Alg 2

6-5 The Quadratic Formula & Discriminant

Used to solve *any* Quadratic Equation

Quadratic Formula

$ax^2 + bx + c = 0$, where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1 - Two Rational Roots

$x^2 - 8x = 33$

$-33 \quad -33$

$x^2 - 8x - 33 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a = 1$
$b = -8$
$c = -33$

$= \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-33)}}{2(1)}$

$= \frac{8 \pm \sqrt{196}}{2}$

$= \frac{8 \pm 14}{2}$

$= \frac{8+14}{2} = 11 \qquad \frac{8-14}{2} = -3$

$\{11, -3\}$

$x^2 - 12x = 28$

$-28 \quad -28$

$x^2 - 12x - 28 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a = 1$
$b = -12$
$c = -28$

$= \frac{12 \pm \sqrt{(-12)^2 - 4(1)(-28)}}{2(1)}$

$= \frac{12 \pm \sqrt{256}}{2}$

$= \frac{12 \pm 16}{2}$

$\frac{12+16}{2} = 14$

$\frac{12-16}{2} = -2$

$\{14, -2\}$

Example 2 - **One Rational Root**

$x^2 - 34x + 289 = 0$

$a = 1$
$b = -34$
$c = 289$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{34 \pm \sqrt{(-34)^2 - 4(1)(289)}}{2(1)}$$

$$= \frac{34 \pm 0}{2} = \frac{34}{2} = \{17\}$$

$x^2 + 22x + 121 = 0$

$a =$
$b =$
$c =$

Example 3 - Irrational Roots

$2x^2 + 4x - 5 = 0$

$a = 2$
$b = 4$
$c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{(4)^2 - 4(2)(-5)}}{2(2)}$$

$$= \frac{-4 \pm \sqrt{16 + 40}}{4}$$

$$= \frac{-4 \pm \sqrt{56}}{4} \quad \sqrt{4}\sqrt{14} \quad = \boxed{\frac{-4 \pm 2\sqrt{14}}{4}}$$

$x^2 - 6x + 2 = 0$

$a =$
$b =$
$c =$

Example 4 - ~~Irrational~~ Roots

Non-real

$x^2 - 4x = -13$

$+13 \quad +13$

$x^2 - 4x + 13 = 0$

a= 1

b= -4

c= 13

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{-36}}{2}$$

$\sqrt{36} \cdot \sqrt{-1} = 6i$

$$= \frac{4 \pm 6i}{2} = \frac{4}{2} \pm \frac{6i}{2}$$

$$= 2 \pm 3i$$

$x^2 + 13 = 6x$

$-6x \quad -6x$

$x^2 - 6x + 13 = 0$

a= 1

b= -6

c= 13

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)}$$

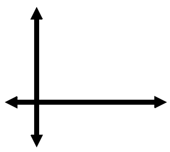
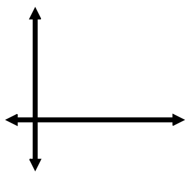
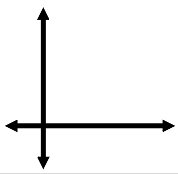
$$= \frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2}$$

$$\frac{6}{2} \pm \frac{4i}{2} = 3 \pm 2i$$

Discriminant

$b^2 - 4ac$ ← the expression under the square root

The value of the discriminant can be used to determine the NUMBER and TYPES of solutions

Value of Discriminant	# & Type of Solutions	Example Graphs
$b^2 - 4ac > 0$ *(positive);* $b^2 - 4ac$ *(perfect square)*	2 Real, Rational Roots	
$b^2 - 4ac > 0$ *(positive);* $b^2 - 4ac$ *(<u>not</u> a perfect square)*	2 Real, Irrational Roots	
$b^2 - 4ac = 0$	1 Real, Rational Root	
$b^2 - 4ac < 0$	2 Complex Roots	

Example 5 - Describe the Roots

$x^2 + 6x + 9 = 0$

$x^2 + 3x + 5 = 0$

$x^2 + 8x - 4 = 0$

$x^2 - 11x + 10 = 0$

Solving Quadratic Equations

METHOD	CAN BE USED	WHEN TO USE
GRAPHING	sometimes	To estimate
FACTORING	sometimes	If the constant term is 0 or if the factors are easily determined $x^2 - 3x = 0$
SQUARE ROOT PROPERTY	sometimes	If a perfect square is equal to a constant $(x + 13)^2 = 9$
COMPLETING THE SQUARE	always	If b is even $x^2 + 14x - 9 = 0$
QUADRATIC FORMULA	always	If other methods fail or are too tedious $3.4x^2 - 2.5x + 7.9 = 0$