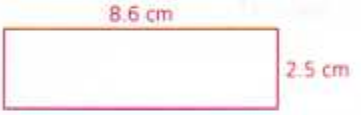
b An angle is the union of two ___?___ with a common ___?___.

9 Draw a number line and label points F, G, H, and J with the coordinates $-4\frac{2}{3}$, 2, 5, and 3.5 respectively. One of these points is the *midpoint* (the halfway point) between two others. Which is it?

10 Given a rectangle with sides 2.5 cm and 8.6 cm long, find

a The rectangle's area

b The rectangle's perimeter (the distance around it)



Problem Set B

11 **a** In $\triangle HJK$, $\overline{HJ}$ is twice as long as $\overline{JK}$ and exactly as long as $\overline{HK}$. If the length of $\overline{HJ}$ is 15, find the perimeter of (the distance around) $\triangle HJK$.

b If the length of $\overline{HJ}$ were $4x$, the length of $\overline{HK}$ were $3x$, the length of $\overline{JK}$ were $2x$, and the perimeter of $\triangle HJK$ were 63, what would the length of $\overline{HJ}$ be?

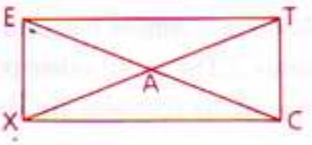
12 Draw a diagram in which $\overline{AB} \cap \overline{CD} = \overline{CB}$.

Problem Set C

13 Draw a diagram in which the intersection of $\angle AEF$ and $\angle DPC$ is $\overrightarrow{ED}$.

14 **a** What percentage of the triangles in the diagram have $\overline{CT}$ as a side?

b What percentage have $\overline{AC}$ as a side?



1.2 Measurement of Segments and Angles

Objectives

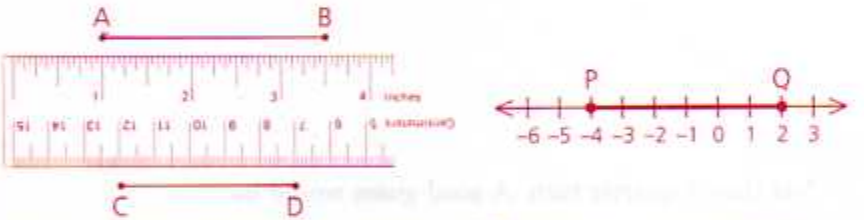
After studying this section, you will be able to

- Measure segments
- Measure angles
- Classify angles by size
- Name the parts of a degree
- Recognize congruent angles and segments

Part One: Introduction

Measuring Segments

We measure segments by using such instruments as rulers or metersticks. We may use any convenient length as a unit of measure. Some of the units that are currently in common use are inches, feet, yards, millimeters, centimeters, and meters. To indicate the measure of $\overline{AB}$, we write AB.

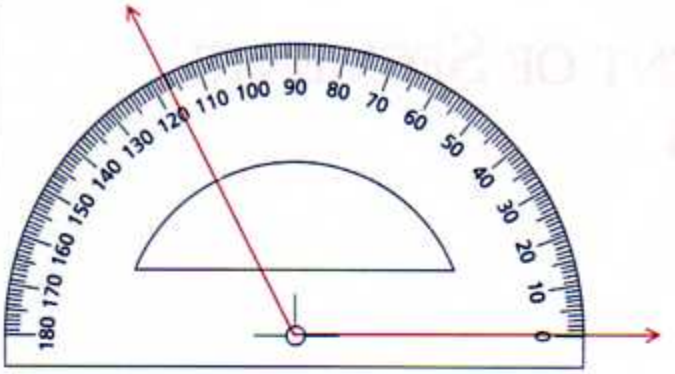


On the ruler shown, find the length of $\overline{AB}$ in inches and the length of $\overline{CD}$ in centimeters. On the number line, find PQ.

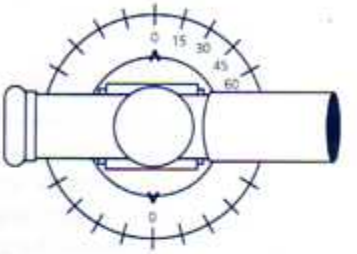
Measuring Angles

Angles are commonly measured by means of a ***protractor***. (The diagram at the top of the next page shows how a protractor can be used to measure a 117° angle.) We shall measure angles ($\angle$s) in ***degrees*** (°). In later courses, you may use other units, such as radians or grads.

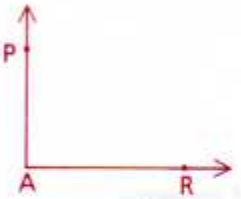
The ***measure***, or size, of an angle is the amount of turning you would do if you were at the vertex, looking along one side, and then turned to look along the other side. (A surveyor's transit works in much the same way.)



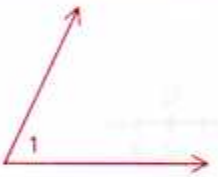
Surveyor's Transit



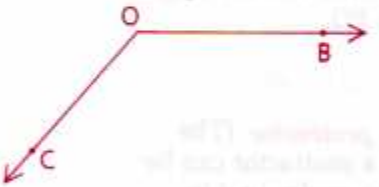
If you turned all the way around (to face your starting direction), you would turn 360°. You can use this fact to estimate the size of an angle.



$\overrightarrow{AP}$ appears to have been turned one fourth of the way around from $\overrightarrow{AR}$, so you might guess that $\angle A$ is approximately a 90° angle.



Angle 1 required less than a quarter turn. A good guess would be that it is a 60° angle.



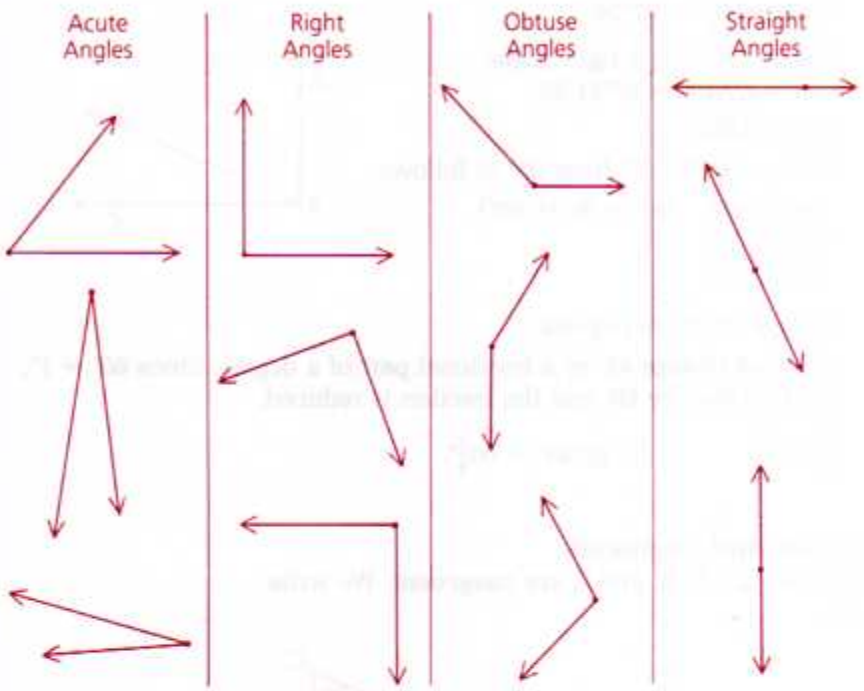
Angle BOC required more than a quarter turn, so its size could be estimated at 130°.

Some math courses deal with negative angles, zero angles, and angles greater than 180°. In this course, you will usually be working with angles greater than 0° and less than or equal to 180°.

$$0 < \text{angle measure} \leq 180$$

Classifying Angles by Size

As shown below, we classify angles into four categories according to their measures.



Definitions

An ***acute angle*** is an angle whose measure is greater than 0 and less than 90.

A ***right angle*** is an angle whose measure is 90.

An ***obtuse angle*** is an angle whose measure is greater than 90 and less than 180.

A ***straight angle*** is an angle whose measure is 180. (As you can see, a straight angle forms a straight line.)

Parts of a Degree

As you know, each hour of the day is divided into 60 minutes, and each minute is divided into 60 seconds. Similarly, each degree (°) of an angle is divided into 60 ***minutes*** (′), and each minute of an angle is divided into 60 ***seconds*** (″).

$60' = 1^\circ$ (60 minutes equals 1 degree.)
$60'' = 1'$ (60 seconds equals 1 minute.)

Thus, $87\frac{1}{2}^\circ = 87^\circ 30'$
$60.4^\circ = 60^\circ 24'$
$90^\circ = 89^\circ 60'$ (since $60' = 1^\circ$)
$180^\circ = 179^\circ 59' 60''$ (since $60'' = 1'$ and $60' = 1^\circ$)

Study the following examples closely.

Example 1 *Change* $41\frac{2}{5}°$ *to degrees and minutes.*

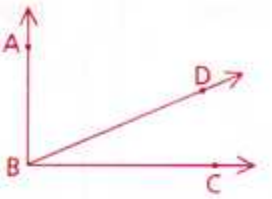
Since there are 60′ in 1°, $\frac{2}{5}°$ is $\frac{2}{5}(60)$ minutes, or 24′.

Hence, $41\frac{2}{5}° = 41°24'$.

Example 2 *Given:* $\angle ABC$ *is a right angle.*

$\angle ABD = 67°21'37''$

Find: $\angle DBC$



Subtract 67°21′37″ from 90° as follows.

$$\begin{array}{r} 89°59'60'' \quad (90° = 89°59'60'') \\ -\,67°21'37'' \\ \hline 22°38'23'' \end{array}$$

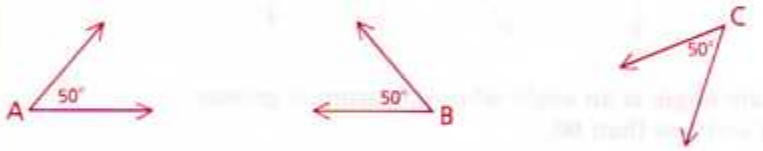
Example 3 *Change* 60°45′ *to degrees.*

We must change 45′ to a fractional part of a degree. Since 60′ = 1°, 45 is divided by 60, and the fraction is reduced.

$\frac{45}{60} = \frac{3}{4}$ So $60°45' = 60\frac{3}{4}°$.

Congruent Angles and Segments

In the diagram below, $\angle$s A, B, and C are ***congruent***. We write $\angle A \cong \angle B \cong \angle C$.



Definition ***Congruent*** (≅) ***angles*** are angles that have the same measure.

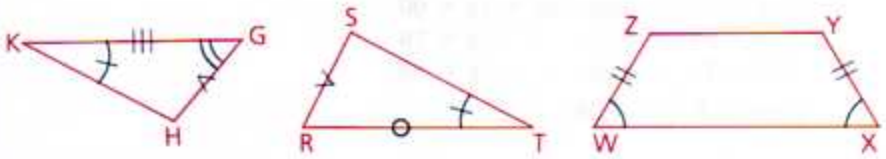
In a similar way, segments can be congruent.

Definition ***Congruent*** (≅) ***segments*** are segments that have the same length.

In the diagram above, segments $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ are congruent. We write $\overline{AB} \cong \overline{CD} \cong \overline{EF}$.

Often, we use identical ***tick marks*** to indicate congruent angles and segments. In the following diagram, the identical tick marks

indicate that there are four pairs of congruent parts. Can you name them?



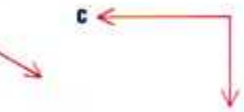
Part Two: Sample Problems

Problem 1 *Classify each of the angles below as acute, right, or obtuse. Then estimate the number of degrees in the angle.*

a

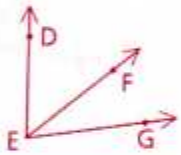
b

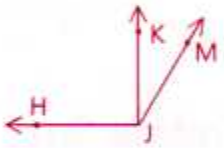
c



Answers **a** Acute; 40° **b** Obtuse; 150° **c** Right; 90°

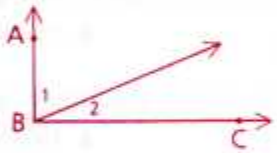
Problem 2 *In the diagram below, ∠DEG = 80°, ∠DEF = 50°, ∠HJM = 120°, and ∠HJK = 90°. Draw a conclusion about ∠FEG and ∠KJM.*





Solution ∠FEG = 30° and ∠KJM = 30°, so ∠FEG ≅ ∠KJM.

Problem 3 *Given:* ∠ABC *is a right angle.*
$\angle 1 = (3x + 4)°$,
$\angle 2 = (x + 6)°$
Find: m∠1 *(the measure of* ∠1*)*

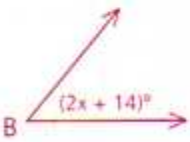


Solution Since ∠ABC is a right ∠, m∠1 + m∠2 = 90.

$$(3x + 4) + (x + 6) = 90$$
$$4x + 10 = 90$$
$$4x = 80$$
$$x = 20$$

Since $m\angle 1 = 3x + 4$, $m\angle 1 = 3(20) + 4$, or 64.

Problem 4 ∠B *is acute.*
a *What are the restrictions on* m∠B?
b *What are the restrictions on* x?



Solution

a Since $\angle B$ is acute, $m\angle B > 0$ and $m\angle B < 90$ $(0 < m\angle B < 90)$.

b
$$\begin{array}{lll} 2x + 14 > 0 & \text{and} & 2x + 14 < 90 \\ 2x > -14 & \text{and} & 2x < 76 \\ x > -7 & \text{and} & x < 38 \end{array}$$

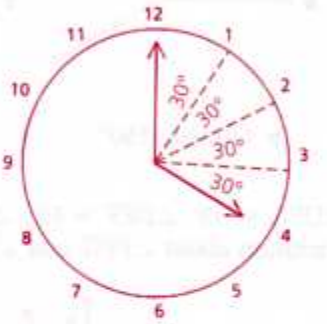
Thus, $-7 < x < 38$.

Problem 5 *Find the angle formed by the hands of a clock at each time.*

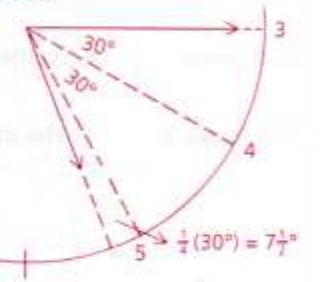
a 4:00 **b** 5:15

Solution

a Since 360° is divided into 12 intervals on a clock, each interval is 30°. From 12 to 4 there are 4 intervals, so the angle is 4(30°), or 120°.



b Remember that the hour hand is on 5 only when the minute hand is on 12. At 5:15 the hour hand is one fourth of the way from 5 to 6. Since $\frac{1}{4}(30°) = 7\frac{1}{2}°$, the hands form an angle of $60 + 7\frac{1}{2}$, or $67\frac{1}{2}$ degrees.



Part Three: Problem Sets

Problem Set A

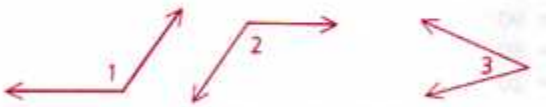
1 Change each of the following to degrees and minutes.

a $61\frac{2}{3}°$ **b** 71.7°

2 Change each of the following to degrees.

a 132°30′ **b** 19°45′

3 Which two of the angles below *appear* to be congruent?



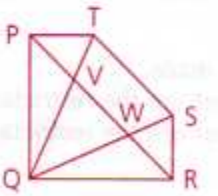
4 **a** $\overrightarrow{QV} \cap \overleftrightarrow{TS} =$?

b $\overline{WP} \cap \overline{VR} =$?

c $\overrightarrow{WP} \cup \overrightarrow{VR} =$?

d $\overrightarrow{SQ} \cup \overrightarrow{SR} =$?

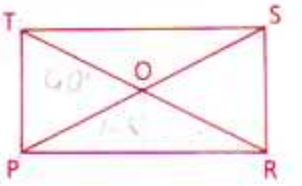
e How many angles have vertex Q?



5 **a** Evaluate 49°32′55″ + 37°27′15″.

b Evaluate 123°15′ − 40°26′.

6 There is a right angle at each corner of PRST. (Later in the course you will learn that PRST is a rectangle.)



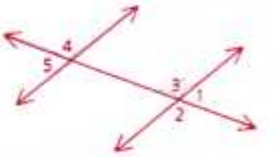
a If ∠TPO = 60°, how large is ∠RPO?

b If ∠PTO = 70°, how large is ∠STO?

c If ∠TOP = 50°, how large is ∠POR?

d Classify ∠TOS as acute, right, or obtuse.

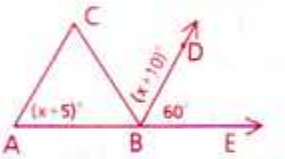
7 **a** Which angle *appears* to have the same measure as ∠1?



b Which angle *appears* larger, ∠2 or ∠3?

c Does ∠3 *appear* to be congruent to ∠4 or to ∠5?

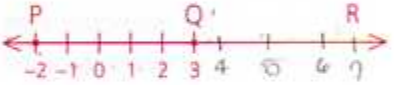
8 If ∠CBD ≅ ∠DBE, find m∠A.



9 Find the measure of the angle formed by the hands of a clock at each time.

a 3:00 **b** 4:30 **c** 7:20 **d** 1:45

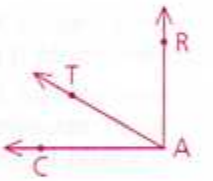
10 **a** Find PQ.



b If R's coordinate is 7, why is $\overline{PQ} \not\cong \overline{QR}$?

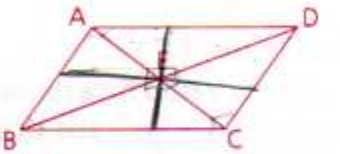
c What must the coordinate of R be in order for Q to be the midpoint of $\overline{PR}$?

11 Given: ∠CAR is a right angle.
m∠CAT = 37°66′10″
Find: m∠RAT



Problem Set B

12 **a** How many triangles (△) are in the diagram?



b How many angles (∠s) in the figure *appear* to be right?

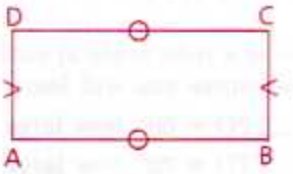
c How many angles in the figure *appear* to be acute?

d How many angles in the figure *appear* to be obtuse?

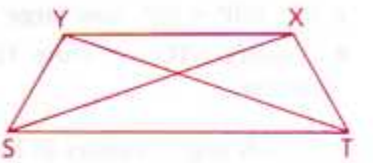
e Name the straight angles in the figure.

Problem Set B, *continued*

13 The perimeter of (the distance around) ABCD is 66, and $\overline{DC}$ is twice as long as $\overline{CB}$. How long is $\overline{AB}$?

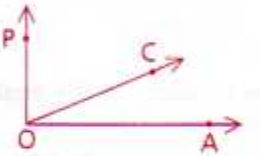


14 Given: $\overline{XS} \cong \overline{YT}$, $\overline{YS} \cong \overline{XT}$,
$XT = 2r + 5$,
$XS = 3m + 7$,
$YS = 3\frac{1}{2}r + 2$,
$YT = 4.2m + 5$
Solve for r and m.



15 Given: $\angle 1 \cong \angle 2$,
$m\angle 1 = x + 14$,
$m\angle 2 = y - 3$
Solve for y in terms of x.

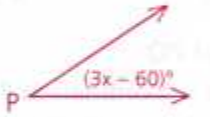
16 If $\angle POA$ is a right angle and if $\angle POC$ is three times as large as $\angle COA$, find $m\angle POC$.



17 $\angle P$ is acute.

a What are the restrictions on $m\angle P$?

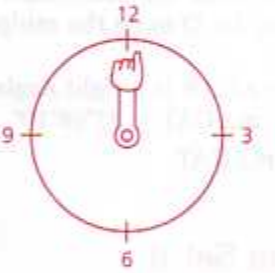
b What are the restrictions on x?



18 The hand is at 12 on the clock.

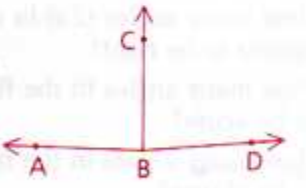
a If the hand were rotated 90° clockwise, at what number would it point?

b If the hand were rotated 150° clockwise and then 30° counterclockwise, at what number would it point?



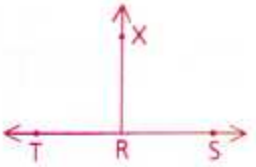
Problem Set C

19 $\angle ABC$ and $\angle CBD$ have the same measure.
If $\angle ABC = \left(\frac{3x}{2} + 2\right)°$ and $\angle CBD = \left(2x - 29\frac{1}{4}\right)°$,
is $\angle ABD$ a straight angle?

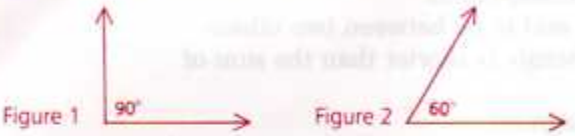


20 Change $15\frac{2}{9}°$ to degrees, minutes, and seconds.

21 Given: $\angle TRS$ is a straight angle.
$\angle TRX$ is a right angle.
$m\angle TRS = 2x + 5y$,
$m\angle XRS = 3x + 3y$
Solve for x and y.



22 Maxie and Minnie were taking a stroll in the Arizona desert when a spaceship from Mars landed. A Martian walked up to them and pointed to Figure 1. "XLr8r, XLr8r, XLr8r plus YBcaws, YBcaws," she said. Pointing to Figure 2, she said, "YBcaws plus XLr8r, XLr8r, XLr8r." What might *XLr8r* mean?



23 Change 72°22′30″ to degrees.

MATHEMATICAL EXCURSION

GEOMETRY IN NATURE

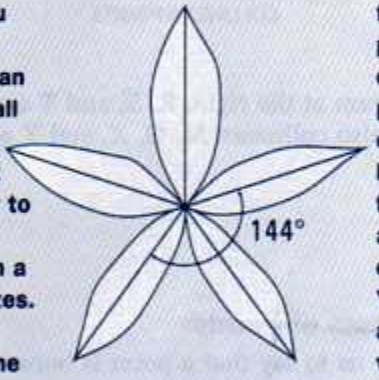
Orange sections and spiraling leaves

If you cut a cross section of an orange, you will see that it is divided into sections that together form a 360° angle. The mathematician Johannes Kepler (1571–1630) thought that all fruits and flowers that grew on trees had five sections or petals. You can see that this isn't true, but the sections of an orange do appear to be the same size and shape.

Flower petals, and leaves on stems grow in a spiral pattern and form angles of consistent sizes.

***Phyllotaxis* is the distribution of leaves around the stem of a plant. The measure of the angle formed by any two leaves in succession on a stem is equal to the measure of the angle between any two other leaves in succession.**

The most common angles seem to be 144° and 135°. A 144° angle is characteristic for rose leaves. Suppose you draw a series of 144° angles with a protractor, using one of the sides of the last angle you drew for each new angle and proceeding in a clockwise direction. You will see that the angles eventually divide a circle into five equal parts.



Botanists say that these angles exist because each bud grows where it will have the most room between the bud before it and the one that will come after it.

1.3 COLLINEARITY, BETWEENNESS, AND ASSUMPTIONS

Objectives

After studying this section, you will be able to

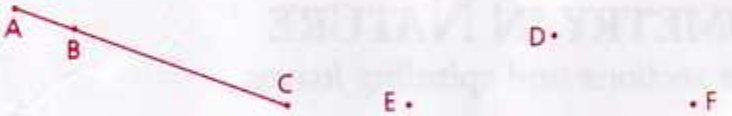
- Recognize collinear and noncollinear points
- Recognize when a point can be said to be between two others
- Recognize that each side of a triangle is shorter than the sum of the other two sides
- Correctly interpret geometric diagrams

Part One: Introduction

Collinearity

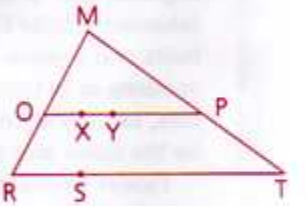
It is often useful to know that a group of points lie on the same line.

Definition Points that lie on the same line are called ***collinear.*** Points that do not lie on the same line are called ***noncollinear.***



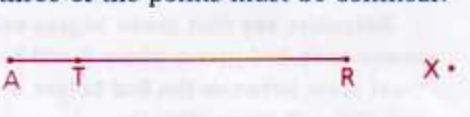
COLLINEAR POINTS NON-COLLINEAR POINTS

In the diagram at the right, R, S, and T are collinear points. P, O, and X are also collinear. M, O, X, and Y are noncollinear.



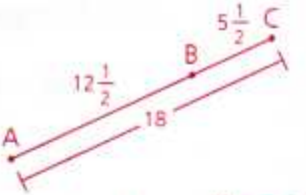
Betweenness of Points

In order for us to say that a point is between two other points, all three of the points must be collinear.



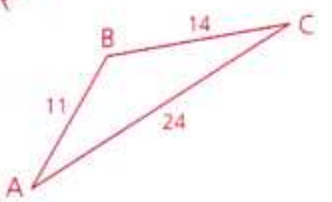
T is between A and R We do *not* say O is between X and Y

Triangle Inequality



For any three points, there are only two possibilities:

1. They are collinear. (One point is between the other two. Two of the distances add up to the third.)
2. They are noncollinear. (The three points determine a triangle.)



Notice that in the triangle, $14 + 11 > 24$. This is an example of an important characteristic of triangles: *The sum of the lengths of any two sides of a triangle is always greater than the length of the third.*

Assumptions from Diagrams

You may wonder what you should and should not assume when you look at a diagram. The chart below gives the general rules you should follow as you work with this book. (There are, however, occasional exceptions, as in Section 1.2, problem 19.)

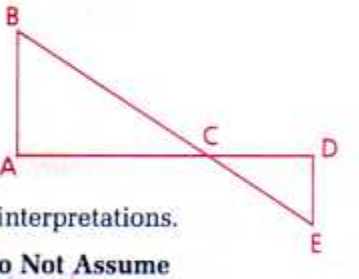
How to Interpret a Diagram

You Should Assume	You Should Not Assume
Straight lines and angles	Right angles
Collinearity of points	Congruent segments
Betweenness of points	Congruent angles
Relative positions of points	Relative sizes of segments and angles

The following example will help you understand what assumptions can be made.

Example *Given: Diagram as shown*

Question: What should we assume?



The following are some of the many valid interpretations.

Do Assume	Do Not Assume
$\overleftrightarrow{ACD}$ and $\overleftrightarrow{BCE}$ are straight lines.	$\angle BAC$ is a right $\angle$.
$\angle BCE$ is a straight angle.	$\overline{CD} \cong \overline{DE}$
C, D, and E are noncollinear.	$\angle B \cong \angle E$
C is between B and E.	$\angle CDE$ is an obtuse angle.
E is to the right of A.	$\overline{BC}$ is longer than $\overline{CE}$.

Reread and study the chart and the example carefully, for it is important that you know what to assume from a diagram.

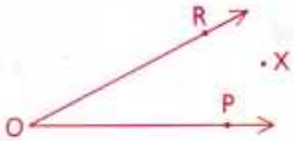
Part Two: Sample Problems

Problem 1 *For each diagram, tell whether X is between P and R. (Answer Yes or No.)*

a

b

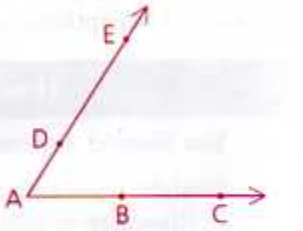
c



Answers **a** Yes **b** No **c** No

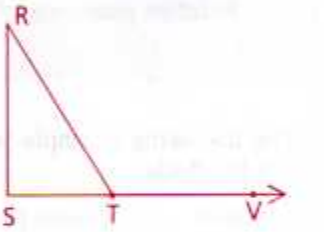
Problem 2 *Draw a diagram in which A, B, and C are collinear, A, D, and E are collinear, and B, C, and D are noncollinear.*

Solution The diagram at the right shows one of the possible solutions.



Problem 3 **a** *Should we assume that S, T, and V are collinear in the diagram?*

b *Should we assume that* $\angle S = 90°$?



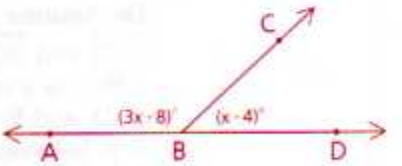
Answers **a** Yes

b No

Part Three: Problem Sets

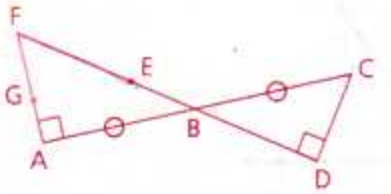
Problem Set A

1 Find $m\angle ABC$ (the measure of $\angle ABC$).



2 Draw a diagram showing four points, no three of which are collinear.

3



a Name all points collinear with E and F.

b Are G, E, and D collinear? Are F and C collinear?

c Which two segments do the tick marks indicate are congruent?

d Is $\angle A \cong \angle D$?

e Is $\angle F \cong \angle ABF$?

f Where do $\overleftrightarrow{AC}$ and $\overleftrightarrow{FE}$ intersect?

g $\overline{AG} \cap \overline{GF} =$?

h $\overline{AG} \cup \overline{GF} =$?

i B lies on a ray whose endpoint is E. Name this ray in all possible ways.

j Name all points between F and D.

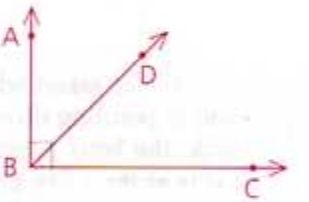
4

a Should we assume that angles E, F, G, and H are right angles? Explain your answer.

b Should we assume that points E, F, and G are noncollinear? Explain your answer.

5 Draw a number line and shade all points that are at or between -5 and 2. Find the length of this shaded segment.

6 $\angle ABC$ is a right angle. The ratio of the measures of $\angle ABD$ and $\angle DBC$ is 3 to 2. Find $m\angle ABD$. (Hint: Let $m\angle ABD = 3x$ and $m\angle DBC = 2x$.)



7 Explain how the sum of two acute angles could be

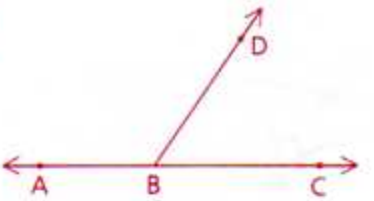
a Acute **b** Obtuse **c** Right

8 **a** Change $124\frac{3}{5}°$ to degrees and minutes.

b Change 84°50′ to degrees.

Problem Set A, *continued*

9 $\angle ABD = (3x)°$
$\angle DBC = x°$
Find: $m\angle ABD$



Problem Set B

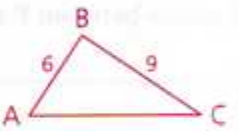
10 A, K, O, and Y are collinear points. K is between O and A, the length of $\overline{AO}$ added to the length of $\overline{AY}$ is equal to the length of $\overline{OY}$ (OA + AY = OY), and A is to the right of O. Draw a diagram that correctly represents this information.

11 Draw a diagram in which F is between A and E, F is also between R and S, and A, E, R, and S are noncollinear.

12 If AB = 16, BC = 8, and AC = 24, which point is between the other two?

13 **a** AC must be smaller than what number?

b AC must be larger than what number?



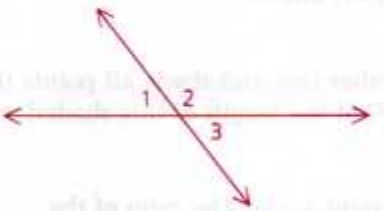
14 Q is between P and R on a number line. P = −8, and R = 4.

a What do we know about the coordinate of Q?

b What do we know about the length PQ + QR?

Problem Set C

15 Given: $m\angle 1 = 2x + 40$,
$m\angle 2 = 2y + 40$,
$m\angle 3 = x + 2y$
Find: $m\angle 1$, $m\angle 2$, and $m\angle 3$



16 When Brock Clock was asked what time it was, he said, "Well, the minute hand is pointing directly at one of the twelve numbers on the clock, the hour hand is pointing toward a spot whose nearest number is at least five greater than the number the minute hand is pointing toward, the angle formed by the hands is acute, the sun is shining in the east, and it is not five minutes past the hour." Wow! What time was it?

17 To the nearest second, what is the first time after 12:00 that the hour hand and the minute hand of a clock are together?

1.4 BEGINNING PROOFS

Objective

After studying this section, you will be able to

- Write simple two-column proofs

Part One: Introduction

Much of the enjoyment and challenge of geometry is found in "proving things." In this section, we shall give examples of ***two-column proofs***. The two-column proof is the major type of proof you will use as you study this book.

We shall also introduce our first ***theorems***.

Definition A ***theorem*** is a mathematical statement that can be proved.

This section also illustrates a procedure that we shall use numerous times in this textbook:

Theorem Procedure

1. We present a theorem or theorems.
2. We prove the theorem(s).

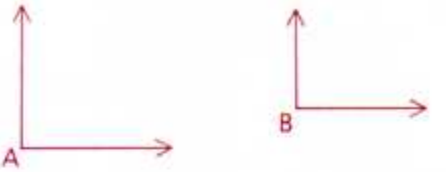
   Note Although all theorems presented can be proved, we shall omit the proofs of certain theorems.

3. We use the theorems to help prove sample problems.
4. You are then given the challenge of using the theorems to prove homework problems. Theorems will save you much time if you learn them and then use them.

We now present our first two theorems.

Theorem 1 ***If two angles are right angles, then they are congruent.***

Given: $\angle A$ is a right $\angle$.
$\angle B$ is a right $\angle$.
Prove: $\angle A \cong \angle B$

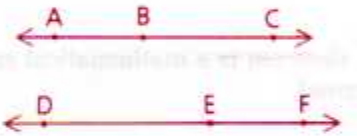


Proof:

Statements	Reasons
1 $\angle A$ is a right angle.	1 Given
2 $m\angle A = 90$	2 If an angle is a right angle, then its measure is 90.
3 $\angle B$ is a right angle.	3 Given
4 $m\angle B = 90$	4 Same as 2
5 $\angle A \cong \angle B$	5 If two angles have the same measure, then they are congruent. (See steps 2 and 4.)

Theorem 2 ***If two angles are straight angles, then they are congruent.***

Given: $\angle ABC$ is a straight angle.
$\angle DEF$ is a straight angle.
Prove: $\angle ABC \cong \angle DEF$



Proof:

Statements	Reasons
1 $\angle ABC$ is a straight angle.	1 Given
2 $m\angle ABC = 180$	2 If an angle is a straight angle, then its measure is 180.
3 $\angle DEF$ is a straight angle.	3 Given
4 $m\angle DEF = 180$	4 Same as 2
5 $\angle ABC \cong \angle DEF$	5 If two angles have the same measure, then they are congruent. (See steps 2 and 4.)

Now that we have presented and proved two theorems, we are ready to use them to help prove some sample problems.

We will use the theorems themselves as reasons in our proofs. You should also use the theorems as reasons in your homework problems.

Remember, the purpose of a theorem is to shorten your work. Therefore, when doing homework problems, do not use the proofs of theorems as a guide. Use the sample problems as a guide.

Part Two: Sample Problems

Problem 1

Given: $\angle A$ *is a right angle.*
$\angle C$ *is a right angle.*
Conclusion: $\angle A \cong \angle C$

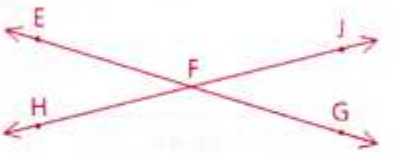
Proof

Statements	Reasons
1 $\angle A$ is a right angle.	1 Given
2 $\angle C$ is a right angle.	2 Given
3 $\angle A \cong \angle C$	3 If two angles are right angles, then they are congruent.

You probably recognize that reason 3 is Theorem 1. Although it may seem easier merely to write "Theorem 1," *do not do so!* Eventually, such a shortcut would make it harder for you to learn the concepts of geometry.

Problem 2

Given: Diagram as shown
Conclusion: $\angle EFG \cong \angle HFJ$

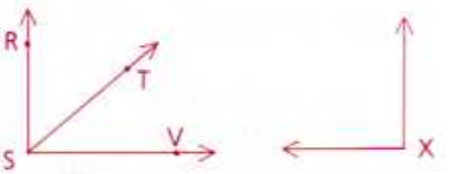


Proof

Statements	Reasons
1 Diagram as shown	1 Given
2 $\angle EFG$ is a straight angle.	2 Assumed from diagram
3 $\angle HFJ$ is a straight angle.	3 Assumed from diagram
4 $\angle EFG \cong \angle HFJ$	4 If two angles are straight angles, then they are congruent.

Problem 3

Given: $\angle RST = 50°$,
$\angle TSV = 40°$;
$\angle X$ *is a right angle.*
Prove: $\angle RSV \cong \angle X$



Proof

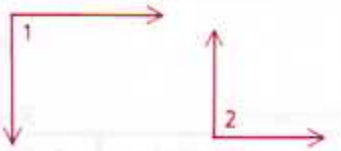
Statements	Reasons
1 $\angle RST = 50°$	1 Given
2 $\angle TSV = 40°$	2 Given
3 $\angle RSV = 90°$	3 Addition ($50° + 40° = 90°$)
4 $\angle RSV$ is a right angle.	4 If an angle is a 90° angle, it is a right angle.
5 $\angle X$ is a right angle.	5 Given
6 $\angle RSV \cong \angle X$	6 If two angles are right angles, then they are congruent.

Part Three: Problem Sets

Problem Set A

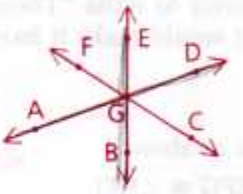
In problems 1 and 2, copy the figure and the incomplete proof. Then complete the proof by filling in the missing reasons.

1 Given: $\angle 1$ is a right $\angle$.
$\angle 2$ is a right $\angle$.
Prove: $\angle 1 \cong \angle 2$



Statements	Reasons
1 $\angle 1$ is a right angle.	1 _____
2 $\angle 2$ is a right angle.	2 _____
3 $\angle 1 \cong \angle 2$	3 _____

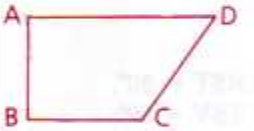
2 Given: Diagram as shown
Prove: $\angle AGD \cong \angle EGB$



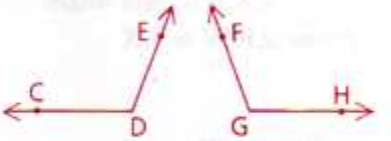
Statements	Reasons
1 Diagram as shown	1 _____
2 $\angle AGD$ is a straight angle.	2 _____
3 $\angle EGB$ is a straight angle.	3 _____
4 $\angle AGD \cong \angle EGB$	4 _____

In problems 3–7, use the two-column form of proof.

3 Given: $\angle A$ is a right angle.
$\angle B$ is a right angle.
Prove: $\angle A \cong \angle B$

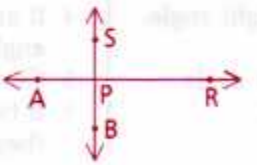


4 Given: $\angle CDE = 110°$,
$\angle FGH = 110°$
Conclusion: $\angle CDE \cong \angle FGH$



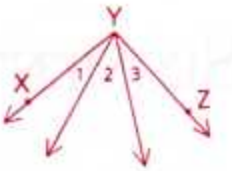
5 Given: JK = 2.5 cm, NO = 2.5 cm
Conclusion: $\overline{JK} \cong \overline{NO}$

6 Given: Diagram as shown
Prove: $\angle APR \cong \angle SPB$



7 Given: $\angle 1 = 20°$,
$\angle 2 = 40°$,
$\angle 3 = 30°$

Prove: $\angle XYZ$ is a right angle.

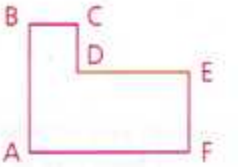


8 Draw the figure ABCDEF.

a Draw its reflection over $\overleftrightarrow{AF}$.

b Draw its reflection over $\overleftrightarrow{AB}$.

c Draw a 90° clockwise rotation of the figure about B.



9 Find the angle formed by the hands of a clock at 11:40.

10 The square has a perimeter of 42.

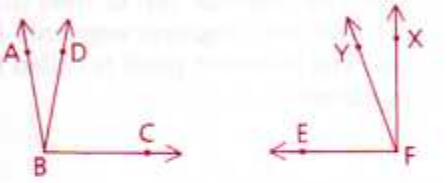
a Solve for x.

b If the perimeter were greater than 42, what would we know about the value of x?

Problem Set B

11 Given: $\angle ABD = 10°$,
$\angle ABC = 100°$,
$\angle EFY = 70°20'$,
$\angle XFY = 19°40'$

Prove: $\angle DBC \cong \angle XFE$



12 Point P has a coordinate of 7 on a number line. If you "slide" P 15 units in the negative direction, what are the coordinates of the resulting point P′?

13 **a** Draw a number line, labeling points A = (−1) and B = (5). Then label point A′, the reflection of A over B.

b Does AB = BA′?

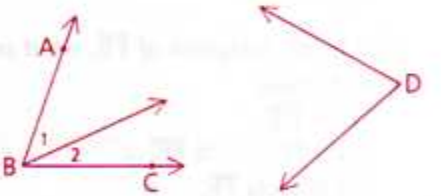
c What do we know about point B?

Problem Set C

14 The measure of an obtuse angle is $5y + 45$. What are the restrictions on y?

15 Given: $\angle 1 = (x + 7)°$,
$\angle 2 = (2x - 3)°$,
$\angle ABC = (x^2)°$,
$\angle D = (5x - 4)°$

Show that $\angle ABC \cong \angle D$.



1.5 Division of Segments and Angles

Objectives

After studying this section, you will be able to

- Identify midpoints and bisectors of segments
- Identify trisection points and trisectors of segments
- Identify angle bisectors
- Identify angle trisectors

Part One: Introduction

Midpoints and Bisectors of Segments

We shall often work with segments that are divided in half.

Definition A point (or segment, ray, or line) that divides a segment into two congruent segments ***bisects*** the segment. The bisection point is called the ***midpoint*** of the segment.

X is not a midpoint Y is not a midpoint

Only segments have midpoints. It does not make sense to say that a ray or a line has a midpoint. Do you understand why?

How many midpoints does $\overline{PQ}$ have?

How many bisectors could $\overline{PQ}$ have?

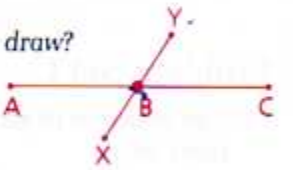
Study the following examples.

Example 1 *If $\overline{XY}$ bisects $\overline{AC}$ at B, what conclusions can we draw?*

Conclusions:

B is the midpoint of $\overline{AC}$.

$\overline{AB} \cong \overline{BC}$



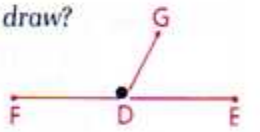
Example 2 *If D is the midpoint of $\overline{FE}$, what conclusions can we draw?*

Conclusions:

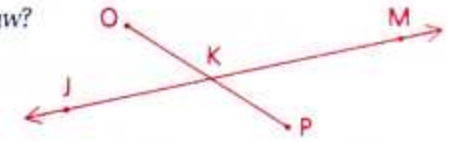
$\overline{FD} \cong \overline{DE}$

Point D bisects $\overline{FE}$.

$\overline{DG}$ bisects $\overline{FE}$.



Example 3 *If* $\overline{OK} \cong \overline{KP}$, *what conclusions can we draw?*



Conclusions:

K is the midpoint of $\overline{OP}$.

$\overleftrightarrow{JM}$ is a bisector of $\overline{OP}$.

Point K bisects $\overline{OP}$.

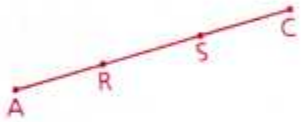
Trisection Points and Trisecting a Segment

A segment divided into *three* congruent parts is said to be ***trisected***.

Definition Two points (or segments, rays, or lines) that divide a segment into three congruent segments ***trisect*** the segment. The two points at which the segment is divided are called the ***trisection points*** of the segment.

Again, only segments have trisection points; rays and lines do not have trisection points.

Example 1 *If* $\overline{AR} \cong \overline{RS} \cong \overline{SC}$, *what conclusions can we draw?*

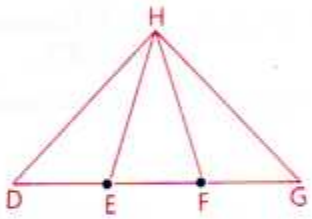


Conclusions:

R and S are trisection points of $\overline{AC}$.

$\overline{AC}$ is trisected by R and S.

Example 2 *If* E *and* F *are trisection points of* $\overline{DG}$, *what conclusions can we draw?*



Conclusions:

$\overline{DE} \cong \overline{EF} \cong \overline{FG}$

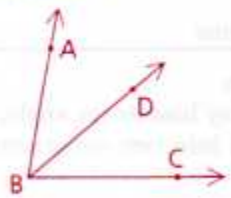
$\overline{HE}$ and $\overline{HF}$ are trisectors of $\overline{DG}$.

Angle Bisectors

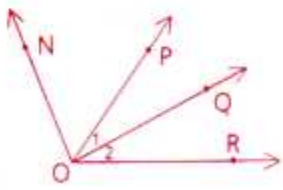
An angle, like a segment, can be bisected.

Definition A ray that divides an angle into two congruent angles ***bisects*** the angle. The dividing ray is called the ***bisector*** of the angle.

If $\angle ABD \cong \angle DBC$, then $\overrightarrow{BD}$ (not $\overrightarrow{DB}$) is the bisector of $\angle ABC$.



If $\angle NOP \cong \angle POR$ and $\overrightarrow{OQ}$ bisects $\angle POR$, then $\overrightarrow{OP}$ (not $\overrightarrow{PO}$) is the bisector of $\angle NOR$, and $\angle 1 \cong \angle 2$.

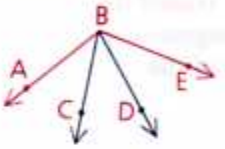


Angle Trisectors

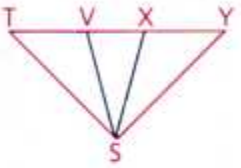
Two rays can divide an angle into three equal parts.

Definition Two rays that divide an angle into three congruent angles ***trisect*** the angle. The two dividing rays are called ***trisectors*** of the angle.

If $\angle ABC \cong \angle CBD \cong \angle DBE$, then $\overrightarrow{BC}$ and $\overrightarrow{BD}$ trisect $\angle ABE$.

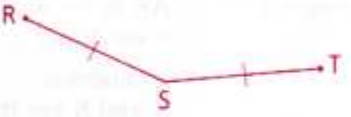


If $\overrightarrow{SV}$ and $\overrightarrow{SX}$ are trisectors of $\angle TSY$, then $\angle TSV \cong \angle VSX \cong \angle XSY$.



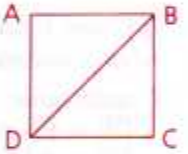
Part Two: Sample Problems

Problem 1 *The tick marks indicate that* $\overline{RS} \cong \overline{ST}$. *Is S the midpoint of* $\overline{RT}$?



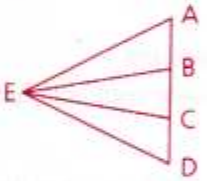
Answer No, the points are not collinear.

Problem 2 *If* $\overrightarrow{BD}$ *bisects* $\angle ABC$, *does* $\overrightarrow{DB}$ *bisect* $\angle ADC$?



Answer No. We need more information.

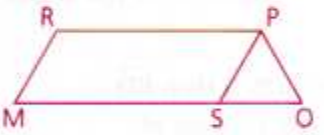
Problem 3 *If* B *and* C *trisect* $\overline{AD}$, *do* $\overrightarrow{EB}$ *and* $\overrightarrow{EC}$ *trisect* $\angle AED$?



Answer No! It is true that $\overline{AB} \cong \overline{BC} \cong \overline{CD}$, but the fact that the *segment* has been trisected does not mean that the *angle* has been trisected.

Problem 4 *Given:* $\overrightarrow{PS}$ *bisects* $\angle RPO$.

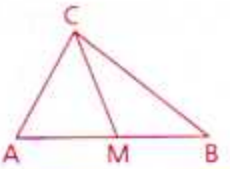
Prove: $\angle RPS \cong \angle OPS$



Proof

Statements	Reasons
1 $\overrightarrow{PS}$ bisects $\angle RPO$.	1 Given
2 $\angle RPS \cong \angle OPS$	2 If a ray bisects an angle, it divides the angle into two congruent angles.

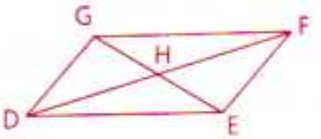
Problem 5 Given: $\overleftrightarrow{CM}$ bisects $\overline{AB}$ (*In Chapter 3 we shall call* $\overline{CM}$ *a* median *of the triangle.*)
Conclusion: $\overline{AM} \cong \overline{MB}$



Proof

Statements	Reasons
1 $\overleftrightarrow{CM}$ bisects $\overline{AB}$.	1 Given
2 $\overline{AM} \cong \overline{MB}$	2 If a line bisects a segment, it divides the segment into two congruent segments.

Problem 6 Given: $\overline{DH} \cong \overline{HF}$
Prove: H *is the midpoint of* $\overline{DF}$.



Proof

Statements	Reasons
1 $\overline{DH} \cong \overline{HF}$	1 Given
2 H is the midpoint of $\overline{DF}$.	2 If a point divides a segment into two congruent segments, it is the midpoint of the segment.

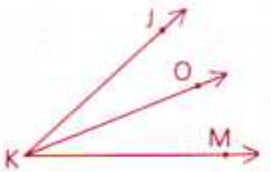
Problem 7 $\overline{EH}$ *is divided by* F *and* G *in the ratio* 5:3:2 *from left to right. If* EH = 30, *find* FG *and name the midpoint of* $\overline{EH}$.

Solution According to the ratio, we can let EF = 5x, FG = 3x, and GH = 2x. First we draw a diagram and place the algebra on it as part of the solution.

$$5x + 3x + 2x = 30$$
$$10x = 30$$
$$x = 3$$

Thus, FG = 3(3), or 9. Since EF = 15 and FH = 15, F is the midpoint of $\overline{EH}$.

Problem 8 Given: $\overrightarrow{KO}$ bisects $\angle JKM$.
$\angle JKM = 41°37'$
Find: $m\angle OKM$



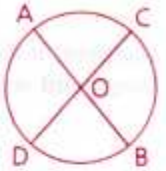
Solution
$$\tfrac{1}{2}(41°37') = 20\tfrac{1}{2}°18\tfrac{1}{2}'$$
$$= 20°48\tfrac{1}{2}' \quad (\text{since } \tfrac{1}{2}° = 30')$$
$$= 20°48'30'' \quad (\text{since } \tfrac{1}{2}' = 30'')$$

Part Three: Problem Sets

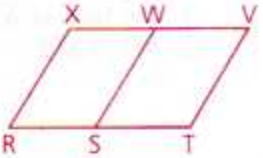
Problem Set A

1 Name the congruent segments.

a O is the midpoint of $\overline{CD}$.

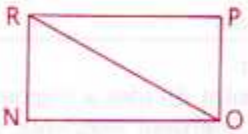


b $\overline{SW}$ bisects $\overline{XV}$.

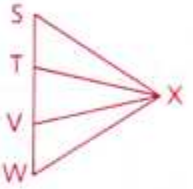


2 Name the congruent angles.

a $\overrightarrow{RO}$ bisects ∠NRP.

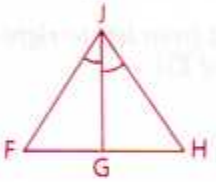


b $\overrightarrow{XT}$ and $\overrightarrow{XV}$ trisect ∠SXW.

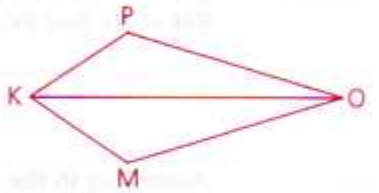


3 Name the angle bisector.

a



b m∠POK = m∠MOK



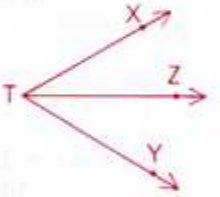
4 Find ∠XTZ if $\overrightarrow{TZ}$ bisects ∠XTY and ∠XTY equals

a 60°

b 48°50′

c $36\frac{1}{2}°$

d 85°74′



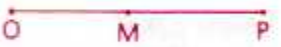
5 B and C trisect $\overline{AD}$.

a Find the coordinates of B and C.

b Find AC.

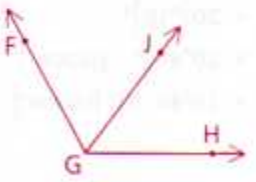
6 Given: OM = x + 8,
MP = 2x − 6,
OP = 44

Is M the midpoint of $\overline{OP}$?

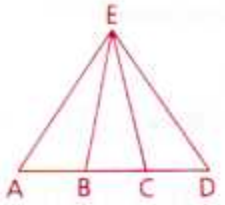


7 Given: m∠FGJ = 3x − 5,
m∠JGH = x + 27;
$\overrightarrow{GJ}$ bisects ∠FGH.

Find: m∠FGJ



8 B and C are trisection points of $\overline{AD}$, and AD = 12.



a Find AB.

b Find AC.

c If AB = x + 3, solve for x.

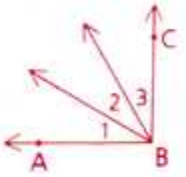
d If AB = x + 3 and AE = 3x + 6, find AE.

e What segment is C the midpoint of?

f Do $\overrightarrow{EB}$ and $\overrightarrow{EC}$ trisect ∠AED?

9 Given: ∠ABC = 90°,
∠1 = (2x + 10)°,
∠2 = (x + 20)°,
∠3 = (3x)°

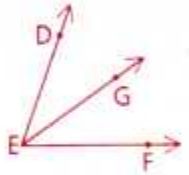
Has ∠ABC been trisected?



In problems 10 and 11, reason 2 in each proof is stated incorrectly. Supply the correct final reason for each problem.

10 Given: ∠DEG ≅ ∠FEG

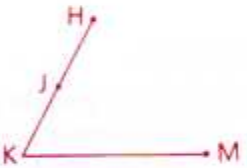
Prove: $\overrightarrow{EG}$ bisects ∠DEF.



Statements	Reasons
1 ∠DEG ≅ ∠FEG	1 Given
2 $\overrightarrow{EG}$ bisects ∠DEF.	2 If a ray divides an angle into two angles, the ray bisects the angle. *(What is the correct reason?)*

11 Given: $\overline{KJ} \cong \overline{HJ}$

Prove: J is the midpoint of $\overline{HK}$.

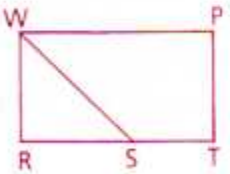


Statements	Reasons
1 $\overline{KJ} \cong \overline{HJ}$	1 Given
2 J is the midpoint of $\overline{HK}$.	2 If a point is the midpoint of a segment, it divides the segment into two congruent segments. *(What is the correct reason?)*

In problems 12–17, write a proof in two-column form.

12 Given: $\overrightarrow{WS}$ bisects ∠RWP.

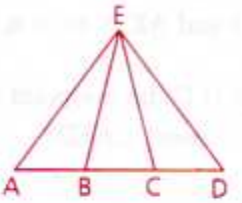
Prove: ∠RWS ≅ ∠PWS



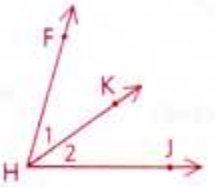
Problem Set A, *continued*

13 Given: $\overline{XY} \cong \overline{YZ}$
Prove: Y is the midpoint of $\overline{XZ}$.

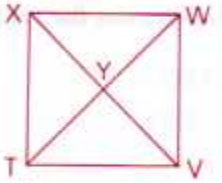
14 Given: $\angle AEB \cong \angle BEC \cong \angle CED$
Conclusion: $\overrightarrow{EB}$ and $\overrightarrow{EC}$ trisect $\angle AED$.



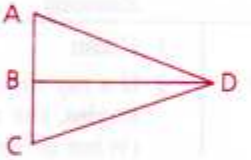
15 Given: $\angle 1 \cong \angle 2$
Conclusion: $\overrightarrow{HK}$ bisects $\angle FHJ$.



16 Given: $\angle TXW$ is a right angle.
$\angle TYV$ is a right angle.
Prove: $\angle TXW \cong \angle TYV$

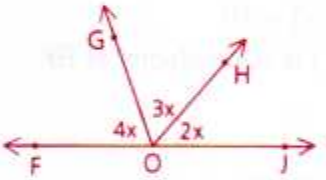


17 Given: B is the midpoint of $\overline{AC}$.
Prove: $\overline{AB} \cong \overline{BC}$

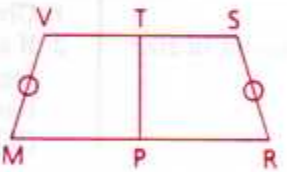


Problem Set B

18 $\overrightarrow{OG}$ and $\overrightarrow{OH}$ divide straight angle FOJ into three angles whose measures are in the ratio 4:3:2. Find $m\angle FOG$.



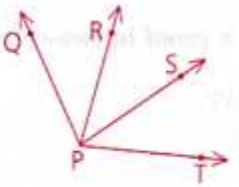
19 Given: $\overleftrightarrow{TP}$ bisects $\overline{VS}$ and $\overline{MR}$.
$\overline{VM} \cong \overline{SR}$,
MP = 9, VT = 6,
perimeter of MRSV = 62
Find: VM



20 $\overrightarrow{PR}$ and $\overrightarrow{PS}$ trisect $\angle QPT$.

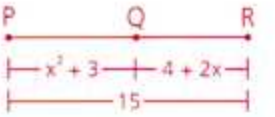
a If $m\angle RPS = 23°50'$, find $m\angle QPT$.

b If $m\angle QPT = 120°48'30''$, find $m\angle QPS$.



21 **a** Find the value of x.

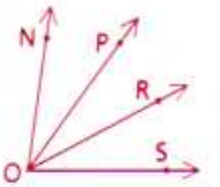
b Is Q the midpoint of $\overline{PR}$?



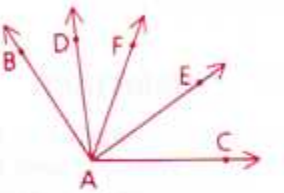
Problem Set C

22 Given: $\overrightarrow{OP}$ and $\overrightarrow{OR}$ trisect $\angle NOS$.
$m\angle NOP = 3x - 4y$,
$m\angle POR = x - y$,
$m\angle ROS = y - 10$

Find: $m\angle ROS$



23 $\angle BAC = 120°$, and points D, E, and F are in the interior of $\angle BAC$ as shown. $\overrightarrow{AD}$ bisects $\angle BAF$. $\overrightarrow{AE}$ bisects $\angle CAF$. Find $m\angle DAE$.



24 The measures of two angles are in the ratio 5:3. The measure of the larger angle is 30 greater than half the difference of the angles. Find the measure of each angle.

CAREER PROFILE

THE SCIENCE OF DEDUCTION

Wendell Griffen objects

Deductive reasoning, the cornerstone of mathematical proof, is responsible for a huge proportion of the scientific and technological achievements of the past three hundred years, but it is equally important in a wide variety of nonmathematical endeavors. Trial lawyer Wendell Griffen believes that the ability to use deductive reasoning is one of the most useful tools a trial lawyer can possess. Why? "Because a trial is an exercise in reason," he explains. Each side in a dispute has different pieces of the puzzle. "When we look at the evidence we find riddles, and riddles within those riddles," he says. "Who is at fault? Which witness is more credible? A trial lawyer's job is to construct a model of events so that the judge and jury can reason their way through to a logical conclusion."

Griffen attended high school in his hometown of Delight, Arkansas, and earned a degree in political science at the University of Arkansas. After three years in the army he entered the University of Arkansas Law School, receiving a law degree in 1979. Now a partner in the general litigation department of a Little Rock law firm, Griffen spends most of his time defending employees in workers' compensation cases. In his rare free moments he enjoys reading. Asked to name his favorite fictional character, he answers without hesitating, "Sherlock Holmes, naturally!"

1.6 PARAGRAPH PROOFS

Objective

After studying this section, you will be able to

- Write paragraph proofs

Part One: Introduction

Although most of the proofs you will encounter this year will be in two-column form, you also need to be familiar with ***paragraph proofs***. They are important because the proofs in journals, more-advanced mathematics courses, and other areas of study are usually in paragraph form.

The sample problems that follow demonstrate how to write paragraph proofs, as well as how to show that a particular conclusion cannot be proved true or can be proved false.

Part Two: Sample Problems

Problem 1 *Given:* $\angle O = 67\frac{1}{2}°$, $\angle P = 67°30'$

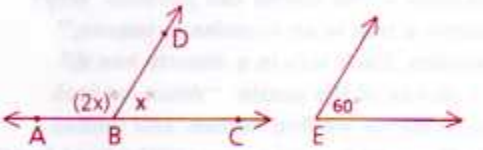
Prove: $\angle O \cong \angle P$

Proof Since there are 60 minutes in 1 degree, $67°30'$ equals $67\frac{1}{2}°$.

Since $\angle O$ and $\angle P$ have the same measure, they are congruent.

Problem 2 *Given: Diagram shown*

Prove: $\angle DBC \cong \angle E$



Proof According to the diagram, $\angle ABC$ is a straight angle. Therefore,

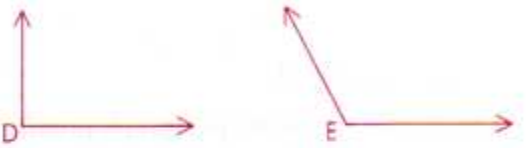
$$2x + x = 180$$
$$3x = 180$$
$$x = 60$$

Since $\angle DBC = 60°$ and $\angle E = 60°$, the angles are congruent.

Problem 3 Given: $\angle 1$ *is acute.*
$\angle 2$ *is acute.*
Conclusion: $\angle 1 \cong \angle 2$

Proof This conclusion *cannot be proved.* For example, if $m\angle 1 = 20$ and $m\angle 2 = 30$, they are both acute but $\angle 1$ is not congruent to $\angle 2$. (An example, like this, of a case in which a conclusion is false is called a ***counterexample***.)

Problem 4 Given: $\angle D = 90°$;
$\angle E$ *is obtuse.*
Prove: $\angle D \cong \angle E$



Proof This conclusion can be proved to be *false.* Since $\angle E$ is obtuse, its measure is greater than 90. Since $\angle D$ and $\angle E$ have different measures, they are not congruent ($\angle D \ncong \angle E$).

Part Three: Problem Sets

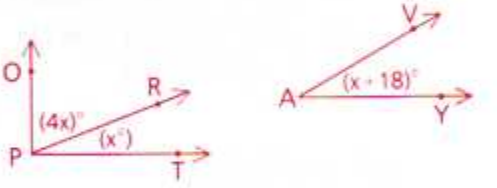
Problem Set A

In problems 1–6, write paragraph proofs.

1 Given: $\angle V = 119\frac{2}{3}°$,
$\angle S = 119°40'$
Conclusion: $\angle V \cong \angle S$

2 Given: Diagram shown
Prove: $\angle FEH \cong \angle JKM$

3 Given: Diagram shown, $\angle OPT = 90°$
Prove: The measure of $\angle VAY$ is twice that of $\angle RPT$.

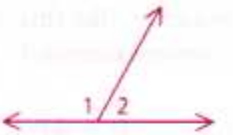


4 Given: $AB = x + 4$,
$BC = 2x$,
$AC = 16$
Conclusion: $\overline{AB} \cong \overline{BC}$

Problem Set A, *continued*

5 Given: $\angle D$ is obtuse.
$\angle C$ is greater than $90°$ ($\angle C > 90°$).
Conclusion: $\angle D \cong \angle C$

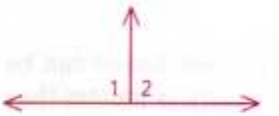
6 Given: $\angle 1$ is obtuse.
$\angle 2$ is acute.
Prove: $\angle 1 \cong \angle 2$



Problem Set B

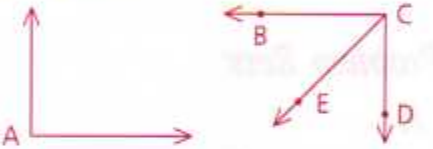
In problems 7–9, write paragraph proofs.

7 Prove that if $\angle 1 \cong \angle 2$, they are both right angles.



8 Prove the following statement: "If an obtuse angle is bisected, each of the two resulting angles is acute."

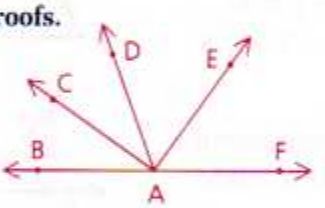
9 Given: $\overrightarrow{CE}$ bisects $\angle BCD$.
$\angle A$ is a right angle.
$m\angle BCE = 45$
Prove: $\angle A \cong \angle BCD$



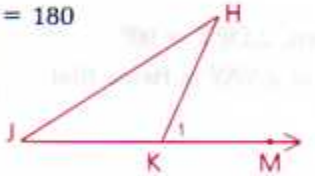
Problem Set C

In problems 10 and 11, write paragraph proofs.

10 Given: Diagram shown;
$\overrightarrow{AC}$ bisects $\angle BAD$.
$\overrightarrow{AE}$ bisects $\angle DAF$.
Prove: $\angle CAE$ is a right angle.

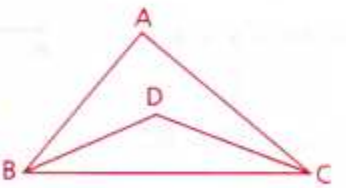


11 Given: $m\angle J + m\angle H + m\angle JKH = 180$
Prove: **a** $m\angle 1 = m\angle J + m\angle H$
b $m\angle 1 > m\angle J$



Problem Set D

12 Given: $m\angle A + m\angle ABC + m\angle ACB = 180$,
$m\angle D + m\angle DBC + m\angle DCB = 180$;
$\overrightarrow{BD}$ bisects $\angle ABC$. $\overrightarrow{CD}$ bisects $\angle ACB$.
Prove: $m\angle D = 90 + \frac{1}{2}(m\angle A)$
(Write a paragraph proof.)



1.7 DEDUCTIVE STRUCTURE

Objectives

After studying this section, you will be able to

- Recognize that geometry is based on a deductive structure
- Identify undefined terms, postulates, and definitions
- Understand the characteristics of theorems and the ways in which they can be used in proofs

Part One: Introduction

The Structure of Geometry

You have just spent a few days writing two-column proofs and paragraph proofs. Since you have learned how to prove a few statements, you may be interested in knowing something about the theory of proofs.

Geometry is based on a ***deductive structure***—a system of thought in which conclusions are justified by means of previously assumed or proved statements. Every deductive structure contains the following four elements.

- Undefined terms
- Assumptions known as ***postulates***
- Definitions
- Theorems and other conclusions

Undefined Terms, Postulates, and Definitions

Undefined terms, postulates, and definitions form the foundation on which the rest of a deductive structure is based. Examples of the undefined terms you have already encountered are *point* and *line*. Although we have not defined these terms, we have described points and lines, so that everyone should have a fairly clear idea of what they are.

As yet, we have not formally presented any postulates. We have, however, used some algebraic postulates in solving problems.

Definition A ***postulate*** is an unproved assumption.

The postulates presented in this book will be preceded by the heading **Postulate**.

You have already seen a number of definitions, such as the definitions of *acute angle*, *right angle*, *obtuse angle*, and *straight angle*.

Definition A ***definition*** states the meaning of a term or idea.

In this book, important definitions are identified by the heading **Definition**.

One very important characteristic of definitions is that they are *reversible*. For example, the definition of *midpoint* (of a segment) can be expressed in either of two ways:

1 If a point is the midpoint of a segment, then the point divides the segment into two congruent segments.
2 If a point divides a segment into two congruent segments, then the point is the midpoint of the segment.

In some problems, form 1 of the definition of *midpoint* must be used. In other problems the definition must be *reversed*, as in form 2 above.

Notice that this definition is stated in the form

"If p, then q"

where p and q are declarative statements. Such a sentence is called a ***conditional statement*** or an ***implication***. The "if" part of the sentence is called the ***hypothesis***. The "then" part of the sentence is called the ***conclusion***. "If p, then q" can be symbolized $p \Rightarrow q$ (also read "p implies q").

The ***converse*** of $p \Rightarrow q$ is $q \Rightarrow p$. To write the converse of a conditional ("If . . . , then . . . ") statement, you reverse parts p and q. The converse of "If p, then q" is "If q, then p," so forms 1 and 2 of the definition of *midpoint* are converses of each other.

Theorems

As you have seen, a theorem is a mathematical statement that can be proved. Almost all the theorems presented in this book will be numbered for ease of reference. Each theorem will be preceded by a heading such as the following:

Theorem 78

You will prove some theorems and other relationships as homework problems. As you work, remember that you must prove conclusions by using conclusions previously assumed or proved. Thus, you cannot use Theorem 1 in order to prove Theorem 1.

Theorems and postulates are not always reversible. For example, "If two angles are right angles, then they are congruent" is true. The converse statement, "If two angles are congruent, then they are right angles," is false.

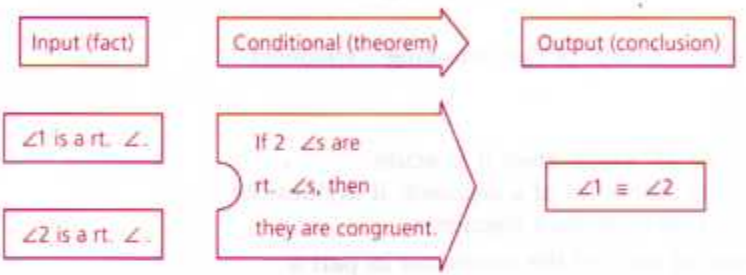
Remember,

- *Definitions* are always reversible
- *Theorems* and *postulates* are not always reversible

If you are to be successful in writing proofs, you must memorize postulates, definitions, and theorems. There is no easier way.

A complete mastery of the deductive structure of geometry is not possible in a short time. However, we do wish to point out the most common error that students make—using the converse of a statement at the wrong time.

It is important to pay attention to the direction of the flow of logic in order to avoid this error. The theorem "If two angles are right angles, then they are congruent" means that whenever we encounter right angles, we can conclude that they are congruent. There is a flow from *right angles* to *congruent*.



In this case, the flow works in only one direction—the converse of the statement, "If two angles are congruent, then they are right angles" is not true. *Remember, only definitions are always reversible. Theorems and postulates are not always reversible.*

The major purpose of this section and the next is to acquaint you with some terminology. As you study Chapter 2 and Chapter 3, you will grow to appreciate and understand these sections even more. The homework problems in these sections are rather different from those you have been solving, and we think you will enjoy them.

Part Two: Sample Problem

Problem *State the converse of each of the following statements and tell whether the converse is true or false.*

a *If an angle contains 90 degrees, it is a right angle.*
b *If Mary received a B on her history test, then she passed the test.*

Answers

a *If an angle is a right angle, it contains 90 degrees. (True)*
b *If Mary passed her history test, she received a B on the test. (False)*

Part Three: Problem Sets

Problem Set A

1 What four elements are found in any deductive structure?

2 Which of the following kinds of statements are always reversible?

a Definitions **b** Theorems **c** Postulates

3 Answer each question Yes or No.

a Do we prove theorems? **b** Do we prove definitions?

4 Tell whether each of the following statements is a theorem or a definition.

a If two angles are right angles, then they are congruent.

b If a ray bisects an angle, it divides the angle into two congruent angles.

5 a Write the converse of each of the following statements.

i If A, then B.
ii Rain $\Rightarrow$ wet
iii If an angle is a 45° angle, then it is acute.
iv If a point is the midpoint of a segment, it divides the segment into two congruent segments.

b Discuss the truth of each of the converses in part **a**.

In problems 6 and 7, comment on the reasoning used.

6 The school colors are orange and black, so I'll wear my orange skirt to the game and everyone will notice me.

7 I've flipped this silver dollar five times and the toss has come up heads each time. Thus, the odds are greater than 50–50 that the toss will come up tails next time.

Problem Set B

In problems 8–12, study each of the arguments and state whether or not the conclusion is deducible. If it is not, comment on the error in the reasoning.

8 If a student at Niles High has room 303 as his or her homeroom, the student is a freshman. Joe Jacobs is a student at Niles High and has room 303 as his homeroom. Therefore, Joe Jacobs is a freshman.

9 If the three angles of a triangle are acute, then the triangle is acute. In triangle ABC, angle A and angle B are acute. Therefore, triangle ABC is acute.

10 All school buses stop at railroad crossings. A vehicle stopped at the Santa Fe railroad crossing. Therefore, that vehicle is a school bus.

11 All cloudy days are depressing. Therefore, since I was depressed on Thursday, Thursday was cloudy.

12 If two angles of a triangle are congruent, then the sides opposite them are congruent. In $\triangle ABC$, $\angle A \cong \angle B$. Therefore, in $\triangle ABC$, $\overline{BC} \cong \overline{AC}$.

Problem Set C

13 Study the following five statements.

1 Spoof is the set of all purrs.
2 Spoof contains at least two distinct purrs.
3 Every lilt is a set of purrs and contains at least two distinct purrs.
4 If *A* and *B* are any two distinct purrs, there is one and only one lilt that contains them.
5 No lilt contains all the purrs.

a Show that each of the following statements is true.
i There is at least one lilt.
ii There are at least three purrs.
iii There are at least three lilts.

b If the lilt "girt" contains the purr "pil" and the purr "til" and if the lilt "mirt" contains the purr "pil" and the purr "til" then the lilt "girt" is the same as the lilt "mirt" except in one case. What is this case?

14 The Bronx Zoo has a green lizard, a red crocodile, and a purple monkey. They are the only animals of their kind in existence. One violently windy Saturday, their name tags blew off, and their keeper's journal was torn to shreds. Inasmuch as they were to appear on television at 7:30 Sunday morning, the night watchman had to replace their name tags. He managed to piece together the following information from the mangled journal.

1 Wendy cannot get along with the lizard.
2 Katie playfully took a bite out of the monkey's ear one month ago.
3 Wendy never casts a red reflection in the mirror.
4 Jody has the personality of a crocodile, but she isn't one.

Match the animals with their names.

1.8 STATEMENTS OF LOGIC

Objective

After studying this section, you will be able to

- Recognize conditional statements
- Recognize the negation of a statement
- Recognize the converse, the inverse, and the contrapositive of a statement
- Use the chain rule to draw conclusions

Part One: Introduction

Review of Conditional Statements

In this section, we will review and extend the discussion of conditional statements in Section 1.7. Recall that a conditional statement is a sentence that is in the form "If . . ., then . . ." Many declarative sentences can be rewritten in conditional form.

Declarative Sentence:	*Conditional Form:*
▪ Two straight angles are congruent.	▪ If two angles are straight angles, then they are congruent.

Remember that

- The clause following the word *if* is called the hypothesis
- The clause following the word *then* is called the conclusion
- The conditional statement "If p, then q" can be written in symbols as $p \Rightarrow q$

Negation

The ***negation*** of any statement p is the statement "not p." (Thus, the negation of "It is raining" is "It is not raining.") The symbol for "not p" is $\sim p$. Notice also that the negation of "It is not raining" is "It is raining"—in general, not (not p) = p, or $\sim\sim p = p$.

Converse, Inverse, and Contrapositive

Every conditional statement "If p, then q" has three other statements associated with it. (You have already been introduced to the first of these—the converse).

1 A converse (If q, then p.)
2 An ***inverse*** (If $\sim p$, then $\sim q$.)
3 A ***contrapositive*** (If $\sim q$, then $\sim p$.)

Example *Find the converse, the inverse, and the contrapositive of the statement "If you live in Atlanta, then you live in Georgia."*

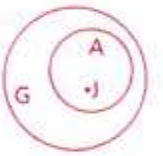
The statement is in the form "If p, then q," with p being "You live in Atlanta" and q being "You live in Georgia."

Converse: "If you live in Georgia, then you live in Atlanta." (If q, then p.)

Inverse: "If you don't live in Atlanta, then you don't live in Georgia." (If $\sim p$, then $\sim q$.)

Contrapositive: "If you don't live in Georgia, then you don't live in Atlanta." (If $\sim q$, then $\sim p$.)

You may have noticed that some of the statements in the preceding example are not necessarily true, although the original statement is true. A useful tool for determining whether or not a conditional statement is true or false is a ***Venn diagram***. Assume that the following statement is true: "If Jenny lives in Atlanta, then Jenny must live in Georgia."



All the people who live in Georgia are represented by points on the large circle and in its interior (G).

All the people who live in Atlanta are represented by points on the small circle and in its interior (A).

Notice that every person in set A, including Jenny (J), is also in set G.

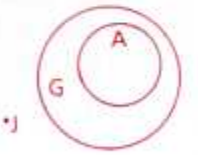
The Venn diagram for this conditional statement may be used to test whether its converse, inverse, and contrapositive are true or false.

Converse: "If Jenny lives in Georgia, then she must live in Atlanta."

This statement is not necessarily true, as shown by the diagram. Notice that point J may lie in G but not in A. This means that Jenny could live in Georgia and yet not live in Atlanta.

In general, the converse of a conditional statement is not necessarily true. Try a similar argument with the same Venn diagram to convince yourself that the inverse of a conditional statement is also not necessarily true.

Contrapositive: "If Jenny does not live in Georgia, then she does not live in Atlanta."



This time point J lies outside of G, so it *cannot* lie in A. Any point that is not in G is also not in A. Therefore, the contrapositive is true.

This analysis suggests the following important theorem:

Theorem 3 ***If a conditional statement is true, then the contrapositive of the statement is also true. (If p, then q $\Leftrightarrow$ If $\sim q$, then $\sim p$.)***

In other words, a statement and its contrapositive are *logically equivalent*.

Chains of Reasoning

Each proof that you do involves a series of steps in a logical sequence. In many cases, the sequence will take the following form.

If $p \Rightarrow q$ and $q \Rightarrow r$, then $p \Rightarrow r$.

This is called the ***chain rule***, and a series of conditional statements so connected is known as a ***chain of reasoning***.

Example *If we accept the two statements "If you study hard, then you will earn a good grade" ($p \Rightarrow q$) and "If you earn a good grade, then your family will be happy" ($q \Rightarrow r$), what can we conclude?*

We can conclude that $p \Rightarrow r$—that is, if you study hard, then your family will be happy.

Part Two: Sample Problems

Problem 1 *Write the converse, the inverse, and the contrapositive of the following true statement: "If two angles are right angles, then they are congruent."*

Solution Converse: "If two angles are congruent, then they are right angles." (The converse is false; for example, each angle may have a measure of 60.)

Inverse: "If two angles are not right angles, then they are not congruent." (The inverse is also false.)

Contrapositive: "If two angles are not congruent, then they are not right angles." (The contrapositive is true—the statements are logically equivalent.)

Problem 2 *Draw a conclusion from the following statements:*

If gremlins grow grapes, then elves eat earthworms.
If trolls don't tell tales, then wizards weave willows.
If trolls tell tales, then elves don't eat earthworms.

Solution First, we rewrite the statements in symbolic form.

(1) $g \Rightarrow e$
(2) $\sim t \Rightarrow w$
(3) $t \Rightarrow \sim e$

To complete the chain of reasoning, we can rearrange the statements and use contrapositives as needed to match symbols. Thus,

(1) $g \Rightarrow e$
(3) $e \Rightarrow \sim t$ ($t \Rightarrow \sim e$ is equivalent to $e \Rightarrow \sim t$.)
(2) $\sim t \Rightarrow w$
$\therefore g \Rightarrow w$ (The symbol $\therefore$ means "therefore.")

Hence, if gremlins grow grapes, then wizards weave willows.

Part Three: Problem Sets

Problem Set A

1 Write each sentence in conditional ("If . . ., then . . .") form.

a Eighteen-year-olds may vote in federal elections.

b Opposite angles of a parallelogram are congruent.

2 Write the converse, the inverse, and the contrapositive of each statement. Determine the truth of each of the new statements.

a If each side of a triangle has a length of 10, then the triangle's perimeter is 30.

b If an angle is acute, then it has a measure greater than 0 and less than 90.

3 If a conditional statement and its converse are both true, the statement is said to be *biconditional*. Which of these statements is biconditional?

a If two angles are congruent, then they have the same measure.

b If two angles are straight angles, then they are congruent.

4 Draw a Venn diagram for the true conditional statement "If a person lives in Chicago, then the person lives in Illinois." Assuming that each of the following "Given . . ." statements is true, determine the truth of the conclusion.

a Given: Penny lives in Chicago.
Conclusion: Penny lives in Illinois.

b Given: Benny lives in Illinois.
Conclusion: Benny lives in Chicago.

c Given: Kenny does not live in Chicago.
Conclusion: Kenny must live in Illinois.

d Given: Denny does not live in Illinois.
Conclusion: Denny lives in Chicago.

Problem Set A, *continued*

5 Write a concluding statement for each of the following chains of reasoning.

a $a \Rightarrow b$
$d \Rightarrow \sim c$
$\sim c \Rightarrow a$
$b \Rightarrow f$

b $p \Rightarrow \sim q$
$r \Rightarrow q$
$s \Rightarrow r$

c If weasels walk wisely, then cougars call their cubs.
If goats go to graze, then horses head for home.
If cougars call their cubs, then goats go to graze.
If bobcats begin to browse, then weasels walk wisely.

Problem Set B

6 Write the converse, the inverse, and the contrapositive of "If M is the midpoint of $\overline{AB}$, then M, A, and B are collinear." Are these statements true or false?

7 Rewrite the following sentence in conditional form and find its converse, inverse, and contrapositive: "A square is a quadrilateral with four congruent sides."

8 Write the converse, the inverse, and the contrapositive of each statement.

a If a ray bisects an angle, it divides the angle into two congruent angles.

b If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

9 What conclusion can be drawn from the following?
$\sim c \Rightarrow \sim f$ $\quad g \Rightarrow b$ $\quad p \Rightarrow f$ $\quad c \Rightarrow \sim b$

Problem Set C

10 What conclusion can be drawn from the following?

If the line is long, then Quincy will go home.
If it is morning, then Quincy will not go home.
If the line is long, then it is morning.

1.9 PROBABILITY

Objective

After studying this section, you will be able to

- Solve probability problems

Part One: Introduction

A knowledge of ***probability*** is obviously important to an insurance company, to a card player or a backgammon expert, and to an operator of a gambling casino. Moreover, setting up and solving probability problems requires the precision and the organized, ordered thinking needed by secretaries, accountants, doctors, filing clerks, computer programmers, and geometry students.

Although probability is not one of the major topics in this book, you will occasionally encounter probability problems in the problem sets. You can analyze such problems by following a simple two-step procedure.

Two Basic Steps for Probability Problems

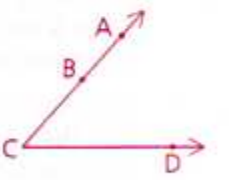
1. Determine all possibilities in a logical manner. Count them.
2. Determine the number of these possibilities that are "favorable." We shall call these *winners*.

You can then calculate the probability by means of the following formula.

$$\text{Probability} = \frac{\text{number of winners}}{\text{number of possibilities}}$$

Part Two: Sample Problems

Problem 1 *If one of the four points is picked at random, what is the probability that the point lies on the angle?*

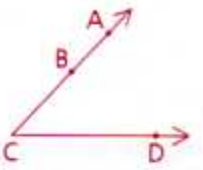


Solution We follow the two basic steps by listing all the possibilities and circling the winners.

Ⓐ Ⓑ Ⓒ Ⓓ

$$\frac{\text{Winners}}{\text{Possibilities}} = \frac{4}{4} = 1$$

Problem 2 *If two of the four points are selected at random, what is the probability that both lie on* $\overrightarrow{CA}$*?*

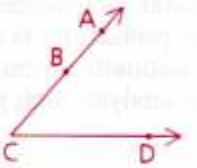


Solution We follow the two basic steps by listing all the possibilities and circling the winners. (Notice how we have attempted to list the possibilities in an orderly manner.)

(AB) (BC) CD
(AC) BD
AD

$$\frac{\text{Winners}}{\text{Possibilities}} = \frac{3}{6} = \frac{1}{2}$$

Problem 3 *If three of the four points are selected in a random order, what is the probability that the ordered letters will correctly name the angle shown?*

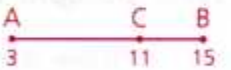


Solution We follow the two basic steps by listing all the possibilities and circling the winners. (This problem is harder than the first two examples because the *order* of the points is important. Notice how we have listed the possibilities in an orderly manner.)

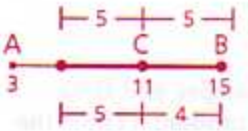
ABC	BAC	CAB	DAB
ABD	BAD	CAD	DAC
ACB	BCA	CBA	DBA
(ACD)	(BCD)	CBD	DBC
ADB	BDA	CDA	(DCA)
ADC	BDC	CDB	(DCB)

$$\frac{\text{Winners}}{\text{Possibilities}} = \frac{4}{24} = \frac{1}{6}$$

Problem 4 *A point Q is randomly chosen on* $\overline{AB}$*. What is the probability that it is within 5 units of C?*



Solution Even though there are infinitely many points on the segment, we can find the probability by comparing the length of the "winning" region with the total length of $\overline{AB}$.



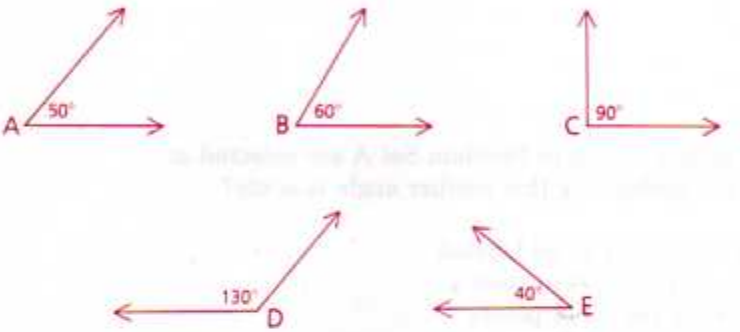
The "winning" region is 9 (not 10) units long. $\overline{AB}$ is 12 units long.

Probability $= \frac{9}{12} = \frac{3}{4}$

Part Three: Problem Sets

Problem Set A

In problems 1–4, refer to the following diagram.



1 If one of the five angles is selected at random, what is the probability that the angle is acute?

2 If one of the five angles is selected at random, what is the probability that the angle is right?

3 If one of the five angles is selected at random, what is the probability that the angle is obtuse?

4 If one of the five angles is selected at random, what is the probability that the angle is straight?

5 If a point is randomly chosen on $\overline{PR}$, what is the probability that it is within 2 units of R?

Problem Set B

In problems 6–9, use the five angles shown at the beginning of Problem Set A.

6 If two of the five angles are selected at random, what is the probability that both are acute?

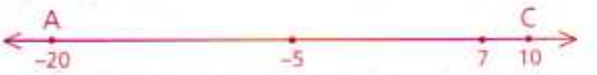
7 If two of the five angles are selected at random, what is the probability that one of them is obtuse?

8 If two of the five angles are selected at random, what is the probability that one is right and the other is obtuse?

Problem Set B, *continued*

9 An angle is selected at random from the five angles and then replaced. A second selection is then made at random. (Thus, the same angle might be selected twice.) What is the probability that an acute angle is selected both times?

10 If a point B is chosen on $\overline{AC}$, what is the probability that $-5 \leq B \leq 7$?



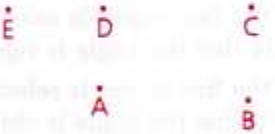
11 The second hand of a clock sweeps continuously around the face of the clock. What is the probability that at any random moment the second hand is between 7 and 12?

Problem Set C

12 If two of the five angles shown in Problem Set A are selected at random, what is the probability that neither angle is acute?

13 If the four points shown are to be labeled with the letters A, B, C, and D in such a way that A and two of the other points are collinear, in how many different ways can the diagram be labeled?

14 Consider points A, B, C, D, and E as shown.



a If two of these points are selected at random, what is the probability that they are collinear?

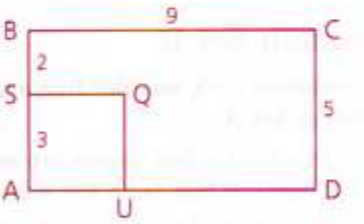
b If three of these points are selected at random, what is the probability that they are collinear?

c If four of these points are selected at random, what is the probability that they are collinear?

15 If a point is chosen at random in rectangle ABCD, what is the probability that

a It is in square SQUA?

b It is not in square SQUA?



1 Chapter Summary

Concepts and Procedures

After studying this chapter, you should be able to

- Recognize points, lines, segments, rays, angles, and triangles (1.1)
- Measure segments and angles (1.2)
- Classify angles and name the parts of a degree (1.2)
- Recognize congruent angles and segments (1.2)
- Recognize collinear and noncollinear points (1.3)
- Recognize when a point is between two other points (1.3)
- Apply the triangle-inequality principle (1.3)
- Correctly interpret geometric diagrams (1.3)
- Write simple two-column proofs (1.4)
- Identify bisectors and trisectors of segments and angles (1.5)
- Write paragraph proofs (1.6)
- Recognize that geometry is based on a deductive structure (1.7)
- Identify undefined terms, postulates, and definitions (1.7)
- Understand the characteristics and application of theorems (1.7)
- Recognize conditional statements and the negation, the converse, the inverse, and the contrapositive of a statement (1.8)
- Use the chain rule to draw conclusions (1.8)
- Solve probability problems (1.9)

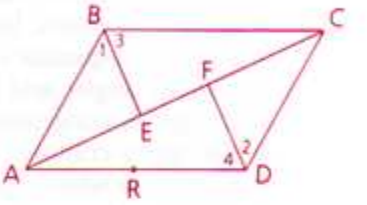
Vocabulary

acute angle (1.2)
angle (1.1)
bisect, bisector (1.5)
chain rule (1.8)
collinear (1.3)
conclusion (1.7)
conditional statement (1.7)
congruent angles (1.2)
congruent segments (1.2)
contrapositive (1.8)
converse (1.7)
counterexample (1.6)
deductive structure (1.7)
definition (1.7)
endpoint (1.1)
hypothesis (1.7)
implication (1.7)
intersection (1.1)
inverse (1.8)
line (1.1)
line segment (1.1)
measure (1.2)
midpoint (1.5)
minute (1.2)
negation (1.8)
noncollinear (1.3)
number line (1.1)
obtuse angle (1.2)
paragraph proof (1.6)
point (1.1)
postulate (1.7)
probability (1.9)
protractor (1.2)
ray (1.1)
right angle (1.2)
second (1.2)
segment (1.1)
straight angle (1.2)
theorem (1.4)
tick mark (1.2)
triangle (1.1)
trisect, trisectors (1.5)
trisection points (1.5)
two-column proof (1.4)
union (1.1)
Venn diagram (1.8)
vertex (1.1)

1 REVIEW PROBLEMS

Problem Set A

1 **a** Name in all possible ways, the line containing A, R, and D.



b Name the sides of ∠ABC.

c What side do ∠2 and ∠4 have in common?

d Name the horizontal ray with endpoint C.

e Estimate the sizes of ∠BAD, ∠2, and ∠ABC.

f Are angles FCD and DCE different angles?

g Which angle in the figure is ∠B?

h $\overrightarrow{EC} \cup \overrightarrow{FA} =$?

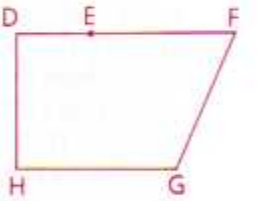
i $\overrightarrow{EC} \cap \overrightarrow{FA} =$?

j $\overrightarrow{BA} \cup \overrightarrow{BE} =$?

k $\overleftrightarrow{AC} \cap \overleftrightarrow{DR} =$?

l $\angle AFD \cap \overline{CE} =$?

2 Tell whether each of the following angles *appears* to be acute, right, obtuse, or straight. Which angle's classification can be assumed from the diagram?



a ∠H

b ∠G

c ∠GFE

d ∠DEF

e ∠HDF

3 **a** $43°15'17'' + 25°49'18'' =$?

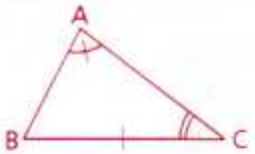
b $90° - 39°17'' =$?

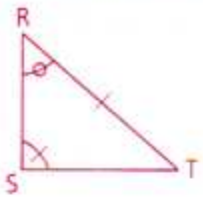
4 **a** Change $46\frac{7}{8}°$ to degrees, minutes, and seconds.

b Change 132°6′ to degrees.

5 **a** According to the diagram, which two segments are congruent?

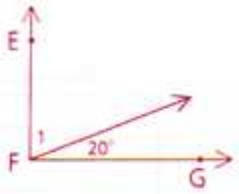
b According to the diagram, which two angles are congruent?

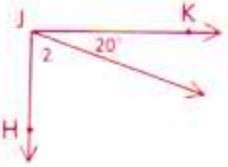




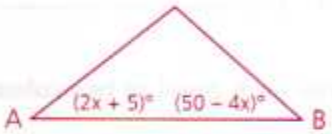
6 **a** If $\angle EFG$ is obtuse and $\angle HJK$ is right, is $\angle 1 \cong \angle 2$?

b If $\angle EFG \cong \angle HJK$, is $\angle 1 \cong \angle 2$?

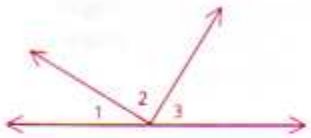




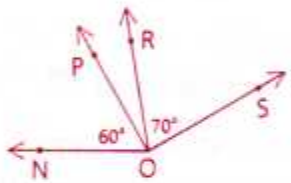
7 If $\angle A \cong \angle B$, find $m\angle A$.



8 The measures of $\angle 1$, $\angle 2$, and $\angle 3$ are in the ratio 1:3:2. Find the measure of each angle.



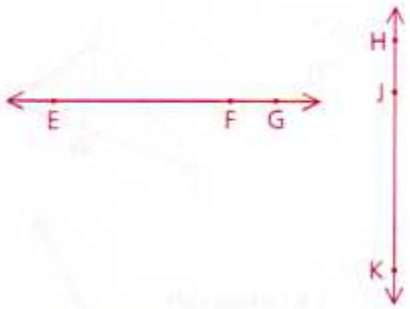
9 Is it possible for both $\angle NOR$ and $\angle POS$ to be right angles?



In problems 10 and 11, copy each figure and incomplete proof. Then complete the proof by filling in the missing reasons.

10 Given: Diagram as shown

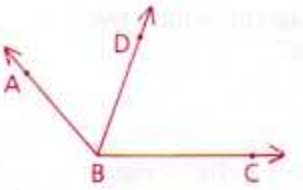
Prove: $\angle EFG \cong \angle HJK$



Statements	Reasons
1 Diagram as shown	1 _____
2 $\angle EFG$ is a straight angle	2 _____
3 HJK is a straight angle	3 _____
4 $\angle EFG \cong \angle HJK$	4 _____

Review Problem Set A, *continued*

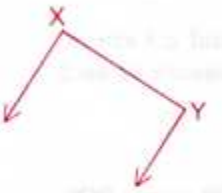
11 Given: $\angle ABC = 130°$,
$\angle ABD = 60°$
Prove: $\angle DBC$ is acute.



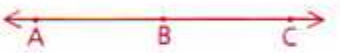
Statements	Reasons
1 $\angle ABC = 130°$	1 _____
2 $\angle ABD = 60°$	2 _____
3 $\angle DBC = 70°$	3 _____
4 $\angle DBC$ is acute	4 _____

In problems 12–15, write each proof in two-column form.

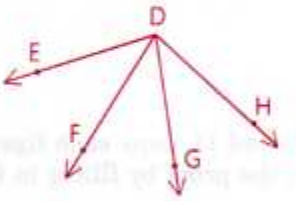
12 Given: $\angle X$ is a right angle.
$\angle Y$ is a right angle.
Prove: $\angle X \cong \angle Y$



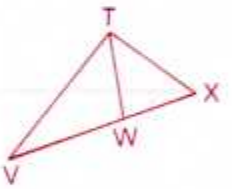
13 Given: $\overline{AB} \cong \overline{BC}$
Prove: B is the midpoint of $\overline{AC}$.



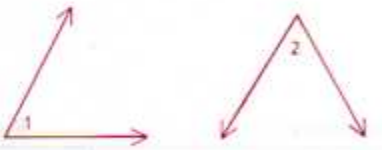
14 Given: $\overrightarrow{DF}$ and $\overrightarrow{DG}$ trisect $\angle EDH$.
Conclusion: $\angle EDF \cong \angle FDG \cong \angle GDH$



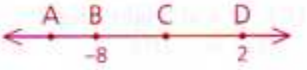
15 Given: $\overrightarrow{TW}$ bisects $\angle VTX$.
Prove: $\angle VTW \cong \angle XTW$



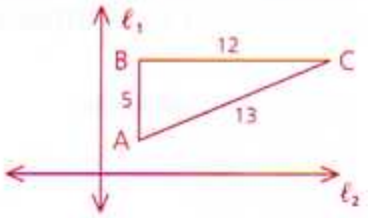
16 Given: $\angle 1 = 61.6°$,
$\angle 2 = 61\frac{3}{5}°$
Prove: $\angle 1 \cong \angle 2$ (Write a paragraph proof.)



17 **a** Find coordinate of C (the midpoint of $\overline{BD}$).
b If AD = 15, find the coordinate of A.

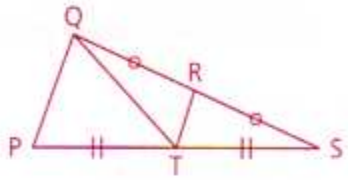


18 Copy the diagram and draw $\triangle A'B'C'$, the reflection of $\triangle ABC$, over ℓ_2. What is the length of $A'B'$?

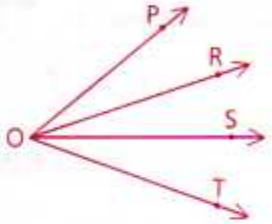


19 **a** If one of the five labeled points is selected at random, what is the probability that it is a midpoint?

b If two of the five points are randomly chosen, what is the probability that both are midpoints?



20 Given: $\overrightarrow{OR}$ and $\overrightarrow{OS}$ trisect $\angle TOP$.
$\angle TOP = 40.2°$
Find: $m\angle POR$



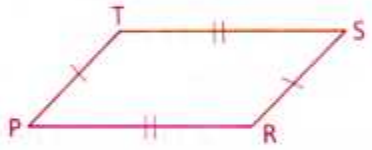
21 Find the angle formed by the hands of a clock at each time.

a 1:00 **b** 11:20 **c** 4:45

22 Write the converse, the inverse, and the contrapositive of the statement "If the time is 2:00, then the angle formed by the hands of a clock is acute." Are these statements true or false?

Problem Set B

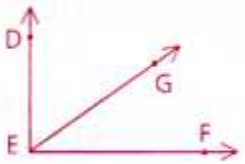
23 The perimeter of PRST is 10 more than 5(RS). If PR = 26, find RS.



24 Given: $\angle DEG = (x + 3y)°$,
$\angle GEF = (2x + y)°$;
$\angle DEF$ is a right angle.

a Solve for y in terms of x.

b If $\angle DEG \cong \angle GEF$, find the values of x and y.



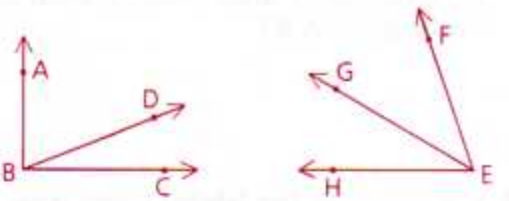
25 Given: WY = 25;
The ratio of WX to XY is 3:2.
Find: WX

26 The measure of $\angle A$ is 6 greater than twice the measure of $\angle B$. If the angles' sum is 42°, find the measure of $\angle A$.

Review Problem Set B, *continued*

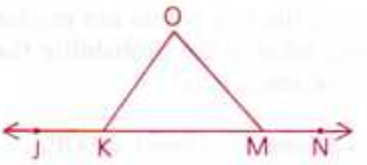
27 Given: $\angle ABC$ is a right angle.
$\angle DBC = 20°$,
$\angle FEG = 40°$,
$\angle GEH = 30°$

Prove: $\angle ABD \cong \angle FEH$
(Write a two-column proof.)

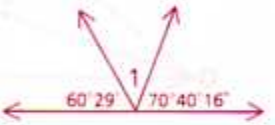


28 Given: $\angle OMK = 50°$,
$\angle OKM = (2x)°$,
$\angle OKJ = (5x + 5)°$

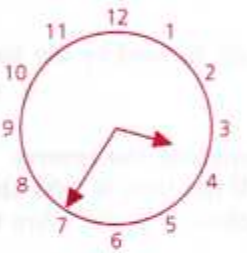
Conclusion: $\angle OKJ \cong \angle OMN$
(Write a paragraph proof.)



29 Find $m\angle 1$.



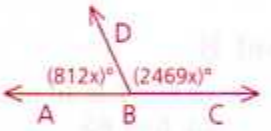
30 The diagram shows Kara's watch. If Kara cannot go home until 4:15, how many degrees must the hour hand travel before she can go home?



31 Find the measure of $\angle ABD$ to

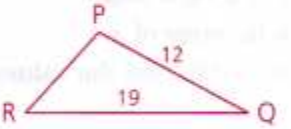
a The nearest tenth of a degree

b The nearest minute



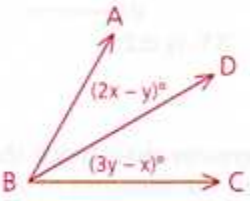
32 If a point is chosen at random on $\overline{PR}$, what is the probability that it is within 6 units of Q?

33 The characteristics of a triangle require that PR be between what two values?

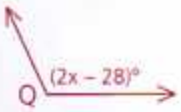


34 Given: $\overrightarrow{BD}$ bisects $\angle ABC$.
$m\angle ABC = 25$

Solve for x and y.



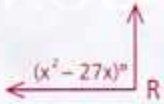
35 $\angle Q$ is obtuse.



a What are the limitations on $m\angle Q$? (Write two inequalities.)

b What are the restrictions on x?

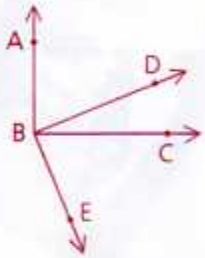
36 Given that $\angle R$ is a right angle, solve for x.



37 The perimeter of a rectangle is 20. If the rectangle's length is less than 4, what is the range of possible values of its width?

Problem Set C

38 Given: $\angle ABC$ is a right angle.
$\angle DBE$ is a right angle.
Prove: $\angle ABD \cong \angle CBE$
(Write a paragraph proof.)



39 Draw a diagram in which $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at E but in which $\angle AEC$ does not appear to be congruent to $\angle DEB$.

40 Jennie's teacher told her to select two problems from a list of two C-level problems, five B-level problems, and one A-level problem. If she selected at random, what is the probability that she selected two B-level problems?

41 At 3:00 the hands of a clock form an angle of 90°. To the nearest second, at what time will the hands of the clock next form a 90° angle?

Problem Set D

42 If six points are represented on a sheet of paper in such a way that any four of them are noncollinear,

a What is the maximum number of lines determined?

b What is the minimum number of lines determined?

43 To the nearest second, what is the first time after 2:00 that the hands of a clock will form an angle $2\frac{1}{2}$ times as great as the angle formed at 2:00?