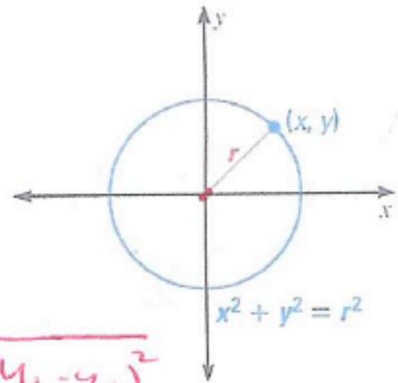
Notes 10.3 – Circles

GOAL 1 GRAPHING AND WRITING EQUATIONS OF CIRCLES

A **circle** is the set of all points (x, y) that are equidistant from a fixed point, called the **center** of the circle. The distance r between the center and any point (x, y) on the circle is the **radius.**



The distance formula can be used to obtain an equation of the circle whose center is the origin and whose radius is r. Because the distance between any point (x, y) on the circle and the center $(0, 0)$ is r, you can write the following.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\left(\sqrt{(x-0)^2 + (y-0)^2}\right)^2 = (r)^2 \qquad \text{Distance formula}$$

$$(x-0)^2 + (y-0)^2 = r^2 \qquad \text{Square both sides.}$$

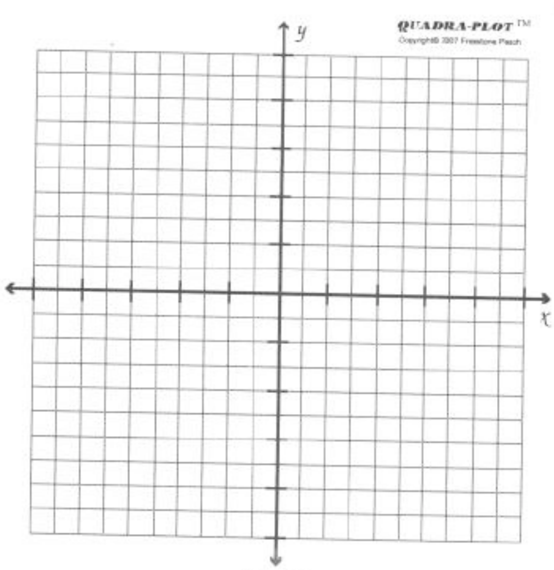
$$x^2 + y^2 = r^2 \qquad \text{Simplify.}$$

Key Concept — **Equations of Circles**

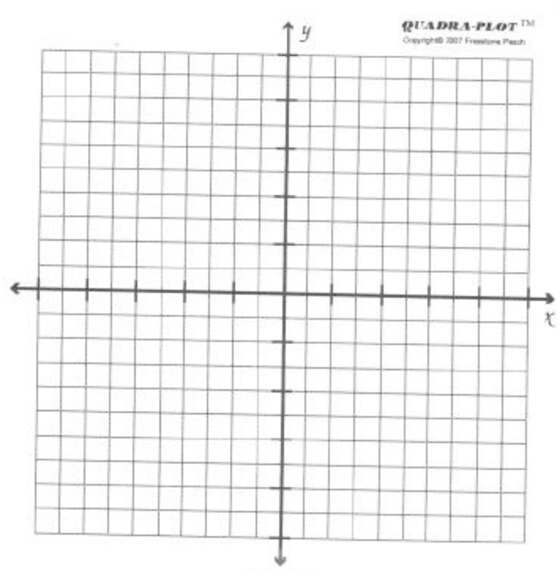
Standard Form of Equation	$x^2 + y^2 = r^2$	$(x-h)^2 + (y-k)^2 = r^2$
Center	(0, 0)	(h, k)
Radius	r	r

EXAMPLE 1 *Graphing an Equation of a Circle*

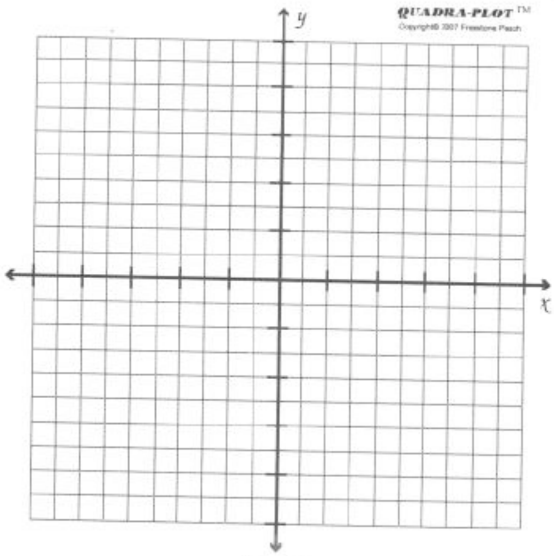
1. $x^2 + y^2 = 9$



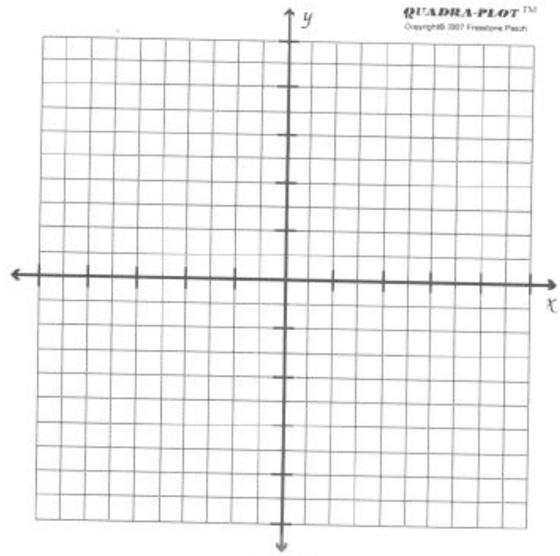
2. $x^2 + y^2 = 16$



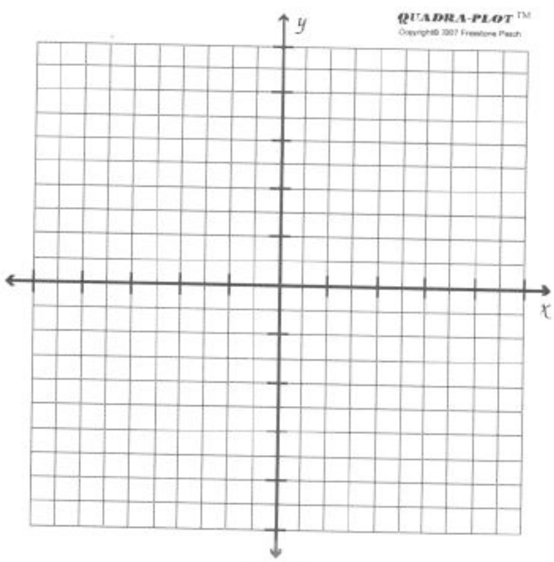
3. $y^2 = 64 - x^2$



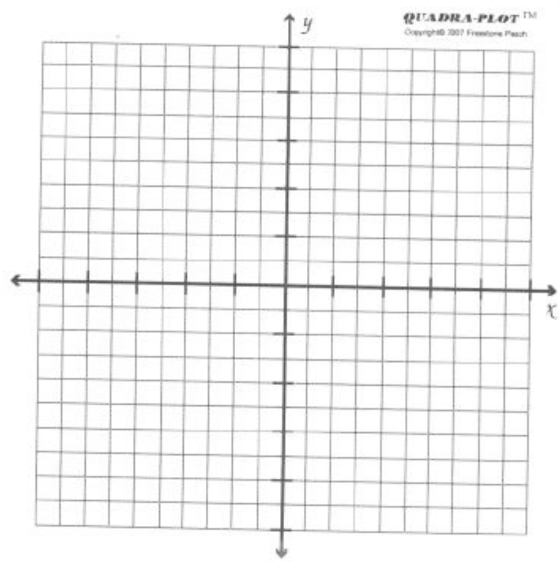
4. $24x^2 + 24y^2 = 96$



5. $(x-3)^2 + (y+2)^2 = 25$



6. $x^2 + y^2 + 6x - 7 = 0$



EXAMPLE 2 *Writing an Equation of a Circle*

Directions: **Write an equation for each circle given the center and radius.**

1. center = (0, 0) and $r = 2\sqrt{7}$

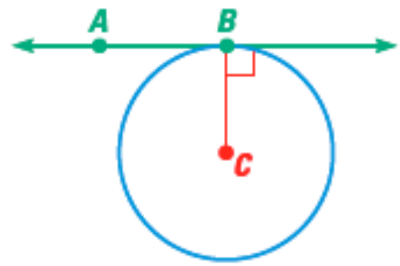
2. center = (3, -4) and $r = \sqrt{15}$

Directions: **Write an equation for each circle given the endpoints of a diameter.**

1. (2, 8) and (2, -2)

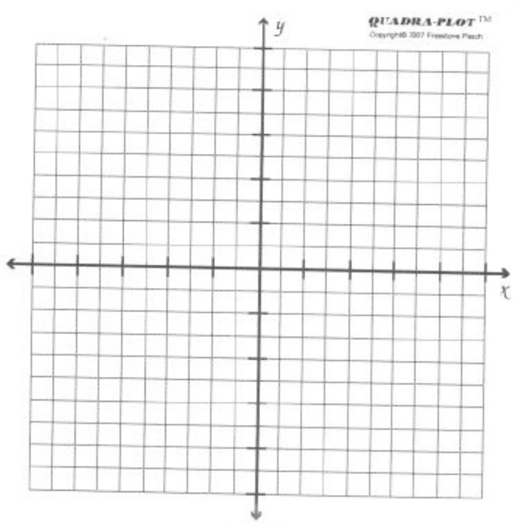
2. (2, 1) and (2, -4)

A theorem in geometry states that a line tangent to a circle is perpendicular to the circle's radius at the point of tangency. In the diagram, $\overleftrightarrow{AB}$ is tangent to the circle with center C at the point of tangency B, so $\overleftrightarrow{AB} \perp \overline{BC}$. This property of circles is used in the next example.



EXAMPLE 3 Finding a Tangent Line

The equation of a circle and a point on the circle is given. Write an equation of the line that is tangent to the circle at that point. $x^2 + y^2 = 20$, (-2, 4)



GOAL 2 USING CIRCLES IN REAL LIFE

COMMUNICATIONS: Suppose an unobstructed radio station broadcast could travel 120 miles. Assume the station is centered at the origin.

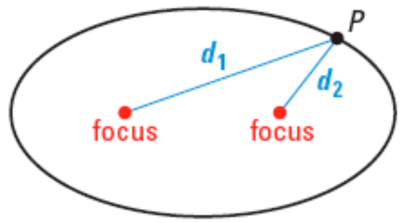
a. Write an equation to represent the boundary of the broadcast area with the origin as the center.

b. If the transmission tower is relocated 40 miles east and 10 miles south of the current location, and an increased signal will transmit signals an additional 80 miles, what is an equation to represent the new broadcast area?

Notes 10.4 – Ellipses

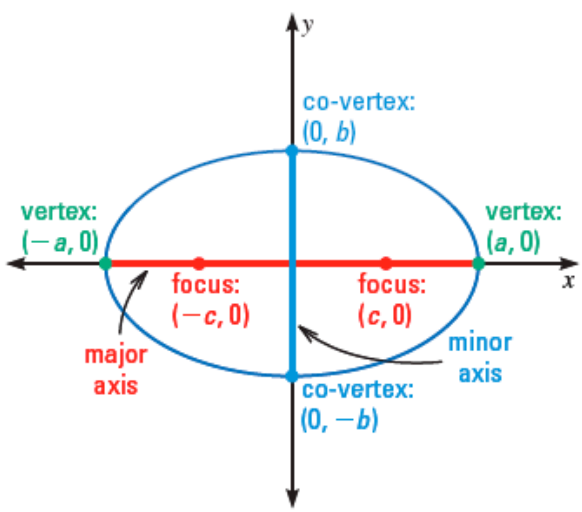
GOAL 1 GRAPHING AND WRITING EQUATIONS OF ELLIPSES

An **ellipse** is the set of all points P such that the sum of the distances between P and two distinct fixed points, called the **foci,** is a constant.



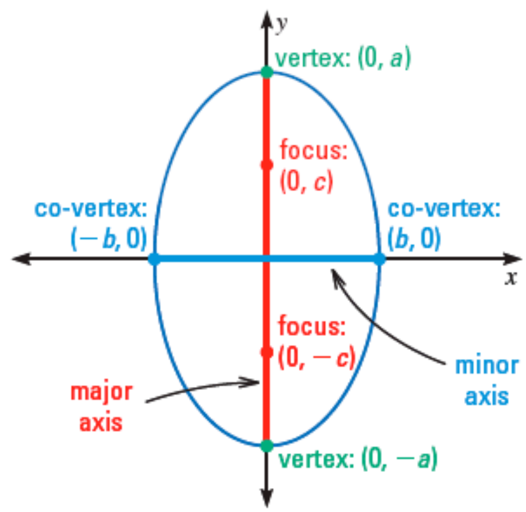
$d_1 + d_2 = \text{constant}$

The line through the foci intersects the ellipse at two points, the **vertices.** The line segment joining the vertices is the **major axis,** and its midpoint is the **center** of the ellipse. The line perpendicular to the major axis at the center intersects the ellipse at two points called the **co-vertices.** The line segment that joins these points is the **minor axis** of the ellipse. The two types of ellipses we will discuss are those with a horizontal major axis and those with a vertical major axis.



Ellipse with horizontal major axis

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Ellipse with vertical major axis

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

CHARACTERISTICS OF AN ELLIPSE (CENTER AT ORIGIN)

The standard form of the equation of an ellipse with center at (0, 0) and major and minor axes of lengths $2a$ and $2b$, where $a > b > 0$, is as follows.

EQUATION	MAJOR AXIS	VERTICES	CO-VERTICES
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	Horizontal	$(\pm a, 0)$	$(0, \pm b)$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	Vertical	$(0, \pm a)$	$(\pm b, 0)$

The foci of the ellipse lie on the major axis, c units from the center where $c^2 = a^2 - b^2$.

Key Concept

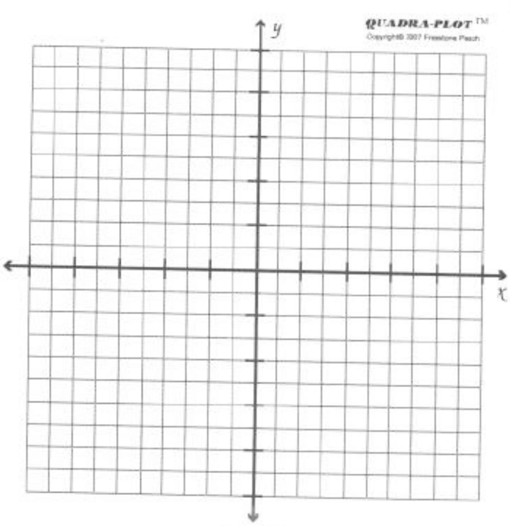
Equations of Ellipses Centered at the Origin

Standard Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
Orientation	horizontal	vertical
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Length of Major Axis	$2a$ units	$2a$ units
Length of Minor Axis	$2b$ units	$2b$ units

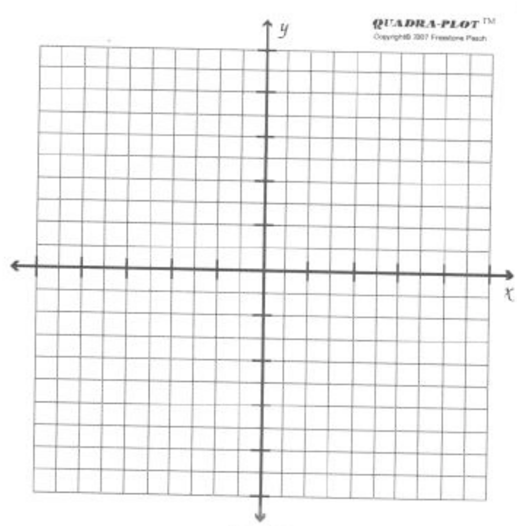
EXAMPLE 1 *Graphing an Equation of an Ellipse*

Directions: Graph and find the foci.

1. $\frac{x^2}{25} + \frac{y^2}{16} = 1$



2. $75x^2 + 36y^2 = 2700$

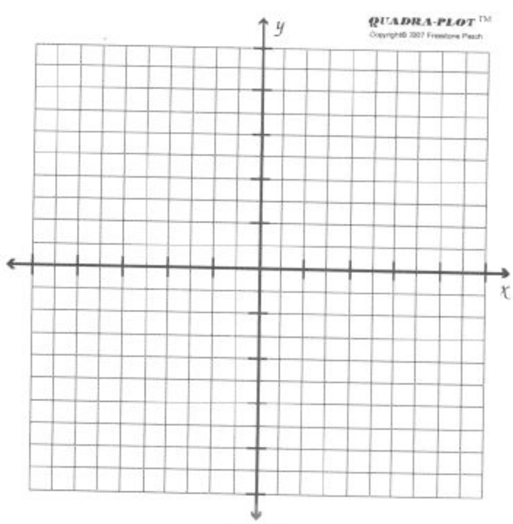


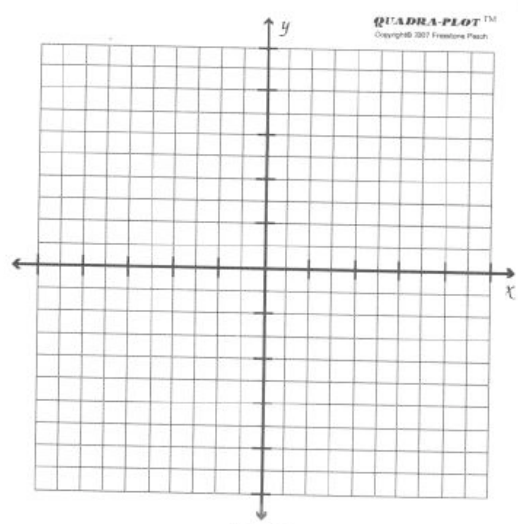
Key Concept — Equations of Ellipses Centered at (h, k)

Standard Form	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$
Orientation	horizontal	vertical
Foci	$(h \pm c, k)$	$(h, k \pm c)$
Vertices	$(h \pm a, k)$	$(h, k \pm a)$
Co-vertices	$(h, k \pm b)$	$(h \pm b, k)$

3. $\frac{(x-3)^2}{25}+\frac{(y+2)^2}{36}=1$

4. $x^2+4y^2-6x-16y-11=0$





EXAMPLE 2 Writing Equations of Ellipses

Directions: Write an equation of the ellipse that satisfied each set of conditions.

1. vertex at: (0,5) & co-vertex at: (4,0)

**2. Vertices at: (-6, 2) & (4, -2)
co-vertices at: (-1, 4) & (-1, 0)**

3. vertex at: (6, 0) & Focus at: (3, 0)

**4. Foci at: (4, 4) & (4, 14)
co-vertex at : (0, 9)**

GOAL 2 USING ELLIPSES IN REAL LIFE

An amusement park has an elliptical garden at its entrance. The garden is 32 feet long and 14 feet wide.

a. Write an equation of the ellipse.

b. The area of an ellipse is $A = \pi ab$. What is the area of the garden?

Assignment:

10.3- pages 634-635 #13-49 every other odd

10.4- pages 644-645 #11-37 odd