

11.2: Simplifying Radicals

Goals: *Simplify radicals using the product property
*Multiply radicals
*Simplify radicals using the quotient property
*Rationalize the denominator

Radicals are simplest form when:

1. The number under the radical has no perfect square factors. (No variables have an exponent greater than 1)
This means that you should not be able to divide the number under the radical by 4, 9, 16, 25...etc because these are perfect squares.

Ex: (Not Simplified)

$$\sqrt{20}$$

Since 20 can be divided by 4 which is a perfect square, then it is not considered to be in simplest form.

Ex: (Simplified)

$$\sqrt{30}$$

Although 30 has several factors (2, 3, 5, 6, 15, & 30) none of them are perfect squares, so it is in simplest form.

2. There are no fractions under the radical sign

3. There are no radicals in the denominator

Properties of Radicals

Product Property: $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$ or $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$

Quotient Property: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ or $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

Simplify:

Ex: $\sqrt{32}$

$$\sqrt{16 \cdot 2}$$

$$\sqrt{16} \cdot \sqrt{2}$$

$$4\sqrt{2}$$

Ex: $\sqrt{9x^2}$

$$\sqrt{9} \cdot \sqrt{x^2}$$

$$3x$$

Ex: $\sqrt{24}$

$$\sqrt{6 \cdot 4}$$

$$\sqrt{6} \cdot \sqrt{4}$$

$$2\sqrt{6}$$

$$\text{Ex: } \sqrt{25x^2}$$

$$5x$$

$$\text{Ex: } \sqrt{48}$$

$$\frac{\sqrt{16 \cdot 3}}{4\sqrt{3}}$$

$$\text{Ex: } \sqrt{36x^4}$$

$$\text{Ex: } \sqrt{25x^3}$$

$$\frac{\sqrt{25} \cdot \sqrt{x^3}}{5 \cdot \sqrt{x^2} \cdot \sqrt{x}} = \frac{5x\sqrt{x}}{5x\sqrt{x}}$$

$$\text{Ex: } \sqrt{6} \cdot \sqrt{6}$$

$$\frac{\sqrt{6 \cdot 6}}{\sqrt{36}} = \frac{6}{6}$$

$$\text{Ex: } \sqrt{3} \cdot \sqrt{6}$$

$$\frac{\sqrt{18}}{\sqrt{9} \cdot \sqrt{2}} = \frac{3\sqrt{2}}{3\sqrt{2}}$$

$$\text{Ex: } \sqrt{3x} \cdot 4\sqrt{x}$$

$$\frac{4\sqrt{3x \cdot x}}{4\sqrt{3x^2}} = \frac{4x\sqrt{3}}{4x\sqrt{3}}$$

$$\text{Ex: } \sqrt{7xy^2} \cdot 3\sqrt{x}$$

$$\frac{3\sqrt{7xy^2 \cdot x}}{3\sqrt{7x^2y^2}} = \frac{3xy\sqrt{7}}{3xy\sqrt{7}}$$

$$\text{Ex: } \sqrt{7} \cdot \sqrt{7}$$

$$\frac{\sqrt{7 \cdot 7}}{\sqrt{49}} = \frac{7}{7}$$

$$\text{Ex: } 3\sqrt{b} \cdot \sqrt{2b^3}$$

$$\frac{3\sqrt{b \cdot 2b^3}}{3\sqrt{2b^4}} = \frac{3b^2\sqrt{2}}{3b^2\sqrt{2}}$$

$$\text{Ex: } 2\sqrt{mn^2} \cdot \sqrt{5m^2}$$

$$\frac{2\sqrt{mn^2 \cdot 5m^2}}{2\sqrt{5m^3n^2}} = \frac{2mn\sqrt{5m}}{2mn\sqrt{5m}}$$

$$\text{Ex: } \sqrt{8y^7}$$

$$\frac{\sqrt{8} \cdot \sqrt{y^7}}{\sqrt{4} \cdot \sqrt{2} \cdot \sqrt{y^6} \cdot \sqrt{y}} = \frac{2y^3\sqrt{2y}}{2y^3\sqrt{2y}}$$

Simplify:

$$\text{Ex: } \sqrt{\frac{16}{25}}$$

$$\frac{\sqrt{16}}{\sqrt{25}}$$

$$\frac{4}{5}$$

$$\text{Ex: } \sqrt{\frac{13}{100}}$$

$$\frac{\sqrt{13}}{\sqrt{100}}$$

$$\frac{\sqrt{13}}{10}$$

$$\text{Ex: } \sqrt{\frac{1}{y^2}}$$

$$\frac{\sqrt{1}}{\sqrt{y^2}}$$

$$\frac{1}{y}$$

$$\text{Ex: } \sqrt{\frac{5}{49}}$$

$$\text{Ex: } \sqrt{\frac{11}{d^4}}$$

$$\frac{\sqrt{5}}{7}$$

$$\frac{\sqrt{11}}{d^2}$$

Rationalizing the denominator:

$$\text{Ex: } \frac{5}{\sqrt{7}}$$

$$\text{Ex: } \frac{\sqrt{2}}{\sqrt{3b}}$$

$$\text{Ex: } \frac{1}{\sqrt{3}}$$

$$\frac{5}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$$

$$\frac{\sqrt{2}}{\sqrt{3b}} \cdot \frac{\sqrt{3b}}{\sqrt{3b}}$$

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{5\sqrt{7}}{7}$$

$$\frac{\sqrt{6b}}{3b}$$

$$\frac{\sqrt{3}}{3}$$

$$\text{Ex: } \frac{1}{\sqrt{x}}$$

$$\text{Ex: } \frac{3}{\sqrt{2x}}$$

$$\text{Ex: } \frac{7}{\sqrt{6}}$$

$$\frac{1}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}}$$

$$\frac{3}{\sqrt{2x}} \cdot \frac{\sqrt{2x}}{\sqrt{2x}}$$

$$\frac{7}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$$

$$\frac{\sqrt{x}}{x}$$

$$\frac{3\sqrt{2x}}{2x}$$

$$\frac{7\sqrt{6}}{6}$$

$$\text{Ex: } \frac{\sqrt{3}}{\sqrt{5a}} \cdot \frac{\sqrt{5a}}{\sqrt{5a}}$$

$$\frac{\sqrt{15a}}{5a}$$