9.8: Factor Polynomials Completely

Goals: *Factor a common binomial
*Factor polynomials by grouping
*Factor polynomials completely

Factor a common binomial:

Ex: $2x(x + 4) – 3(x + 4)$

$(x + 4)(2x – 3)$

Ex: $4x(x – 3) + 5(x – 3)$

$(x – 3)(4x + 5)$

Ex: $3y^2(y – 2) + 5(2 – y)$

$3y^2(y – 2) + –5(y – 2)$
$(y – 2)(3y^2 – 5)$

Ex: $2y^2(y – 5) – 3(5 – y)$

$2y^2(y – 5) + 3(y – 5)$
$(2y^2 + 3)(y – 5)$

Ex: $x(x – 2) + (x – 2)$

$(x + 1)(x – 2)$

Factor by grouping:

Ex: $x^3 + 3x^2 + 5x + 15$

$(x^3 + 3x^2) + (5x + 15)$
$x^2(x + 3) + 5(x + 3)$
$(x^2 + 5)(x + 3)$

Ex: $y^2 + y + yx + x$

$(y^2 + y) + (yx + x)$
$y(y + 1) + x(y + 1)$
$(y + x)(y + 1)$

Ex: $a^3 + 3a^2 + a + 3$

$(a^2 + 1)(a + 3)$

Ex: $x^3 + 2x^2 + 8x + 16$

$(x^2 + 8)(x + 2)$

Ex: $r^2 + 4r + rs + 4s$

$(r + s)(r + 4)$

Factoring by grouping (rearrange first):

Ex: $x^3 - 6 + 2x - 3x^2$

$x^3 - 3x^2 + 2x - 6$
$(x^2 + 2)(x - 3)$

Ex: $y^2 + 2x + yx + 2y$

$y^2 + 2y + yx + 2x$
$(y + x)(y + 2)$

Ex: $x^3 - 10 - 5x + 2x^2$

$(x^2 - 5)(x + 2)$

Factor completely:

Ex: $n^2 + 2n - 1$

Prime- the only factors of –1 are 1 and –1, and they don't add up to 2. Can't be factored.

Ex: $4x^3 - 44x^2 + 96x$

$4x(x^2 - 11x + 24)$
$4x(x - 8)(x - 3)$

Ex: $50h^4 - 2h^2$

$2h^2(25h^2 - 1)$ *difference of two squares
$2h^2(5h - 1)(5h + 1)$

Ex: $3x^3 - 12x$

$3x(x^2 - 4)$ *difference of two squares
$3x(x + 2)(x - 2)$

Ex: $2y^3 - 12y^2 + 18y$

$2y(y^2 - 6y + 9)$ *perfect square trinomial
$2y(y - 3)^2$

Ex: $m^3 - 2m^2 - 8m$

$m(m^2 - 2m - 8)$
$m(m - 4)(m + 2)$

Ex: $x^2 - 4x - 3$

Prime, 3 and –1 and –3 and 1 can't add up to –4

Ex: $3x^3 - 21x^2 - 54x$

$3x(x^2 - 7x - 18)$
$3x(x - 9)(x + 2)$

Ex: $x^4 - x^2 + 3x^2 - 3$

Group: $(x^4 - x^2) + (3x^2 - 3)$
Factor: $x^2(x^2 - 1) + 3(x^2 - 1)$
$(x^2 + 3)(x^2 - 1)$ *difference of two squares
$(x^2 + 3)(x - 1)(x + 1)$

Ex: $8d^3 + 24d$

$8d(d^2 + 3)$

Solve:

Ex: $3x^3 + 18x^2 = -24x$

$3x^3 + 18x^2 + 24x = 0$
$3x(x^2 + 6x + 8) = 0$
$3x(x + 4)(x + 2) = 0$
$x = 0, x = -4, x = -2$

Ex: $w^3 - 8w^2 + 16w = 0$

$w(w^2 - 8w + 16) = 0$
$w(w - 4)(w - 4) = 0$
$w = 0, w = 4$

Ex: $c^3 - 7c^2 + 12c = 0$

$c = 0, c = 3, c = 4$

Ex: $2x^3 - 18x^2 = -36x$

$x = 0, x = 3, x = 6$

Terrarium A terrarium in the shape of a rectangular prism has a volume of 4608 cubic inches. Its length is more than 10 inches. The dimensions of the terrarium are shown. Find the length, width and the height of the terrarium.

$V = lwh$
$4608 = w(w + 4)(36 - w)$
$4608 = 32w^2 + 144w - w^3$
$-w^3 + 32w^2 + 144w - 4608 = 0$
$(-w^3 + 32w^2) + (144w - 4608) = 0$
$-w^2(w - 32) + 144(w - 32) = 0$
$(-w^2 + 144)(w - 32) = 0$
$-1(w^2\ 0 - 144)(w - 32) = 0$
$-1(w - 12)(w + 12)(w - 32) = 0$
$w = 12, w = 0 - 12, w = 32$

w can't be negative so -12 is not a solution
if $w = 32$, then the length would be $36 - 32 = 4$ (this is not more than 10)

So, $w = 12$