7.5: Special Types of Linear Systems

Goals: *Solve and identify when a system of equations has one solution, no solution or an infinite number of solutions
*Arrange systems so you can state the number of solutions without solving

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****Review/Preview****

- What is a solution to a linear system?
  1) An ordered pair that when substituted in works for both equations

  2) A point on both lines (the point of intersection)

- Then what would you say is the solution if you graphed two lines and they happened to be parallel?
  When are two lines parallel?
  If two lines are parallel they never intersect, which means there would be no solution to the system.
  Two lines are parallel when their slopes are the same but their y-intercepts are different.

- What would you say if you graphed two lines and they were the exact same line?
  When are two lines **exactly the same**?
  If two lines are the same then they have infinite points in common, which means there are infinite solutions.
  Two lines are exactly the same line if they have the same slope and same y-intercept.

- If two lines are **not** parallel, then what must be true about them?
  When are lines **not** parallel?
  They intersect exactly once which means they have one solution.
  If they are not parallel lines their slopes must be different. Their y-intercepts are irrelevant.

****RECALL****

Solve each equation or inequality.

Ex: $3(x + 4) = 3x + 16$

$3x + 12 = 3x + 16$
$12 = 16$
No solution

Ex: $4(2x + 6) = 8(x + 3)$

$8x + 24 = 8x + 24$
$24 = 24$
Any number

Ex: $2x - 3x + 6 \leq -(x - 10)$
$-x + 6 \leq -x + 10$
$6 \leq 10$
Any number

Ex: $3(6x - 1) > 2(9x - 1)$
$18x - 3 > 18x - 2$
$-3 > -2$
No Solution

***Regardless of if you are solving an equation or an inequality what is the general rule that applies to both types of problems?**

If you get a true statement then the solution is "any number"
If you get a false statement then the solution is "no solution"

Now we are going to apply this same concept to systems of equations…

Solve each system using the method of your choice:

Ex: $3x + 2y = 10$

$-\ 3x + 2y = 2$

$0 = 8$

No Solution

Ex: $x - 2y = -4$

$y = \frac{1}{2}x + 2$

Substitute: $x - 2\left(\frac{1}{2}x + 2\right) = -4$

$x - x - 4 = -4$

$-4 = -4$

Infinite Solutions

Ex: $5x + 3y = 6$

$-5x - 3y = 3$

No Solution

Ex: $y = 2x - 4$

$-6x + 3y = -12$

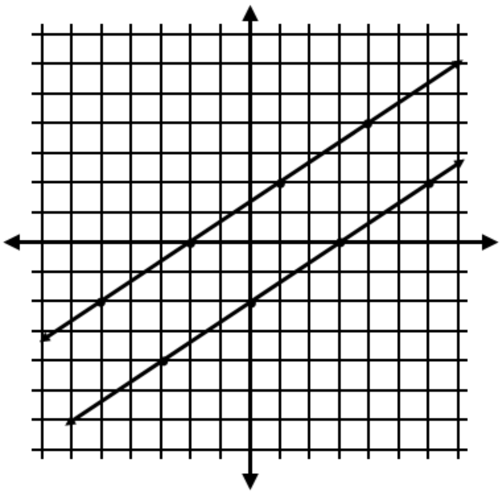
Infinite Solutions

Solve each system by graphing.

Ex: $2x - 3y = 6$

$2x - 3y = -4$

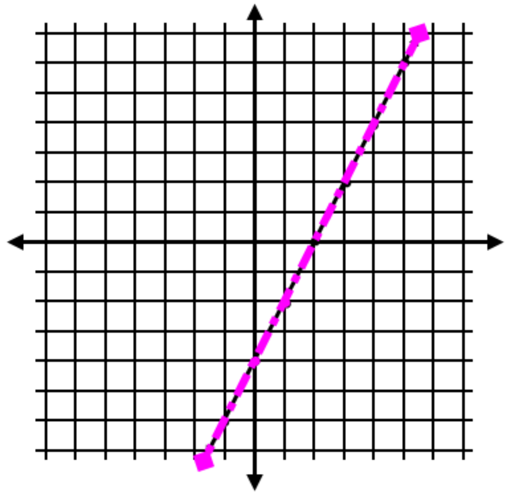
No solution



Ex: $4x - 2y = 8$

$y = 2x - 4$

(infinite solutions)



Identify the number of solutions of a linear system:

· A system of equations will have **no solution** when the two lines are: Parallel

They are ______parallel________________ when: They have the same slope but different *y*-intercepts

· A system of equations will have an **infinite number of solutions** when the two lines are: exactly the same

They are ____the same line___________________ when: they have the same slope and *y*-intercept

· A system of equations will have exactly **one solution** when the two lines are: Not Parallel

They are ______not Parallel___________________ when: Their slopes are different. The *y*-intercept is irrelevant

Number of Solutions	**Slopes and *y*-intercepts**
One	m = different b = same or different
None	m = same b = different
Infinite	m = same b = same

If you can quickly identify the slope and *y*-intercept of each line, then you can state how many solutions the system has **without solving.**

- What do you need to do to be able to quickly identify the slope and *y*-intercept of a line?

  The line needs to be in slope-intercept form

Without solving the system, tell whether there is one solution, no solution or infinitely many solutions.

Ex: $5x + y = -2$
$-10x - 2y = 4$

$y = -2 - 5x$
$y = -2 - 5x$

Infinite Solutions

Ex: $6x + 2y = 3$
$6x + 2y = -5$

$y = -3x + 1.5$
$y = -3x - 2.5$

No solution

Ex: $-3x + 5y = 6$
$6x - 10y = -12$

Infinite Solutions

Ex: $9x - 5y = 12$
$9x - 5y = 8$

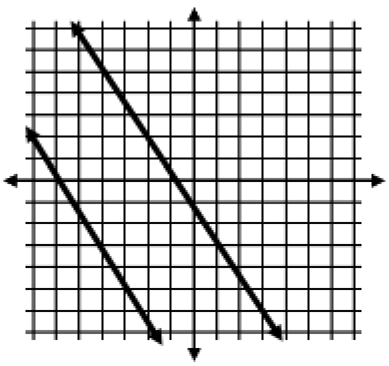
No solution

Ex: $x - 3y = -15$
$2x - 3y = -18$

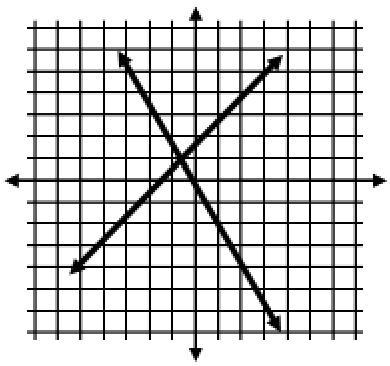
One Solution

Use the graphs below to show a system of equations with:

a. No solution



b. One solution



c. Infinitely many solutions