Find the probability of an event with or without replacement :

The probability of an outcome of an event is the ratio of the number of ways that outcome can occur to the total number of different possible outcomes of the event.

$$\text{Probability of an outcome} = \frac{\textit{number of ways the outcome can occur}}{\textit{total number of possible outcomes}}$$

Whether or not an item is replaced in, or returned to, a group affects the probability. If the item is not replaced, the possible outcomes for items being chosen later are different from the original probability of choosing an item.

Example
Marc is choosing a marble from a bag containing 6 red marbles, 3 blue marbles, and 5 green marbles. He picks a green marble. Then Angelina picks a marble. What is the probability that Angelina picked a green marble if Marc replaced his marble?

Remember that probability is the ratio of desired outcomes to all outcomes. For this experiment all outcomes are the total number of marbles in the bag. Namely, $6+3+5=14$ marbles. The desired outcome for this experiment is green. Since Marc put his back, there are still 5 green marbles, so the probability that Angelina would draw a green marble would be $\frac{5}{14}$.

Now let's figure the probability that Angelina picked a green marble if Marc did not replace his marble.

If Marc keeps his green marble, then what is left in the bag is decreased by one marble so, there are only 13 marbles left in the bag. Likewise because he kept a green marble there are only 4 green marble left. So the probability that Angelina would draw a green marble is only $\frac{4}{13}$.

Predict the outcome of a probability experiment :

Experimental probability is the probability of an event based on an actual experiment. Theoretical probability is used when an observation is not based on an actual experiment. In other words, you will infer, predict, or guess what will

happen. A probability of 0 means that the event will not occur. A probability of 1 means that the event is certain to happen.

The difference between experimental probability and theoretical probability is that with experimental probability, you actually record all the outcomes. The greater the number of trials in an experiment, the closer the experimental probability will come to the theoretical probability. To find the theoretical probability of an event, use the following formula:

$$P(E) = \frac{\textit{number of favorable outcomes}}{\textit{total number of outcomes}}$$

Example:
Let's take a look at the rolling of a die. A die is sometimes called a number cube and it is numbered 1 through 6.

- Let's find the probability that you roll a 3.

  The number of favorable outcomes is 1 (3). The total number of outcomes is 6 (1, 2, 3, 4, 5, 6). So, $P(E) = \frac{1}{6}$.

- Now find the probability that you roll an 8.

  The number of favorable outcomes is 0 because 8 does not appear on a die. The total number of outcomes is 6 (1, 2, 3, 4, 5, 6). So, $P(E) = \frac{0}{6} = 0$. This event cannot happen.

- Now find the probability that you roll a number less than 7.

  The number of favorable outcomes is 6 because numbers less than 7 on a die are 1, 2, 3, 4, 5, 6. The total number of outcomes is also 6 (1, 2, 3, 4, 5, 6). So, $P(E) = \frac{6}{6} = 1$. This event is certain to occur.

- Now find the probability that you roll a number greater than 4.

The number of favorable outcomes is 2 because the numbers on a die that are greater than 4 are 5 and 6. The total number of outcome is 6 (1, 2, 3, 4, 5, 6). So, $P(E) = \frac{2}{6} = \frac{1}{3}$. Make sure you reduce all your fractions.

Independent and Dependant events :

Independent events

Independent events are events that have no influence on each other. To find the probability of independent events occurring, multiply the probabilities of the individual events.

Example
There are 25 pairs of socks in Catherine's sock drawer: 6 are black, 10 are white, and 9 are brown. What is the probability of picking a brown pair out of the drawer, replacing it, and then picking a black pair our of the drawer?

Does picking the first pair have any effect on picking the second pair? No. Since the first pair is replaced, the total number of pairs in the drawer is the same for each drawing.

- The probability of a brown pair is $\frac{9}{25}$
- The probability of a black pair is $\frac{6}{25}$
- The probability of a brown pair first then a black pair is $\frac{9}{25} \cdot \frac{6}{25} = \frac{54}{625}$

Dependent events

Dependent events are events that are influenced by other events. To find the probability of dependent events occurring, multiply the probabilities of the individual events.

Example
Using the same drawer of socks as in the previous example, what is the probability of picking a brown pair out of a drawer, leaving it out, and then picking a black pair out of the drawer?

Does picking the first pair have any effect on picking the second pair? Yes. When you pick the first pair out of the drawer without replacing it, the total number of pairs left is reduced by 1.

- Probability of a brown pair is $\frac{9}{25}$
- Probability of a black pair is $\frac{6}{24} = \frac{1}{4}$ (reduce all fractions)

- The probability of a brown pair first then a black pair is $\frac{9}{25} \cdot \frac{1}{4} = \frac{9}{100}$

Determine the number of possible outcomes :

Counting Techniques

You can use different techniques to find the number of possible outcomes of an event. Tree diagrams and the Fundamental Counting Principle are two techniques that work well.

Tree Diagrams

Tree diagrams use branches to show the outcome set of a situation

Fundamental counting principle

When you need to find too many possible outcomes for a tree diagram, you should use the Fundamental Counting Principle. The Fundamental Counting Principle uses multiplication to find the number of possible outcomes.

Combinations and Permutations :

When counting the number of ways to choose a group of items or events, the result will be different depending on whether or not the order in which the objects are chosen is important.

Permutation

A permutation is an arrangement of items or events in a particular order. A different order of the same items is a different permutation. The number of permutations of n items or events taken g at a time is calculated with the formula: $P(n, g) = \frac{n!}{(n-g)!}$.

Combination

If the order of the items or events does not matter, the arrangement is a combination. The number of combinations of n items or events taken g at a time is calculated with the formula: $C(n, g) = \frac{n!}{(n-g)!\, g!}$.