Lesson 9.4-Graphing Rational Functions

Vertical and Horizontal Asymptotes:

- A rational function has an equation of the form $f(x)=\dfrac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ are polynomial functions and $b(x)\neq 0$.
- In order to graph a rational function, it is helpful to locate the zeros and asymptotes.
- A **zero** of a rational function $f(x)=\dfrac{a(x)}{b(x)}$ occurs at every value of x for which $a(x)=0$ (page 577).

  numerator
- $f(x)$ has a **vertical asymptote** whenever $b(x)=0$.

  denominator
- $f(x)$ has at most one **horizontal asymptote:**
  a. If the degree $a(x)$ > the degree of $b(x)$, there is **no horizontal asymptopte** (but it may have a slant or oblique asymptote).
  b. If the degree of $a(x)$ < the degree of $b(x)$, the **horozontal asymptote is the line y = 0**.
  c. If the degree of $a(x)$ = the degree of $b(x)$, **the horozontal asymptote is the line y = leading coefficient of $a(x)$/ leading coefficient of $b(x)$**.

Directions: Graph and analyze each function.

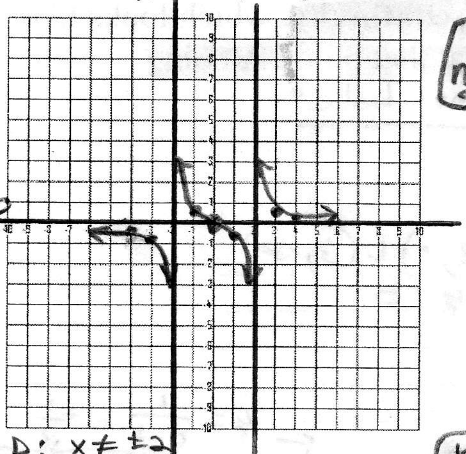
1. $f(x)=\dfrac{x}{x^2-4}$ option (b), $\boxed{HA: y=0}$

$\dfrac{x}{(x+2)(x-2)}$ $\boxed{VA: x=\pm 2}$

X=-2 X=2

xintercept: numerator: set equal to 0 X=0

y=0



X	Y
1	-1/3
-1	1/3
3	3/5
4	1/3
-3	-3/5
-4	-1/3

D: $x\neq\pm 2$

R: R

x-int: + y-int: (0,0)

VA: $x=\pm 2$

HA/SA: y=0

End behaviors:

as $x\to-\infty$, $y\to 0$

as $x\to -2$, $y\to\infty$

as $x\to -2$, $y\to\infty$

as $x\to 2$, $y\to-\infty$

as $x\to 2$, $y\to\infty$

as $x\to\infty$, $y\to 0$

y intercept: make x=0 and solve:

$\dfrac{0}{0^2-4}=\dfrac{0}{-4}=\boxed{0}$

2. $f(x)=\dfrac{1x^2+2}{1x^2-x-6}=\dfrac{x^2+2}{(x-3)(x+2)}$ option c: $\boxed{HA: y=1}$ $\boxed{VA: x\neq 3,\ x\neq -2}$

xintercept

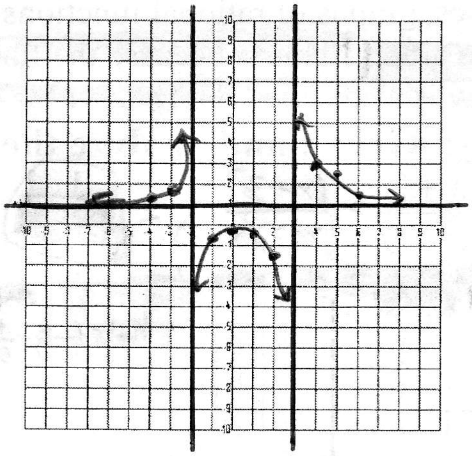
$x^2+2=0$

$x^2=-2$

$\sqrt{x^2}=-2$

$x=\pm i\sqrt{2}$

not real, so no xinter.



y intercept:

$\dfrac{0^2+2}{0^2-0-6}=-\dfrac{1}{3}$

$\boxed{0,-\frac{1}{3}}$

X	Y
-1	-3/4
+1	-1/2
2	-1.5
-3	1 5/6
-4	9/7=1 2/7
4	3
5	27/14=1 13/14
6	38/24= 17/12

D: $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$

R: $(-\infty,-1/3]\cup(1,\infty)$

VA: $x\neq -2$, $x\neq 3$

HA/SA: y=1

x-int: none

y-int: (0, -1/3)

End behaviors:

as $x\to-\infty$, $y\to 1$; as $x\to -2$, $y\to\infty$

as $x\to -2$, $y\to-\infty$; as $x\to 3$, $y\to-\infty$

as $x\to 3$, $y\to\infty$; as $x\to\infty$, $y\to 1$

Oblique (slant) Asymptotes and Point Discontinuity: $y=mx+b$

- An **oblique asymptote**, sometimes called a ***slant asymptote***, is an asymptote that is neither horizontal nor vertical. This occur in the rational function $f(x)=\frac{a(x)}{b(x)}$, if the degree of $a(x)$ minus the degree of $b(x)=1$. The equation of the asymptote is $f(x)=\frac{a(x)}{b(x)}$ with no remainder (page 579).

Ex: $\frac{x^3-8}{x^2-9}$ would have SA

$\frac{x^2-9}{2}$ would not have SA

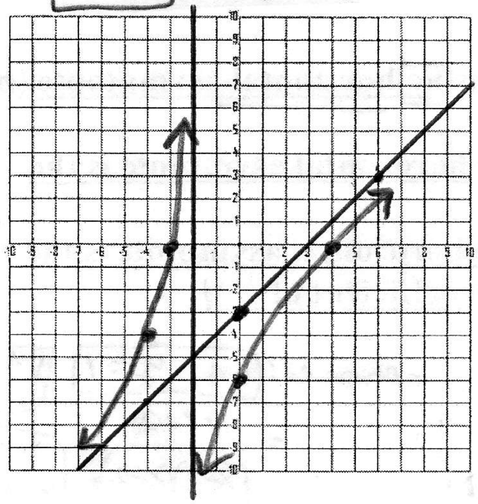
Directions: Graph and analyze the function.

3. $f(x)=\frac{x^2-x-12}{x+2} = \frac{(x-4)(x+3)}{(x+2)}$ ← x-intercept zeros: $x=4$, $x=-3$

← VA: $x \neq -2$

NO HA bc $a(x) > b(x)$

y intercept $\frac{(0-4)(0+3)}{0+2} = \frac{(-4)(3)}{2} = \frac{-12}{2} = -6$, so $(0,-6)$



D: $(-\infty,-2) \cup (-2,\infty)$

R: $\mathbb{R}$

VA: $x=-2$

HA/SA: $y=x-3$

x-int: $(4,0)$ $(-3,0)$

y-int: $(0,-6)$

~~interval:~~

x	y
-4	8/-2 = -4

SA: (long division)

$$\begin{array}{r} x-3 \\ x+2 \overline{\big) x^2-x-12} \\ -(x^2+2x) \\ \hline -3x-12 \\ -(-3x-6) \\ \hline -6 \end{array}$$

SA is line: $y=x-3$

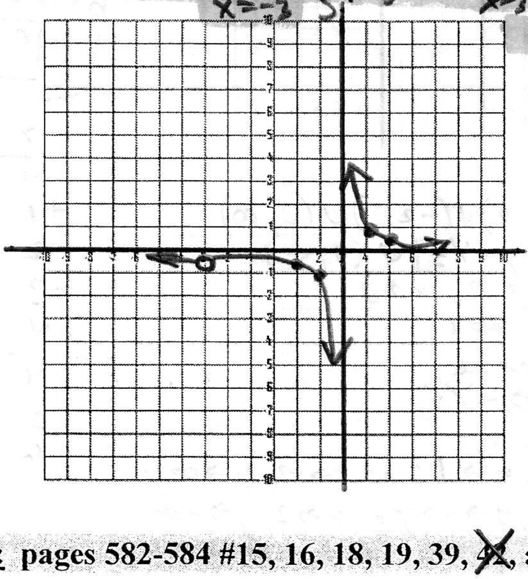
-6 ignore remainder

- In some cases, graphs of rational functions may have **point discontinuity**, which looks like a hole in the graph. This is because the function is undefined at that point. A point discontinuity is also called the **removable discontinuity**. "donut hole"

Directions: Graph and analyze the function.

4. $f(x)=\frac{x+3}{x^2-9} = \frac{\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} = \frac{1}{(x-3)}$

HOLE $x+3=0$, $x=-3$ } plug into $\frac{1}{x-3} = -\frac{1}{6}$, so hole $(-3,-\frac{1}{6})$



D: $(-\infty,-3) \cup (-3,3) \cup (3,\infty)$

R: $y \neq -1/6$, $y \neq 0$

VA: $x=3$

HA/SA: $y=0$

x-int: none

y-int: $(0,-\frac{1}{3})$ ← $y=\frac{1}{0-3}=\frac{-1}{3}$

Hole: $(-3,-\frac{1}{6})$

Using $\frac{1}{x-3}$

X	Y
2	-1
1	-1/2
4	1
5	1/2

simplified equation

HW: pages 582-584 #15, 16, 18, 19, 39, ~~42~~, and #51-61 ~~all~~ odd