4.6: Model Direct Variation

Goals: *Identify a direct variation equation given an *x*/*y* relationship
*Graph a direct variation equation
*Write a direct variation equation given data

Direct Variation: a linear equation where the ratio of $\frac{y}{x}$ remains constant. Two variables show direct variation if they can be written in the form.

$$y = ax$$

a = constant of variation and cannot equal 0.

Similar to: $y = mx + b$

but: $b = 0$

Since: $b = 0$

Graph will always: Pass through the origin

1. Decide whether the equation represents direct variation. If so, identify the constant of variation.

Ex: $2x - 3y = 0$

Can the equation be rewritten so it is in the form $y = ax$?
Try and rewrite the equation in that form.

$$\begin{aligned} 2x - 3y &= 0 \\ \underline{-2x \quad\quad} & \underline{\;-2x} \\ \frac{-3y}{-3} &= \frac{-2x}{-3} \\ y &= \frac{2}{3}x \end{aligned}$$

Yes, it is a direct variation equation and $a = \frac{2}{3}$

Ex: $-x + y = 4$

$$\begin{aligned} -x + y &= 4 \\ \underline{+x \quad\quad} & \underline{\;+x} \\ y &= 4 + x \end{aligned}$$

Ex: $-x + y = 1$

$$\begin{aligned} -x + y &= 1 \\ \underline{+x \quad\quad} & \underline{\;+x} \\ y &= x + 1 \end{aligned}$$

No, neither of these equations are direct variation because they are not in the form $y = ax$

Ex: $2x + y = 0$

$$\begin{aligned} 2x + y &= 0 \\ \underline{-2x \quad\quad} & \underline{\;-2x} \\ y &= -2x \end{aligned}$$

Yes, $a = -2$

Ex: $4x - 5y = 0$

$$\begin{aligned} 4x - 5y &= 0 \\ \underline{-4x \quad\quad} & \underline{\;-4x} \\ \frac{-5y}{-5} &= \frac{-4x}{-5} \\ y &= \frac{4}{5}x \end{aligned}$$

Yes, $a = \frac{4}{5}$

For the graphs draw below, which equation represents direct variation? How do you know?

Graph B because it passes through the origin

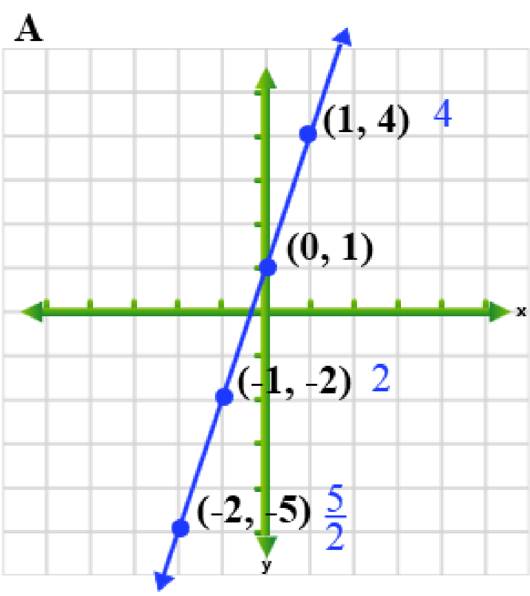
Now we will *prove* why this equation represents direct variation, based on its $\frac{y}{x}$ ratio.

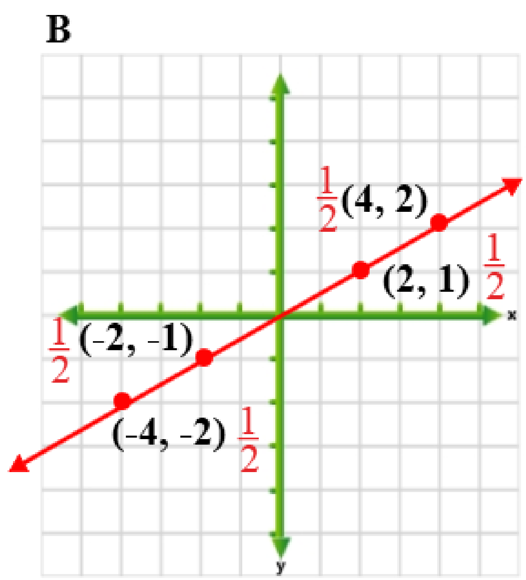
1. Fill in each ordered pair

2. For each ordered pair, divide *y* by *x*. $\left(\frac{y}{x}\right)$ Write your answer next to each ordered pair.

3. What do you notice about the ratios in graph A compared to the ratios in graph B?

In graph B the ratios are *constant* at ½ (therefore this is the *constant of variation*) while in graph A they are not constant. This is why graph B represents direct variation and graph A does not.



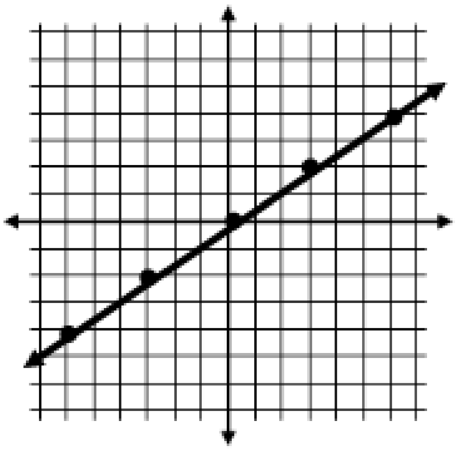


4. Now go back to graph A and this time instead of dividing *y* by *x*, pick any two consecutive ordered pairs and find $\frac{\Delta y}{\Delta x}$. Do this for all segments of the line. What do you notice?

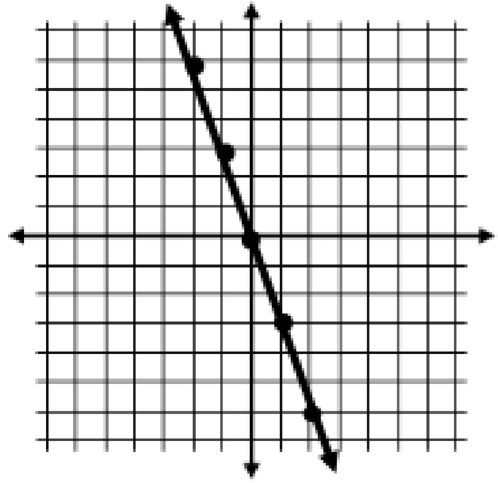
All have a ratio of 3. This is the slope. This equation is *linear*, but not direct variation.

2. Graph a direct variation equation. (Graph the same way as: $y = mx + b$)

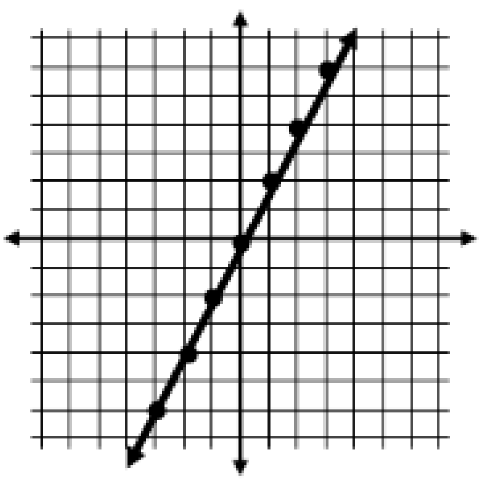
Ex: $y = \frac{2}{3}x$



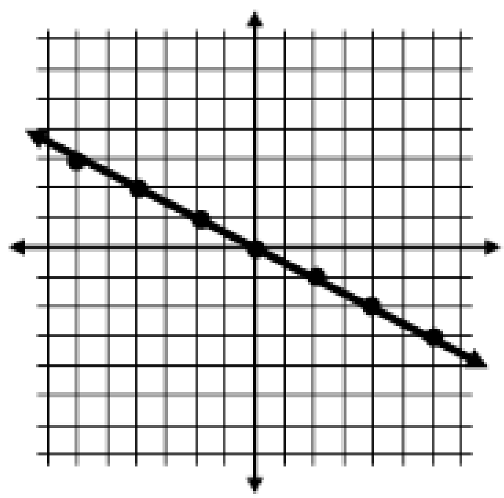
Ex: $y = -3x$



Ex: $y = 2x$

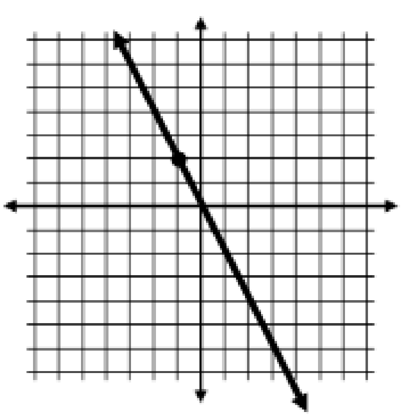


Ex: $y = -\frac{1}{2}x$



3. Write a direct variation equation.

Ex:



$y = ax$

$2 = a(-1)$

$-2 = a$

$y = -2x$

1. Start with general D.V. equation
2. Substitute in everything you know
3. Find constant of variation
4. Write final D.V. equation

Ex: The graph of a direct variation equation passes through the point (4, 6).

a) Write a direct variation equation relating x and y.

$y = ax$

$\frac{6}{4} = \frac{a(4)}{4}$

$\frac{3}{2} = a$

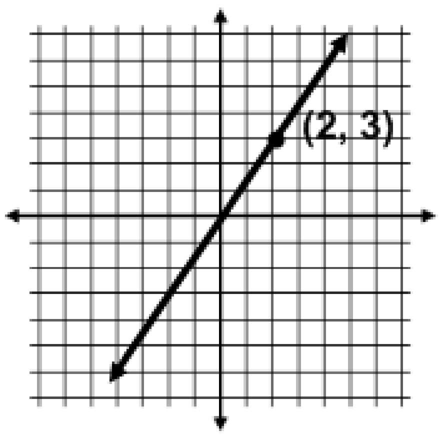
$y = \frac{3}{2}x$

b) Find y when $x = 24$.

$y = \frac{3}{2}(24)$

$y = 36$

Ex: Write a direct variation equation and find y when $x = 14$.



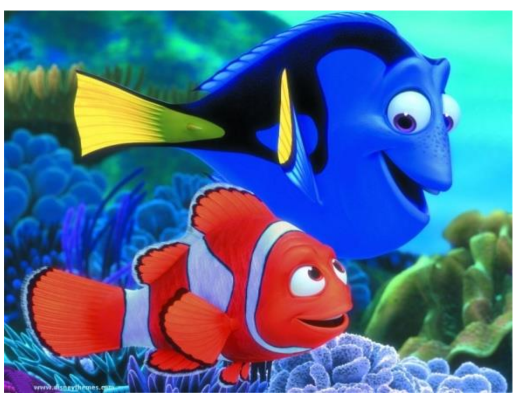
$y = ax$

$3 = a(2)$

$y = \frac{3}{2}x$

$y = 21$ when $x = 14$

Ex: The number s, of tablespoons of sea salt needed in a saltwater fish tank varies directly with the number w, of gallons of water in the tank. A pet shop owner recommends adding 100 tablespoons of sea salt to a 20 gallon tank.



a) Write a direct variation equation relating w and s.

$s = aw$

$100 = a(20)$

$5 = a$

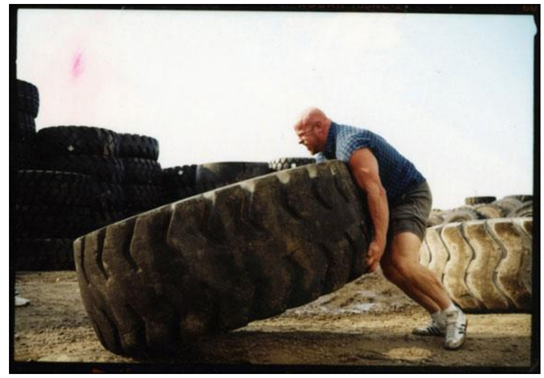
$s = 5w$

b) Find the number of tablespoons needed in a 30 gallon tank.

$s = 5(30)$

$s = 150$ tablespoons

Ex: An object that weighs 100 pounds on Earth would weigh just 6 pounds on Pluto. Assume that weight *p*, on Pluto varies directly with weight *e*, on Earth.



a) Write a direct variation equation relating *e* and *p*.

$p = ae$
$6 = a(100)$
$0.06 = a$
$p = 0.06e$

b) What would a 750 pound rock weigh on Pluto?

$p = 0.06(750)$
$p = 45$ pounds

Ex: The table shows the total cost *c*, of downloading *s* songs at an internet music site. Explain why *c* varies directly with *s*. Then write the direct variation equation.

y varies with *x* → $y = ax$
c varies with *s* → $c = as$

s	***c* ($)**
3	2.97
5	4.95
7	6.93

x and *y* vary directly if every time you do $\frac{y}{x}$ the answer is constant. In this case, we are dealing with *c* and *s*, so to check for direct variation you do $\frac{c}{s}$ to see if the answer is constant every time.

$\frac{2.97}{3} = 0.99$ $\frac{4.95}{5} = 0.99$ $\frac{6.93}{7} = 0.99$

Since every time you divide *c* by *s* you get 0.99, then this is direct variation with a constant of variation (*a*) of 0.99 so the equation is:

$c = 0.99s$

Ex: The table shows the total cost *c*, of buying *d* used DVD's at a music store.

y varies with *x* → $y = ax$
c varies with *d* → $c = ad$

d	***c* ($)**
3	25.77
6	51.54
9	77.31

a) Explain why *c* varies directly with *d*.

x and *y* vary directly if every time you do $\frac{y}{x}$ the answer is constant. In this case, we are dealing with *c* and *d*, so to check for direct variation you do $\frac{c}{d}$ to see if the answer is constant every time.

$\frac{25.77}{3} = 8.59$ $\frac{51.54}{6} = 8.59$ $\frac{77.31}{9} = 8.59$

Since every time you divide c by d you get 8.59, then this is direct variation with a constant of variation (a) of 8.59

b) Write the direct variation equation.

$c = 8.59d$