Notes 7.5- Operations with Radical Expressions

Why?

Golden rectangles are used by artists and architects to create beautiful designs. Many golden rectangles appear in the Parthenon in Athens, Greece. The ratio of the lengths of the sides of a golden rectangle is $\frac{2}{\sqrt{5}-1}$. In this lesson, you will learn to simplify radical expressions like $\frac{2}{\sqrt{5}-1}$.

Simplify Radicals: The properties you have used to simplify radical expressions involving square roots also hold true espressions involving *n*th roots. (page 439).

n	n^2	n^3	n^4	n^5
2				
3				
4				
5				

Key Concept **Product Property of Radicals**

Words For any real numbers a and b and any integer $n > 1$, $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$, if n is even and a and b are both nonnegative or if n is odd.

Examples $\sqrt{2} \cdot \sqrt{8} = \sqrt{16}$ or 4 and $\sqrt[3]{3} \cdot \sqrt[3]{9} = \sqrt[3]{27}$ or 3

For a radical to be in simplest form, the radical must contain no factors that are *n*th powers of an integer or polynomial.

EXAMPLE 1 Simplify Expressions with the Product Property

Directions: **Simplify**

a. $\sqrt{25a^4b^9}$ **b.** $\sqrt[3]{125m^{30}p^{20}}$ **c.** $\sqrt[3]{-108x^9y^{12}z^{17}}$

The quotient Property of Radicals is another property used to simplify radicals.

Key Concept **Quotient Property of Radicals**

Words For any real numbers a and $b \neq 0$ and any integer $n > 1$,

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, if all roots are defined.

Examples $\frac{\sqrt{27}}{\sqrt{3}} = \sqrt{9}$ or 3 $\sqrt[3]{\frac{x^6}{8}} = \frac{\sqrt[3]{x^6}}{\sqrt[3]{8}} = \frac{x^2}{2}$ or $\frac{1}{2}x^2$

To eliminate radicals from a denominator or fractions from a radicand, **rationale the denominator**. To rationalize, multiply the numerator and denominator by a quantity so that the radicand has an exact root.

If the denominator is:	**Multiply the numerator & denominator by:**	**Examples**
$\sqrt{b}$	$\sqrt{b}$	$\frac{2}{\sqrt{5}} =$
$\sqrt[n]{b^x}$	$\sqrt[n]{b^{n-x}}$	$\frac{3}{\sqrt[4]{2}} =$

EXAMPLE 2 Simplify Expressions with the Quotient Property

Directions: **Simplify**

a. $\sqrt{\frac{y^8}{x^7}}$

b. $\sqrt[3]{\frac{2}{9x}}$

c. $\sqrt[3]{\frac{8}{25}}$

Here is a summary of the rules used to simplify radicals:

Concept Summary — Simplifying Radical Expressions

A radical expression is in simplified form when the following conditions are met.

- The index *n* is as small as possible.
- The radicand contains no factors (other than 1) that are *n*th powers of an integer or polynomial.
- The radicand contains no fractions.
- No radicals appear in a denominator.

Operations with Radicals: You can use the Product and Quotient Properties to multiply and divide some radicals.

EXAMPLE 3 Multiply Radicals

Directions: **Simplify**

a. $5\sqrt[3]{100a^2} \bullet \sqrt[3]{10a}$

b. $2\sqrt[4]{8x^3y^2} \bullet 3\sqrt[4]{2x^5y^2}$

Radicals can be added and subtracted if the radicals are like terms. Radicals are **like radical expressions** if both the index and the radicand are identical.

Like: $\sqrt{3b}$ and $4\sqrt{3b}$ Unlike: $\sqrt{3b}$ and $\sqrt[3]{3b}$ Unlike: $\sqrt{2b}$ and $\sqrt{3b}$

EXAMPLE 4 Add and Subtract Radicals

Directions: Simplify

a. $3\sqrt{45} - 5\sqrt{80} + 4\sqrt{20}$ **b.** $2\sqrt[3]{125a^4} - 5\sqrt[3]{8a}$

You can also multiply radicals using the FOIL method as you do when multiplying binomials.

EXAMPLE 5 Multiply Radicals

Directions: Simplify

a. $(2\sqrt{3} + 3\sqrt{5})(3 - \sqrt{3})$ **b.** $(7\sqrt{2} - 3\sqrt{3})(7\sqrt{2} + 3\sqrt{3})$

Binomials of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$, where a, b, c, and d are rational numbers, are called **conjugates** of each other. You can use conjugates to rationalize denominators.

Real-World EXAMPLE 6 Use a Conjugate to Rationalize a Denominator

ARCHITECTURE: Refer to the beginning of the lesson and simplify the expression $\frac{2}{\sqrt{5}-1}$.

Simplify: $\frac{6-\sqrt{3}}{\sqrt{3}+4}$

Notes 7.6- Rational Exponents

Rational Exponents and Radicals:

Key Concept $b^{\frac{1}{n}}$

Words For any real number b and any positive integer n, $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and n is even. When $b < 0$ and n is even, a complex root may exist.

Examples $27^{\frac{1}{3}} = \sqrt[3]{27}$ or 3 $\quad (-16)^{\frac{1}{2}} = \sqrt{-16}$ or $4i$

EXAMPLE 1 Radical and Exponential Forms

a. Write $a^{\frac{1}{7}}$ in radical form. **b. Write $a^{\frac{7}{4}}$ in radical form.**

c. Write $\sqrt{w}$ in exponential form.

d. Write $\sqrt[3]{c^{-5}}$ in exponential form.

The rules for negative exponents also apply to negative rational exponents.

Rational Exponents

Let $b^{\frac{1}{n}}$ be an nth root of b, and let m be a positive integer.

- $b^{\frac{1}{n}} = \sqrt[n]{b}$
- $b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m$
- $b^{\frac{-m}{n}} = \frac{1}{b^{\frac{m}{n}}} = \frac{1}{\left(\sqrt[n]{b}\right)^m}, b \neq 0$

EXAMPLE 2 Evaluate Expressions with Rational Exponents

Directions: Evaluate each expression.

a. $49^{\frac{-1}{2}}$ **b.** $32^{\frac{2}{5}}$ **c.** $64^{\frac{-2}{3}}$ **d.** $(-243)^{\frac{-3}{5}}$ **e.** $\sqrt[3]{10^{-9}}$

Simplify Expressions: Properties of powers you learned in Lesson 6-1 apply to rational exponents. Write each expression with all positive exponents. Also, any exponents in the denominator of a fraction must be positive integers. It may be necessary to rationalize the denominator.

Property	Example
Product of Power Property $a^m \cdot a^n = a^{m+n}$	$3^{\frac{1}{4}} \bullet 3^{\frac{3}{4}}$
Power of a Power Property $(a^m)^n = a^{mn}$	$(5^{\frac{2}{3}})^{\frac{1}{2}}$
Power of a Product Property $(ab)^m = a^m b^m$	$(64^{\frac{1}{3}} \bullet 8^{\frac{1}{3}})^2$
Negative Exponent Property $a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{\frac{-1}{2}}$
Quotient of Powers Property $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{7^{\frac{3}{4}}}{7^{\frac{1}{4}}}$
Power of a Quotient Property $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{5^2}{8^2}\right)^{\frac{1}{2}}$

EXAMPLE 4 Simplify Expressions with Rational Exponents

<u>Directions:</u> Simplify each expression.

a. $p^{\frac{1}{4}} \bullet p^{\frac{9}{4}}$ **b.** $x^{\frac{-2}{3}}$ **c.** $\frac{b^3}{c^{\frac{1}{2}}} \bullet \frac{c}{b^{\frac{1}{3}}}$

EXAMPLE 5 Simplify Radical Expressions

Directions: Simplify each expression.

a. $\sqrt[3]{16x^4}$ **b.** $(27x^5y^4)^{\frac{1}{2}}$ **c.** $\sqrt[3]{-64x^9y^{12}x^{17}}$ **d.** $\dfrac{y^{\frac{1}{2}}+1}{y^{\frac{1}{2}}-1}$

Concept Summary

Expressions with Rational Exponents

An expression with rational exponents is simplified when all of the following conditions are met.

- It has no negative exponents.
- It has no exponents that are not positive integers in the denominator.
- It is not a complex fraction.
- The index of any remaining radical is the least number possible.

HW: Page 443 #19-49 odd and Page 450 #17-37 odd and #45-51 odd