3.4: Solve Equations with Variables on Both Sides

Goals: *Solve an equation with variables on both sides

*Decide if an equation has one solution, no solution or an infinite number of solutions

---

To solve an equation with variables on both sides:

1. Simplify each side of the equation first. Then get variable terms together on one side of equals sign. Can only move a variable by adding or subtracting.

2. Solve like normal.

Ex: $7 - 8x = 4x - 17$

$\underline{+8x \quad +8x}$

$7 = 12x - 17$

$\underline{+17 \qquad +17}$

$\dfrac{24}{12} = \dfrac{12x}{12}$

$x = 2$

Ex: $13 + 5x = 2x - 7$

$\underline{-2x \quad -2x}$

$13 + 3x = -7$

$\underline{-13 \qquad -13}$

$\dfrac{3x}{3} = \dfrac{-20}{3}$

$x = -\dfrac{20}{3}$

Ex: $9x - 5 = \frac{1}{4}(16x + 60)$

$9x - 5 = 4x + 15$

$\underline{-4x \qquad -4x}$

$5x - 5 = 15$

$\underline{+5 \quad +5}$

$\dfrac{5x}{5} = \dfrac{20}{5}$

$x = 4$

Ex: $4x - 5 = \frac{1}{5}(5x + 20)$

$4x - 5 = x + 4$

$\underline{-x \qquad -x}$

$3x - 5 = 4$

$\underline{+5 \quad +5}$

$\dfrac{3x}{3} = \dfrac{9}{3}$

$x = 3$

Ex: A car dealership sold 78 new cars and 67 used cars this year. The number of new cars sold by the dealership has been increasing by 6 cars each year. The number of used cars sold by the dealership has been decreasing by 4 cars each year. If these trends continue, in how many years will the number of new cars sold be twice the number of used cars sold?

New = Twice Used   (x = number of years)

$78 + 6x = 2(67 - 4x)$

$78 + 6x = 134 - 8x$

$78 + 14x = 134$

$14x = 56$

$x = 4$ 4 years

Ex: A music website sold 94 single songs and 67 albums today. The number of single downloads has been increasing by 22 each day and the number of album downloads has been decreasing by 5 each day. If these trends continue, in how many days will the number of single downloads be ten times the number of album downloads?

$$\text{Singles} = 10 \text{ times Albums} \qquad x = \text{number of days}$$
$$94 + 22x = 10(67 - 5x)$$
$$94 + 22x = 670 - 50x$$
$$94 + 72x = 670$$
$$72x = 576$$
$$x = 8 \text{ days}$$

RECALL (from 1.4)

SOLUTION: The value of the variable that makes the equation true.

The directions "Solve" really mean: Tell me what x can be

When solving equations with variables on both sides the equation can have:

1 solution: $x = \#$ (The number you get x equaling is the only number that x can be)

No solutions: There is no number that x can possibly be ($0 = 10$ or any false statement indicates this)

or

All Real Numbers (could be solutions): x can be any number ($0 = 0$ or any true statement indicates this)

Solve:

Ex: $3x = 3(x + 4)$

$$3x = 3x + 12$$
$$\underline{-3x \quad -3x}$$
$$0 = 12$$

No Solution

Ex: $2x + 10 = 2(x + 5)$

$$2x + 10 = 2x + 10$$
$$\underline{-2x \qquad -2x}$$
$$10 = 10$$

Any Number

Ex: $5x - 6 = (x - 1)5$

$5x - 6 = 5x - 5$
$-6 = -5$
No Solution

Ex: $4(3x + 2) = 2(6x + 4)$

$12x + 8 = 12x + 8$
$8 = 8$
Any Number

Ex: $3(4x + 6) = 9(2x + 2)$

$12x + 18 = 18x + 18$
$18 = 6x + 18$
$0 = 6x$
$0 = x$

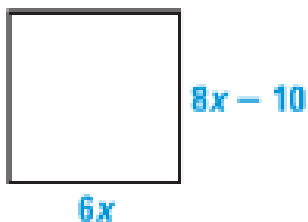**Ex:** $-3(2x - 7) = 6(4 - x)$

$-6x + 21 = 24 - 6x$
$21 = 24$
No Solution

Find the perimeter of the square.

Ex:

Since it's a square you know that both sides are equal

$8x - 10 = 6x$
$-10 = -2x$
$5 = x$
If $x = 5$, then one side is 30 units.
The perimeter is 120 units