Completing the Square

The Square Root Property can only be used to solve quadratic equations when the side containing thequadratic expression is a perfect square. However, FEW quadratic equations are perfect squares!

Therefore, we use a method called COMPLETING THE SQUARE to solve these equations!

To complete the square for any quadratic expression of the form $x^2 + bx$, follow the steps below

Step 1 - Find one half of *b*, the coefficient of the *x* term

Step 2 - Square the result of Step One.

Step 3 - Add the result of Step Two to $x^2 + bx$

Example 3 - Complete the Square

Find the value of *c* that makes the expression a perfect square.

$x^2 + 12x + c$

$x^2 + 12x + 36$

1) $\frac{12}{2} = 6$

2) $6^2 = 36$

Step 1 - Find one half of *b*, the coefficient of the *x* term

Step 2 - Square the result of Step One.

Step 3 - Add the result of Step Two to $x^2 + bx$

$x^2 + 16x + c$

$x^2 + 16x + 64$

1) $\frac{16}{2} = 8$

2) $8^2 = 64$

Step 1 - Find one half of *b*, the coefficient of the *x* term

Step 2 - Square the result of Step One.

Step 3 - Add the result of Step Two to $x^2 + bx$

Example 4 - Solve an Equation by Completing the Square

$x^2 + 8x - 20 = 0$

$+20 \quad +20$

$x^2+8x=20$

1) $\frac{8}{2} = 4$

2) $4^2 = 16$

$x^2+8x+16 = 20+16$

$\sqrt{(x+4)^2} = \sqrt{36}$

$x+4 = \pm 6$

$-4 \quad -4$

$x = -4 \pm 6$

$x = -4+6 = 2$

$x = -4-6 = -10$

$\{2, -10\}$

$x^2 + 4x - 12 = 0$

$+12 \quad +12$

$x^2+4x=12$

1) $\frac{4}{2} = 2$

2) $2^2 = 4$

$x^2+4x+4 = 12+4$

$\sqrt{(x+2)^2} = \sqrt{16}$

$x+2 = \pm 4$

$-2 \quad -2$

$x = -2 \pm 4$

$x = -2+4 = 2$

$x = -2-4 = -6$

$\{2, -6\}$

$x^2 - 6x + 4 = 0$

$-4 \quad -4$

$x^2 - 6x = -4$

1) $\frac{-6}{2} = -3$

2) $(-3)^2 = 9$

$x^2 - 6x + 9 = -4 + 9$

$\sqrt{(x-3)^2} = \sqrt{5}$

$x - 3 = \pm\sqrt{5}$

$+3 \quad +3$

$x = 3 \pm \sqrt{5}$

$x^2 - 8x + 8 = 0$

$-8 \quad -8$

$x^2 - 8x = -8$

1) $\frac{-8}{2} = -4$

2) $(-4)^2 = 16$

$x^2 - 8x + 16 = -8 + 16$

$\sqrt{(x-4)^2} = \sqrt{8} = \sqrt{4}\cdot\sqrt{2} = 2\sqrt{2}$

$x - 4 = \pm 2\sqrt{2}$

$+4 \quad +4$

$x = 4 \pm 2\sqrt{2}$

$x^2 - 4x - 13 = 0$

$+13 \quad +13$

$x^2 - 4x = 13$

1) $\frac{-4}{2} = -2$

2) $(-2) = 4$

$x^2 - 4x + 4 = 13 + 4$

$\sqrt{(x-2)^2} = \sqrt{17}$

$x - 2 = \pm\sqrt{17}$

$+2 \quad +2$

$x = 2 \pm \sqrt{17}$

$x^2 - 14x + 19 = 0$

$-19 \quad -19$

$x^2 - 14x = -19$

1) $\frac{-14}{2} = -7$

2) $(-7)^2 = 49$

$x^2 - 14x + 49 = -19 + 49$

$\sqrt{(x-7)^2} = \sqrt{30}$

$x - 7 = \pm\sqrt{30}$

$+7 \quad +7$

$x = 7 \pm \sqrt{30}$

Example 5 - Equation with $a \neq 1$

$$3x^2 - 2x - 1 = 0$$

$$2x^2 - 5x + 3 = 0$$

Example 6 - Equations with Complex Solutions

Remember Complex #'s ?
What are they????

$$x^2 + 4x + 11 = 0$$

$$x^2 + 2x + 3 = 0$$