CHAPTER 10

Algebra: Nonlinear Functions and Polynomials

BIG Ideas

- **Standard 7AF2.0** Interpret and evaluate expressions involving integer powers and simple roots.
- **Standard 7NS1.0** Use exponents, powers, and roots and use exponents in working with fractions.

Key Vocabulary

cube root (p. 554)
nonlinear function (p. 522)
quadratic function (p. 528)

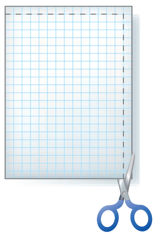
Real-World Link

Fountains Many real-world situations, such as this fountain at Paramount's Great America theme park in Santa Clara California, cannot be modeled by linear functions. These can be modeled using nonlinear functions.

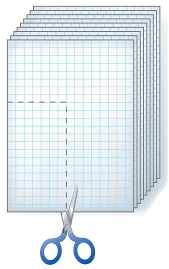
FOLDABLES Study Organizer

Algebra: Nonlinear Functions and Polynomials Make this Foldable to help you organize your notes. Begin with eight sheets of grid paper.

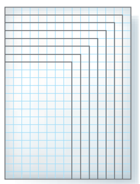
1. **Cut** off one section of the grid paper along both the long and short edges.



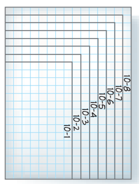
2. **Cut** off two sections from the second sheet, three sections from the third sheet, and so on to the 8th sheet.



3. **Stack** the sheets from narrowest to widest.



4. **Label** each of the right tabs with a lesson number.



GET READY for Chapter 10

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

Math Online Take the Online Readiness Quiz at ca.gr7math.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

Graph each equation. (Lesson 11-2)

1. $y = x - 4$
2. $y = 2x$
3. $y = x + 2$
4. **MEASUREMENT** The equation $y = 2.54x$ describes about how many centimeters y are in x inches. Graph the function. (Lessons 11-2)

Write each expression using a positive exponent. (Lesson 2-9)

5. a^{-9}
6. 6^{-4}
7. x^{-5}
8. 5^{-2}

Write each expression using exponents. (Lesson 2-9)

9. $6 \cdot 6 \cdot 6 \cdot 6$
10. $3 \cdot 7 \cdot 7 \cdot 3 \cdot 7$
11. **FUND-RAISER** The students at Hampton Middle School raised $8 \cdot 8 \cdot 2 \cdot 8 \cdot 2$ dollars to help build a new community center. How much money did they raise? (Lesson 2-9)

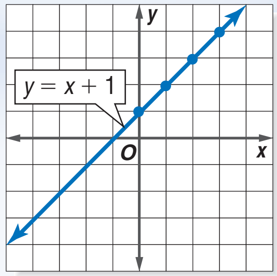
QUICK Review

Example 1

Graph $y = x + 1$.

First, make a table of values. Then, graph the ordered pairs and connect the points.

x	y	(x, y)
0	1	(0, 1)
1	2	(1, 2)
2	3	(2, 3)
3	4	(3, 4)



Example 2

Write n^{-3} using a positive exponent.

$n^{-3} = \frac{1}{n^3}$ definition of negative exponent

Example 3

Write $5 \cdot 4 \cdot 5 \cdot 4 \cdot 5$ using exponents.

5 is multiplied by itself 3 times and 4 is multiplied by itself 2 times.

So, $5 \cdot 4 \cdot 5 \cdot 4 \cdot 5 = 5^3 \cdot 4^2$.

10-1 Linear and Nonlinear Functions

Main IDEA

Determine whether a function is linear or nonlinear.

Preparation for AF1.5 Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

NEW Vocabulary

nonlinear function

GET READY for the Lesson

ROCKETRY The tables show the flight data for a model rocket launch. The first table gives the rocket's height at each second of its ascent, or upward flight. The second table gives its height as it descends back to Earth using a parachute.

Ascent	
Time (s)	Height (m)
0	0
1	38
2	74
3	106
4	128
5	138
6	142

Descent	
Time (s)	Height (m)
7	140
8	130
9	120
10	110
11	100
12	90
13	80

1. During its ascent, did the rocket travel the same distance each second? Justify your answer.
2. During its descent, did the rocket travel the same distance each second? Justify your answer.
3. Graph the ordered pairs (time, height) for the rocket's ascent and descent on separate axes. Connect the points with a straight line or smooth curve. Then compare the graphs.

REVIEW Vocabulary

constant rate of change occurs when the rate of change between any two data points is proportional. (Lesson 4-10)

In Lesson 9-2, you learned that linear functions have graphs that are straight lines. These graphs represent constant rates of change. **Nonlinear functions** are functions that do not have constant rates of change. Therefore, their graphs are not straight lines.

EXAMPLES Identify Functions Using Tables

Determine whether each table represents a *linear* or *nonlinear* function. Explain.

1

	x	y	
	2	50	
+2			−15
	4	35	
+2			−15
	6	20	
+2			−15
	8	5	

As x increases by 2, y decreases by 15 each time. The rate of change is constant, so this function is linear.

2

	x	y	
	1	1	
+3			+15
	4	16	
+3			+33
	7	49	
+3			+51
	10	100	

As x increases by 3, y increases by a greater amount each time. The rate of change is not constant, so this function is nonlinear.

CHECK Your Progress Determine whether each table represents a *linear* or *nonlinear* function. Explain.

a.

x	0	5	10	15
y	20	16	12	8

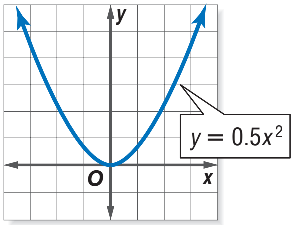
b.

x	0	2	4	6
y	0	2	8	18

EXAMPLES Identify Functions Using Graphs

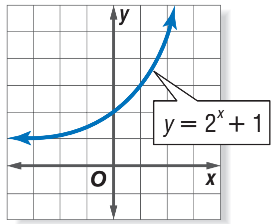
Determine whether each graph represents a *linear* or *nonlinear* function. Explain.

3



The graph is a curve, not a straight line. So, it represents a nonlinear function.

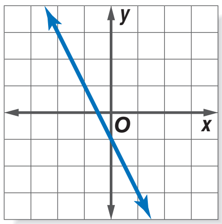
4



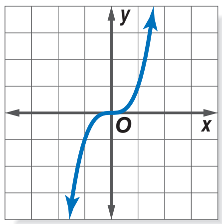
This graph is also a curve. So, it represents a nonlinear function.

CHECK Your Progress Determine whether each graph represents a *linear* or *nonlinear* function. Explain.

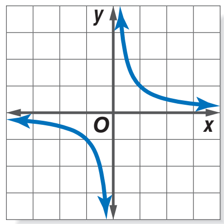
c.



d.



e.



STUDY TIP

Identifying Linear Equations Always examine an equation after it has been solved for y to see that the power of x is 1 or 0. Then check to see that x does not appear in the denominator.

Recall that the equation for a linear function can be written in the form $y = mx + b$, where m represents the constant rate of change.

EXAMPLES Identify Functions Using Equations

Determine whether each equation represents a *linear* or *nonlinear* function. Explain.

5 $y = x + 4$

Since the equation can be written as $y = 1x + 4$, this function is linear.

6 $y = \frac{6}{x}$

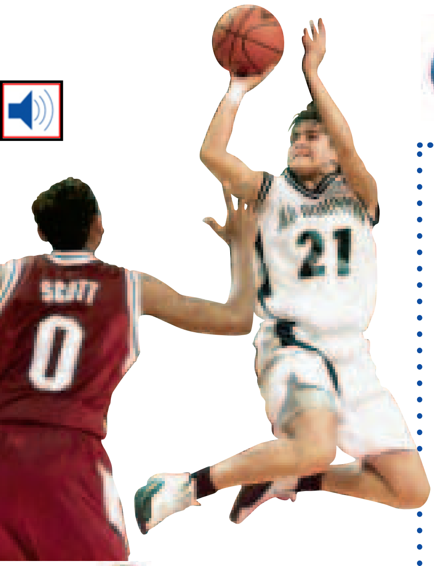
The equation cannot be written in the form $y = mx + b$. So, this function is nonlinear.

CHECK Your Progress

f. $y = 2x^3 + 1$

g. $y = 3x$

h. $y = \frac{x}{5}$



Real-World Link The NCAA women's basketball tournament begins with 64 teams and consists of 6 rounds of play.

Real-World EXAMPLE

7 **BASKETBALL** **Use the table to determine whether the number of teams is a linear function of the number of rounds of play.**

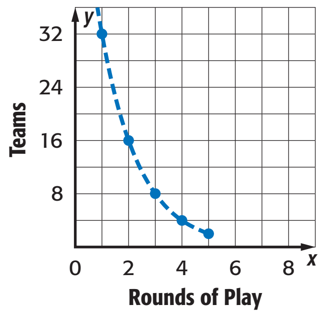
Round(s) of play	Teams
1	32
2	16
3	8
4	4
5	2

Examine the differences between the number of teams for each round.

$16 - 32 = -16 \qquad 8 - 16 = -8$
$4 - 8 = -4 \qquad 2 - 4 = -2$

While there is a pattern in the differences, they are not the same. Therefore, this function is nonlinear.

Check Graph the data to verify the ordered pairs do not lie on a straight line.



CHECK Your Progress

i. **TICKETS** Tickets to the school dance cost \$5 per student. Are the ticket sales a linear function of the number of tickets sold? Explain.

Number of Tickets Sold	1	2	3
Ticket Sales	\$5	\$10	\$15

Personal Tutor

Online Personal Tutor at ca.gr7math.com

CHECK Your Understanding

Determine whether each table, graph, or equation or represents a *linear* or *nonlinear* function. Explain.

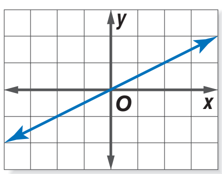
Examples 1–6 (pp. 522–523)

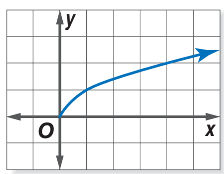
1.

x	0	1	2	3
y	1	3	6	10

2.

x	0	3	6	9
y	−3	9	21	33

3. 

4. 

5. $y = \frac{x}{3}$

6. $y = 2x^2$

Example 7 (p. 524)

7. **MEASUREMENT** The table shows the measures of the sides of several rectangles. Are the widths of the rectangles a linear function of the lengths? Explain.

Length (in.)	1	4	8	10
Width (in.)	64	16	8	6.4

Exercises

HOMEWORK HELP	
For Exercises	**See Examples**
8–13	1, 2
14–19	3, 4
20–25	5, 6
26–29	7

Determine whether each table, graph, or equation or represents a *linear* or *nonlinear* function. Explain.

8.

x	3	6	9	12
y	12	10	8	6

9.

x	1	2	3	4
y	1	4	9	16

10.

x	5	10	15	20
y	13	28	43	58

11.

x	1	3	5	7
y	−2	−18	−50	−98

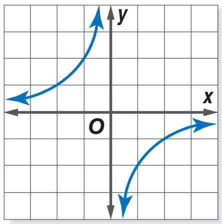
12.

x	2	4	6	8
y	10	12	16	24

13.

x	4	8	12	16
y	3	0	−3	−6

14.



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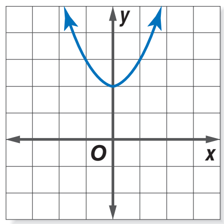

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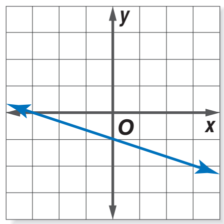
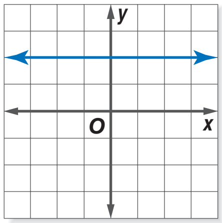
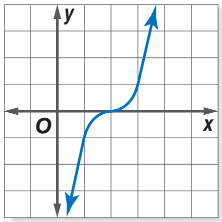


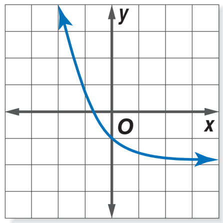
17.



18.



19.



20. $y = x^3 - 1$

21. $y = 4x^2 + 9$

22. $y = 0.6x$

23. $y = \frac{3x}{2}$

24. $y = \frac{4}{x}$

25. $y = \frac{8}{x} + 5$

26. **TRAVEL** The Guzman family drove from Sacramento to Yreka. Use the table to determine whether the distance driven is a linear function of the hours traveled. Explain.

Time (h)	1	2	3	4
Distance (mi)	65	130	195	260

27. **BUILDINGS** The table shows the height of several buildings in Chicago, Illinois. Use the table to determine whether the height of the building is a linear function of the number of stories. Explain.

Building	Stories	Height (ft)
Harris Bank III	35	510
One Financial Place	40	515
Kluczynski Federal Building	45	545
Mid Continental Plaza	50	582
North Harbor Tower	55	556

Source: *The World Almanac*

MEASUREMENT **For Exercises 28 and 29, use the following information.** Recall that the circumference of a circle is equal to pi times its diameter and that the area of a circle is equal to pi times the square of its radius.

28. Is the circumference of a circle a linear or nonlinear function of its diameter? Explain your reasoning.

29. Is the area of a circle a linear or nonlinear function of its radius? Explain your reasoning.

For Exercises 30–34, determine whether each equation or table represents a *linear* or *nonlinear* function. Explain.

30. $y - x = 1$

31. $xy = -9$

32. $y = 2^x$

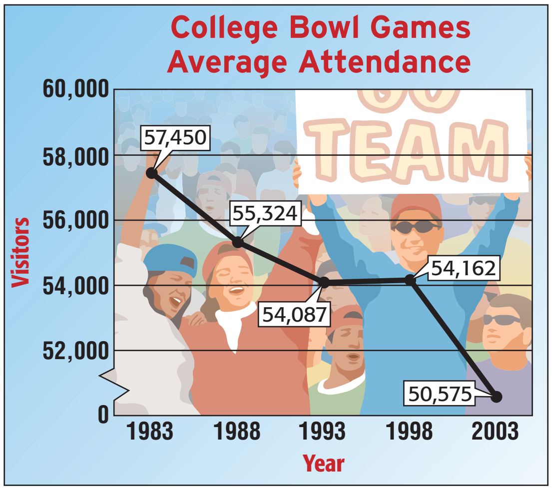
33.

x	0.5	1	1.5	2
y	15	8	1	−6

34.

x	−4	0	4	8
y	2	1	−1	−4

35. **FOOTBALL** The graphic shows the decrease in the average attendance at college bowl games from 1983 to 2003. Would you describe the decline as linear or nonlinear? Explain.



Source: USA Today

36. **MEASUREMENT** Make a graph showing the area of a square as a function of its perimeter. Explain whether the function is linear.

EXTRA PRACTICE
See pages 702, 717.
Math Online
Self-Check Quiz at ca.gr7math.com

37. **GRAPHING** Water is poured at a constant rate into the vase at the right. Draw a graph of the water level as a function of time. Is the water level a linear or nonlinear function of time? Explain.

H.O.T. Problems

38. **CHALLENGE** *True* or *false?* All graphs of straight lines are linear functions. Explain your reasoning or provide a counterexample.

39. **Which One Doesn't Belong?** Identify the function that is not linear. Explain your reasoning.

$y = 2x$ $\quad$ $y = x^2$ $\quad$ $y - 2 = x$ $\quad$ $x - y = 2$

40. **OPEN ENDED** Give an example of a nonlinear function using a table of values.

41. **WRITING IN MATH** Describe two methods for determining whether a function is linear given its equation.

STANDARDS PRACTICE

42. Which equation describes the data in the table?

x	-7	-5	-3	0	4
y	50	26	10	1	17

A $5x + 1 = y$
B $xy = 68$
C $x^2 + 1 = y$
D $-2x^2 + 8 = y$

43. Which equation represents a nonlinear function?

F $y = 3x + 1$
G $y = \frac{x}{3}$
H $2xy = 10$
J $y = 3(x - 5)$

Spiral Review

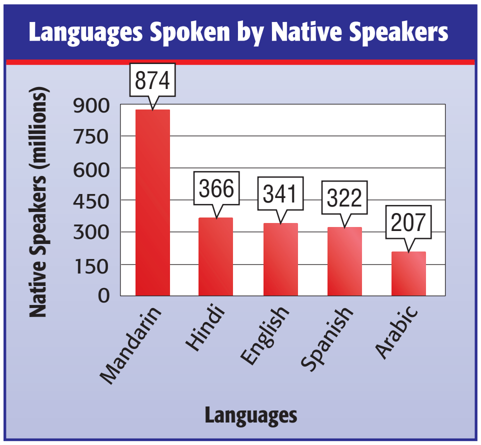
STATISTICS **Determine whether a scatter plot of the data for the following might show a *positive*, *negative*, or *no* relationship.** (Lesson 9-8)

44. grade on a test and amount of time spent studying

45. age and number of siblings

46. number of Calories burned and length of time exercising

47. LANGUAGES The graph shows the top five languages spoken by at least 100 million native speakers worldwide. What conclusions can you make about the number of Mandarin native speakers and the number of English native speakers? (Lesson 9-7)



Source: *The World Almanac For Kids*

Solve each equation. Check your solution. (Lesson 8-4)

48. $1 - 3c = 9c + 7$

49. $7k + 12 = 8 - 9k$

50. $13.4w + 17 = 5w - 4$

51. $8.1a + 2.3 = 5.1a - 3.1$

52. $4.1x - 23 = -3.9x - 1$

53. $3.2n + 3 = -4.8n - 29$

54. PARKS A circular fountain in a park has a diameter of 8 feet. The park director wants to build a fountain that has an area four times that of the current fountain. What will be the diameter of the new fountain? (Lesson 7-1)

55. MEASUREMENT The cylindrical air duct of a large furnace has a diameter of 30 inches and a height of 120 feet. If it takes 15 minutes for the contents of the duct to be expelled into the air, what is the volume of the substances being expelled each hour? (Lesson 7-5)

GET READY for the Next Lesson

PREREQUISITE SKILL **Graph each equation.** (Lesson 9-2)

56. $y = 2x$

57. $y = x + 3$

58. $y = 3x - 2$

59. $y = \frac{1}{3}x + 1$

10-2 Graphing Quadratic Functions

Main IDEA

Graph quadratic functions.

Standard 7AF1.5 **Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.**
Standard 7AF3.1 **Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.**

NEW Vocabulary

quadratic function

MINI Lab

You know that the area A of a square is equal to the length of a side s squared, $A = s^2$.

STEP 1 Copy and complete the table.

STEP 2 Graph the ordered pairs from the table. Connect them with a smooth curve.

s	s^2	(s, A)
0	0	(0, 0)
1	1	(1, 1)
2		
3		
4		
5		
6		

1. Is the relationship between the side length and the area of a square linear or nonlinear? Explain.
2. Describe the shape of the graph.

A **quadratic function**, like $A = s^2$, is a function in which the greatest power of the variable is 2. Its graph is U-shaped, opening upward or downward. The graph opens upward if the number in front of the variable that is squared is *positive*, downward if it is *negative*.

STUDY TIP

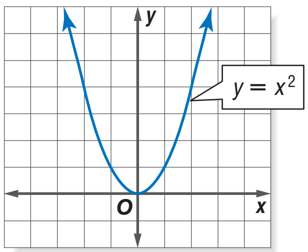
Quadratic Fuctions The graph of a quadratic function is called a *parabola.*

EXAMPLES Graph Quadratic Functions

1 Graph $y = x^2$.

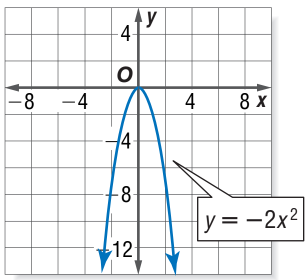
To graph a quadratic function, make a table of values, plot the ordered pairs, and connect the points with a smooth curve.

x	x^2	y	(x, y)
−2	$(-2)^2 = 4$	4	(−2, 4)
−1	$(-1)^2 = 1$	1	(−1, 1)
0	$(0)^2 = 0$	0	(0, 0)
1	$(1)^2 = 1$	1	(1, 1)
2	$(2)^2 = 4$	4	(2, 4)



2 Graph $y = -2x^2$.

x	$-2x^2$	y	(x, y)
−2	$-2(-2)^2 = -8$	−8	(−2, −8)
−1	$-2(-1)^2 = -2$	−2	(−1, −2)
0	$-2(0)^2 = 0$	0	(0, 0)
1	$-2(1)^2 = -2$	−2	(1, −2)
2	$-2(2)^2 = -8$	−8	(2, −8)



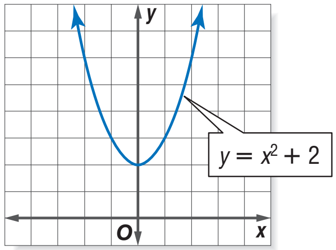
Math Online **Extra Examples at** ca.gr7math.com

READING in the Content Area

For more strategies in reading this lesson, visit ca.gr7math.com.

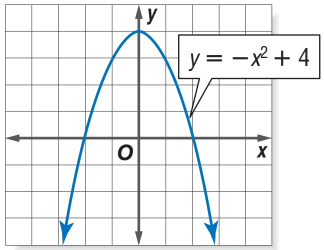
3 Graph $y = x^2 + 2$.

x	$x^2 + 2$	y	(x, y)
-2	$(-2)^2 + 2 = 6$	6	$(-2, 6)$
-1	$(-1)^2 + 2 = 3$	3	$(-1, 3)$
0	$(0)^2 + 2 = 2$	2	$(0, 2)$
1	$(1)^2 + 2 = 3$	3	$(1, 3)$
2	$(2)^2 + 2 = 6$	6	$(2, 6)$



4 Graph $y = -x^2 + 4$.

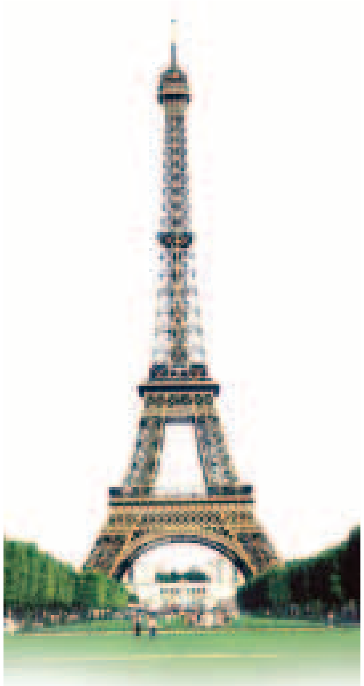
x	$-x^2 + 4$	y	(x, y)
-2	$-(-2)^2 + 4 = 0$	0	$(-2, 0)$
-1	$-(-1)^2 + 4 = 3$	3	$(-1, 3)$
0	$-(0)^2 + 4 = 4$	4	$(0, 4)$
1	$-(1)^2 + 4 = 3$	3	$(1, 3)$
2	$-(2)^2 + 4 = 0$	0	$(2, 0)$



CHECK Your Progress Graph each function.

a. $y = 6x^2$ **b.** $y = x^2 - 2$ **c.** $y = -2x^2 - 1$

Real-World EXAMPLE

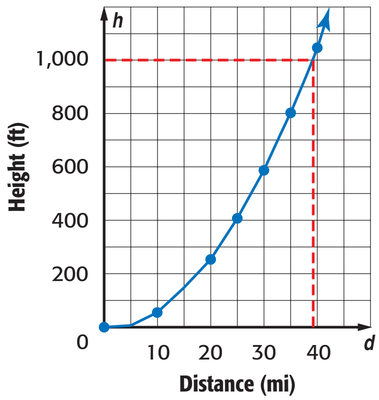


Real-World Link The Eiffel Tower in Paris, France, opened in 1889 as part of the World Exposition. It is about 986 feet tall.
Source: structurae.de

5 **MONUMENTS** **The function $h = 0.66d^2$ represents the distance d in miles you can see from a height of h feet. Graph this function. Then use your graph and the information at the left to estimate how far you could see from the top of the Eiffel Tower.**

Distance cannot be negative, so use only positive values of d.

d	$h = 0.66d^2$	(d, h)
0	$0.66(0)^2 = 0$	$(0, 0)$
10	$0.66(10)^2 = 66$	$(10, 66)$
20	$0.66(20)^2 = 264$	$(20, 264)$
25	$0.66(25)^2 = 412.5$	$(25, 412.5)$
30	$0.66(30)^2 = 594$	$(30, 594)$
35	$0.66(35)^2 = 808.5$	$(35, 808.5)$
40	$0.66(40)^2 = 1{,}056$	$(40, 1{,}056)$



At a height of 986 feet, you could see approximately 39 miles.

CHECK Your Progress

d. **TOWERS** The outdoor observation deck of the Space Needle in Seattle, Washington, is 520 feet above ground level. Estimate how far you could see from the observation deck.

Personal Tutor

Online **Personal Tutor at** ca.gr7math.com

CHECK Your Understanding

Examples 1–4 (pp. 528–529)

Graph each function.

1. $y = 3x^2$
2. $y = -5x^2$
3. $y = -4x^2$
4. $y = -x^2 + 1$
5. $y = x^2 - 3$
6. $y = -x^2 + 2$

Example 5 (p. 529)

7. **CARS** The function $d = 0.006s^2$ represents the braking distance d in meters of a car traveling at a speed s in kilometers per second. Graph this function. Then use your graph to estimate the speed of the car if its braking distance is 12 meters.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
8–11	1, 2
12–19	3, 4
20, 21	5

Graph each function.

8. $y = 4x^2$
9. $y = 5x^2$
10. $y = -3x^2$
11. $y = -6x^2$
12. $y = x^2 + 6$
13. $y = x^2 - 4$
14. $y = -x^2 + 2$
15. $y = -x^2 - 5$
16. $y = 2x^2 - 1$
17. $y = 2x^2 + 3$
18. $y = -4x^2 - 1$
19. $y = -3x^2 + 2$

20. **RACING** The function $d = \frac{1}{2}at^2$ represents the distance d that a race car will travel over an amount of time t given the rate of acceleration a. Suppose a car is accelerating at a rate of 5 feet per second every second. Graph this function. Then use your graph to find the time it would take the car to travel 125 feet.

21. **WATERFALLS** The function $d = -16t^2 + 182$ models the distance d in feet a drop of water falls t seconds after it begins its descent from the top of the 182-foot high American Falls in New York. Graph this function. Then use your graph to estimate the time it will take the drop of water to reach the river at the base of the falls.

Graph each function.

22. $y = 0.5x^2 + 1$
23. $y = 1.5x^2$
24. $y = 4.5x^2 - 6$
25. $y = \frac{1}{3}x^2 - 2$
26. $y = \frac{1}{2}x^2$
27. $y = -\frac{1}{4}x^2 + 1$

MEASUREMENT **For Exercises 28 and 29, write a function for each of the following. Then graph the function in the first quadrant.**

28. The surface area of a cube is a function of the edge length a. Use your graph to estimate the edge length of a cube with a surface area of 54 square centimeters.

29. The volume V of a rectangular prism with a square base and a fixed height of 5 inches is a function of the base edge length s. Use your graph to estimate the base edge length of a prism whose volume is 180 cubic inches.

EXTRA PRACTICE

See pages 702, 717.

Math Online

Self-Check Quiz at ca.gr7math.com

H.O.T. Problems

CHALLENGE The graphs of quadratic functions may have exactly one highest point, called a *maximum*, or exactly one lowest point, called a *minimum*. Graph each quadratic equation. Determine whether each graph has a maximum or a minimum. If so, give the coordinates of each point.

30. $y = 2x^2 + 1$ **31.** $y = -x^2 + 5$ **32.** $y = x^2 - 3$

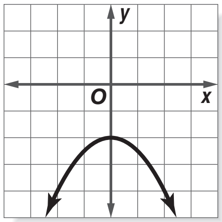
33. **OPEN ENDED** Write and graph a quadratic function that opens upward and has its minimum at $(0, -3.5)$.

34. **WRITING IN MATH** Write a quadratic function of the form $y = ax^2 + c$ and explain how to graph it.

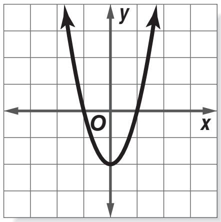
STANDARDS PRACTICE

35. Which graph represents the function $y = -0.5x^2 - 2$?

A



B



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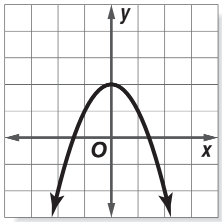
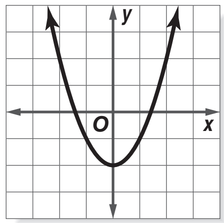


D



Spiral Review

Determine whether each equation represents a *linear* or *nonlinear* function. (Lesson 10-1)

36. $y = x - 5$ **37.** $y = 3x^3 + 2$ **38.** $x + y = -6$ **39.** $y = -2x^2$

STATISTICS **For Exercises 40–42, use the information at the right.** (Lesson 9-8)

40. Draw a scatter plot of the data and draw a line of fit.

41. Does the scatter plot show a *positive, negative,* or *no* relationship?

42. Use your graph to estimate the population of the whooping crane at the refuge in 2005.

Whooping Cranes

Year	Population
2000	172
2001	171
2002	181
2003	194
2004	197

43. **SAVINGS** Anna's parents put \$750 into a college savings account. After 6 years, the investment had earned \$540. Write an equation that you could use to find the simple interest rate. Then find the simple interest rate. (Lesson 5-9)

GET READY for the Next Lesson

44. **PREREQUISITE SKILL** A section of a theater is arranged so that each row has the same number of seats. You are seated in the 5th row from the front and the 3rd row from the back. If your seat is 6th from the left and 2nd from the right, how many seats are in this section of the theater? Use the *draw a diagram* strategy. (Lesson 4-4)

10-3 Problem-Solving Investigation

MAIN IDEA: Solve problems by making a model.

Standard 7MR2.5 **Use a variety of methods,** such as words, numbers, symbols, charts, graphs, tables, diagrams, and **models, to explain mathematical reasoning.** **Standard 7AF1.1** **Use** variables and **appropriate operations to write an expression, an equation,** an inequality, or a system of equations or inequalities **that represents a verbal description** (e.g. three less than a number, half as large as area *A*.

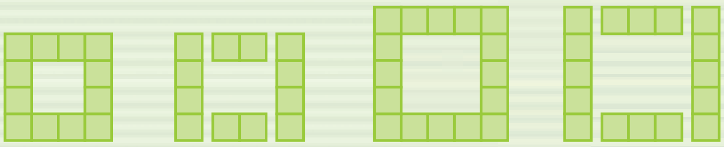
P.S.I. TEAM +

e-Mail: MAKE A MODEL

YOUR MISSION: Make a model to solve the problem.

THE PROBLEM: Determine if there are enough tables to make a 10-by-10 square arrangement.

Tonya: We have 35 square tables. We need to arrange them into a square that is open in the middle and has 10 tables on each side.

EXPLORE	You know Tonya has 35 square tables.
PLAN	Start by making models of a 4-by-4 square and of a 5-by-5 square. Then look for a pattern.
SOLVE	 4-by-4 square; 2 groups of 4 and 2 groups of 2; 5-by-5 square; 2 groups of 5 and 2 groups of 3 For a 10-by-10 square, Tonya needs $2 \cdot 10 + 2 \cdot 8$ or 36 tables. She has 35 tables, so she needs one more.
CHECK	You can estimate that Tonya needs 4×10 or 40 tables. But each of the corner tables is counted twice. So, she needs $40 - 4$ or 36 tables.

Analyze The Strategy

1. Draw a diagram showing another way the students could have grouped the tiles to solve this problem. Use a 4-by-4 square.
2. WRITING IN MATH Write a problem that can be solved by making a model. Describe the model. Then solve the problem.

Laura Sifferlin

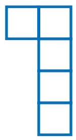
Mixed Problem Solving

For Exercises 3–5, solve by making a model.

3. **STICKERS** In how many different ways can three rectangular stickers be torn from a sheet of 3 × 3 stickers so that all three stickers are still attached? Draw each arrangement.

4. **MEASUREMENT** A 10-inch by 12-inch piece of cardboard has a 2-inch square cut out of each corner. Then the sides are folded up and taped together to make an open box. Find the volume of the box.

5. **GEOMETRY** A computer game requires players to stack arrangements of five squares arranged to form a single shape. One arrangement is shown at the right. How many different arrangements are there if touching squares must border on a full side?



Use any strategy to solve Exercises 6–11. Some strategies are shown below.

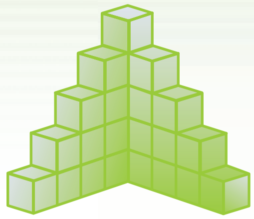
PROBLEM-SOLVING STRATEGIES
- Use the four-step plan.
- Draw a diagram.
- Guess and Check.
- Make a model.

6. **CAMP** The camp counselor lists 21 chores on separate pieces of paper and places them in a basket. The counselor takes one piece of paper, and each camper takes one as the basket is passed around the circle. There is one piece of paper left when the basket returns to the counselor. How many people could be in the circle if the basket goes around the circle more than once?

7. **PARKING** Parking space numbers consist of 3 digits. They are typed on a slip of paper and given to students at orientation. Tara accidentally read her number upside-down. The number she read was 795 more than her actual parking space number. What is Tara's parking space number?

8. **PETS** Mrs. Harper owns both cats and canaries. Altogether, her pets have thirty heads and eighty legs. How many cats does she have?

GEOMETRY For Exercises 9 and 10, use the figure at the right.



9. How many cubes would it take to build this tower?

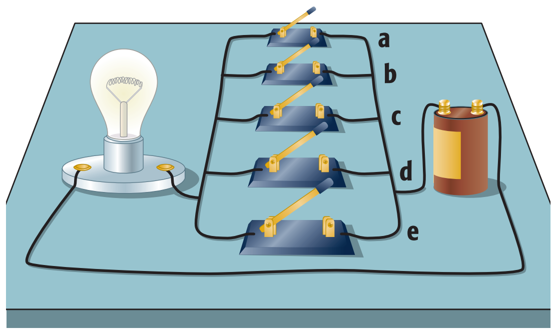
10. How many cubes would it take to build a similar tower that is 12 cubes high?

11. **CARS** Yesterday you noted that the mileage on the family car read 60,094.8 miles. Today it reads 60,099.1 miles. Was the car driven about 4 or 40 miles?

Select the Operation

For Exercises 12 and 13, select the appropriate operation(s) to solve the problem. Justify your selections(s) and solve the problem.

12. **SCIENCE** The light in the circuit will turn on if one or more switches are closed. How many combinations of open and closed switches will result in the light being on?



13. **HOBBIES** Lorena says to Angela, "If you give me one of your baseball cards, I will have twice as many baseball cards as you have." Angela answers, "If you give me one of your cards, we will have the same number of cards." How many cards does each girl have?

10-4 Graphing Cubic Functions

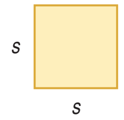
Main IDEA

Graph cubic functions.

Standard 7AF3.1 **Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.**
Standard 7AF3.2 **Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths** or a triangle prism with a fixed height and an equilateral triangle base of varying lengths).

GET READY for the Lesson

MEASUREMENT You can find the area A of a square by squaring the length of a side s. This relationship can be represented in different ways.



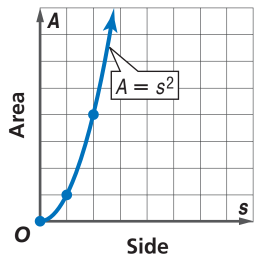
Words and Equation

$$\underbrace{\text{Area}}_{A}\ \underbrace{\text{equals}}_{=}\ \underbrace{\text{length of a side squared.}}_{s^2}$$

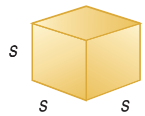
Table

s	s^2	(s, A)
0	$0^2 = 0$	(0, 0)
1	$1^2 = 1$	(1, 1)
2	$2^2 = 4$	(2, 4)

Graph



1. The volume V of a cube is found by cubing the length of a side s. Write a formula to represent the volume of a cube as a function of side length.



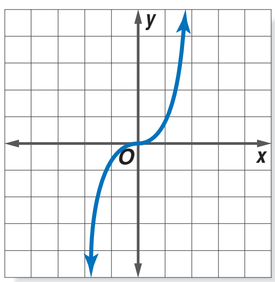
2. Graph the volume as a function of side length. (*Hint*: Use values of s such as 0, 0.5, 1, 1.5, 2, and so on.)
3. Would it be reasonable to use negative numbers for x-values in this situation? Explain.

You can graph cubic functions such as the formula for the volume of a cube by making a table of values.

EXAMPLE Graph a Cubic Function

1 Graph $y = x^3$.

x	$y = x^3$	(x, y)
-1.5	$(-1.5)^3 \approx -3.4$	$(-1.5, -3.4)$
-1	$(-1)^3 = -1$	$(-1, -1)$
0	$(0)^3 = 0$	(0, 0)
1	$(1)^3 = 1$	(1, 1)
1.5	$(1.5)^3 \approx 3.4$	(1.5, 3.4)



STUDY TIP

Graphing
It is often helpful to substitute decimal values of x in order to graph points that are closer together.

CHECK Your Progress

Graph each function.

a. $y = x^3 - 1$ **b.** $y = -4x^3$ **c.** $y = x^3 + 4$

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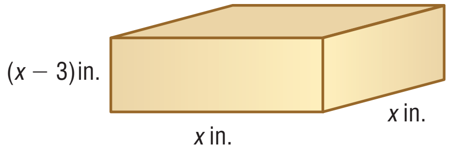
Real-World Link

Packaging is the nation's third largest industry, with over $130 billion in sales each year.

Source: San Jose State University

Real-World EXAMPLE

2 **PACKAGING** **A packaging company wants to manufacture a cardboard box with a square base of side length x inches and a height of $(x-3)$ inches as shown.**

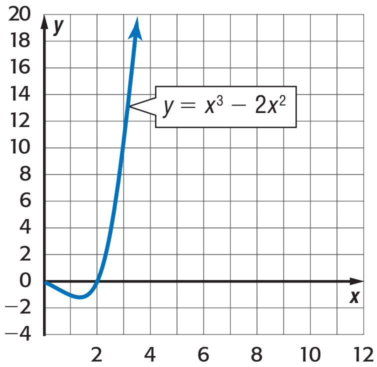


Write the function for the volume V of the box. Graph the function. Then estimate the dimensions of the box that would give a volume of approximately 8 cubic inches.

$V = lwh$	Volume of a rectangular prism
$V = x \cdot x \cdot (x-3)$	Replace l with x, w with x, and h with $(x-3)$.
$V = x^2(x-3)$	$x \cdot x = x^2$
$V = x^3 - 3x^2$	Distributive Property

The function for the volume V of the box is $V = x^3 - 3x^2$. Make a table of values to graph this function. You do not need to include negative values of x since the side length of the box cannot be negative.

x	$V = x^3 - 3x^2$	(x, V)
0	$(0)^3 - 3(0)^2 = 0$	(0, 0)
0.5	$(0.5)^3 - 3(0.5)^2 \approx -0.6$	(0.5, –0.6)
1	$(1)^3 - 3(1)^2 = -2$	(1, –2)
1.5	$(1.5)^3 - 3(1.5)^2 \approx -3.8$	(1.5, –3.8)
2	$(2)^3 - 3(2)^2 = -4$	(2, –4)
2.5	$(2.5)^3 - 3(2.5)^2 \approx -3.1$	(2.5, –3.1)
3	$(3)^3 - 3(3)^2 = 0$	(3, 0)
3.5	$(3.5)^3 - 3(3.5)^2 \approx 6.1$	(3.5, 6.1)
4	$(4)^3 - 3(4)^2 = 16$	(4, 16)



Looking at the graph, we see that the volume of the box is approximately 8 cubic inches when x is about 3.6 inches.

The dimensions of the box when the volume is about 8 cubic inches are 3.6 inches, 3.6 inches, and 3.6 – 3 or 0.6 inch.

STUDY TIP

Analyze the Graph
Notice that the graph is *below* the x-axis for values of $x < 3$. This means that the "volume" of the box is negative for $x < 3$. To have a box with a positive height and a positive volume, x must be greater than 3.

CHECK Your Progress

d. **PACKAGING** A packaging company wants to manufacture a cardboard box with a square base of side length x feet and a height of $(x-2)$ feet. Write the function for the volume V of the box. Graph the function. Then estimate the dimensions of the box that would give a volume of about 1 cubic foot.

CHECK Your Understanding

Example 1 (p. 534)

Graph each function.

1. $y = -x^3$
2. $y = 0.5x^3$
3. $y = x^3 - 2$
4. $y = 2x^3 + 1$

Example 2 (p. 535)

5. **MEASUREMENT** A rectangular prism with a square base of side length x centimeters has a height of $(x + 1)$ centimeters. Write the function for the volume V of the prism. Graph the function. Then estimate the dimensions of the box that would give a volume of approximately 9 cubic centimeters.

Exercises

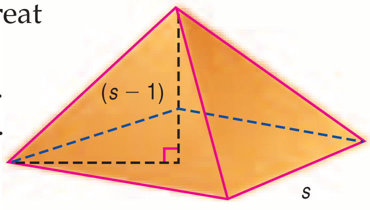
HOMEWORK HELP

For Exercises	See Examples
6–17	1
18, 19	2

Graph each function.

6. $y = -2x^3$
7. $y = -3x^3$
8. $y = 0.2x^3$
9. $y = 0.1x^3$
10. $y = x^3 + 1$
11. $y = 2x^3 + 1$
12. $y = x^3 - 3$
13. $y = 2x^3 - 2$
14. $y = \frac{1}{4}x^3$
15. $y = \frac{1}{3}x^3 + 2$
16. $y = -x^3 - 2$
17. $y = -x^3 + 1$

18. **MEASUREMENT** Jorge built a scale model of the Great Pyramid. The base of the model is a square with side length s and the model's height is $(s - 1)$ feet. Write the function for the volume V of the model. Graph this function. Then estimate the length of one side of the square base of the model if the model's volume is approximately 8 cubic feet.



EXTRA PRACTICE

See pages 703, 717.

Math Online

Self-Check Quiz at ca.gr7math.com

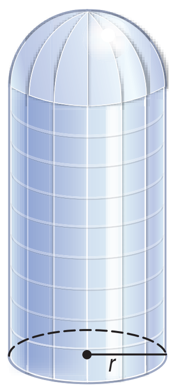
19. **MEASUREMENT** The formula for the volume V of a tennis ball is given by the equation $V = \frac{4}{3}\pi r^3$ where r represents the radius of the ball. Graph this function. Use 3.14 for π. Then estimate the length of the radius if the volume of the tennis ball is approximately 11 cubic inches.

Graph each pair of equations on the same coordinate plane. Describe their similarities and differences.

20. $y = x^3$; $y = 3x^3$
21. $y = x^3$; $y = x^3 - 3$
22. $y = 0.5x^3$; $y = 2x^3$
23. $y = 2x^3$; $y = -2x^3$

FARMING For Exercises 24 and 25, use the following information.

A grain silo consists of a cylindrical main section and a hemispherical roof. The cylindrical main section has a radius of r units and a height h equivalent to the radius. The volume V of a cylinder is given by the equation $V = \pi r^2 h$.



24. Write the function for the volume V of the cylindrical main section of the grain silo in terms of its radius r.
25. Graph this function. Use 3.14 for π. Then estimate the radius and height in meters of the cylindrical main section of the grain silo if the volume is approximately 15.5 cubic meters.

H.O.T. Problems

26. **OPEN ENDED** Write the equation of a cubic function whose graph in the first quadrant shows faster growth than the function $y = x^3$.

CHALLENGE The *zero* of a cubic function is the x-coordinate at which the function crosses the x-axis. Find the zeros of each function below.

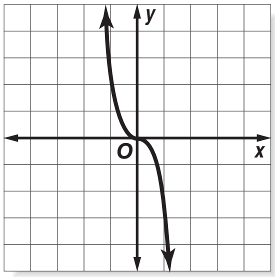
27. $y = x^3$

28. $y = x^3 + 1$

29. **WRITING IN MATH** The volume V of a cube with side length s is given by the equation $V = s^3$. Explain why negative values are not necessary when creating a table or a graph of this function.

STANDARDS PRACTICE

30. Which equation could represent the graph shown below?



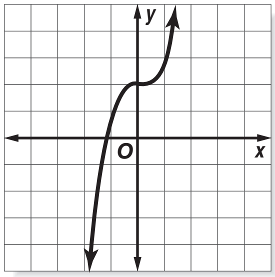
A $y = x^3$

B $y = -x^3$

C $y = 2x^3$

D $y = -2x^3$

31. Which equation could represent the graph shown below?



F $y = x^3 - 2$

G $y = x^3 + 2$

H $y = -2x^3$

J $y = 2x^3 + 1$

Spiral Review

32. **MANUFACTURING** A company packages six small books for a children's collection in a decorated 4-inch cube. They are shipped to bookstores in cartons. Twenty cubes fit in a carton with no extra space. What are the dimensions of the carton? Use the *make a model* strategy. (Lesson 10-3)

Graph each function. (Lesson 10-2)

33. $y = -2x^2$

34. $y = x^2 + 3$

35. $y = -3x^2 + 1$

36. $y = 4x^2 + 3$

Estimate each square root to the nearest whole number. (Lesson 3-2)

37. $\sqrt{54}$

38. $-\sqrt{126}$

39. $\sqrt{8.67}$

40. $-\sqrt{19.85}$

GET READY for the Next Lesson

PREREQUISITE SKILL **Write each expression using exponents.** (Lesson 2-9)

41. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

42. $5 \cdot 4 \cdot 5 \cdot 5 \cdot 4$

43. $7 \cdot (7 \cdot 7)$

44. $(2 \cdot 2) \cdot (2 \cdot 2 \cdot 2)$

Extend 10-4

Graphing Calculator Lab
Families of Nonlinear Functions

Main IDEA

Use a graphing calculator to graph families of nonlinear functions.

Standard 7AF3.1 **Graph functions of the form $y = nx^2$ and** $y = nx^3$ and use in solving problems.
Standard 7MR3.3 Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.

Families of nonlinear functions share a common characteristic based on a parent function. The parent function of a family of quadratic functions is $y = x^2$. You can use a graphing calculator to investigate families of quadratic functions.

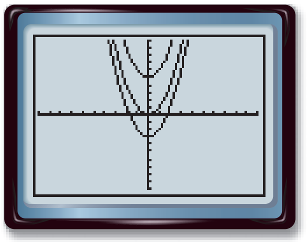
ACTIVITY

Graph $y = x^2$, $y = x^2 + 5$, and $y = x^2 - 3$ on the same screen.

STEP 1 Clear any existing equations from the Y= list by pressing [Y=] [CLEAR].

STEP 2 Enter each equation. Press
[X,T,θ,*n*] [x^2] [ENTER],
[X,T,θ,*n*] [x^2] [+] 5 [ENTER], and
[X,T,θ,*n*] [x^2] [−] 3 [ENTER].

STEP 3 Graph the equations in the standard viewing window. Press [ZOOM] 6.



Analyze the Results

1. Compare and contrast the three equations you graphed.
2. Describe how the graphs of the three equations are related.
3. **MAKE A CONJECTURE** How does changing the value of c in the equation $y = x^2 + c$ affect the graph?
4. Use a graphing calculator to graph $y = 0.5x^2$, $y = x^2$, and $y = 2x^2$.
5. Compare and contrast the three equations you graphed in Exercise 4.
6. Describe how the graphs of the three equations are related.
7. **MAKE A CONJECTURE** How does changing the value of a in the equation $y = ax^2$ affect the graph?
8. Use a graphing calculator to graph $y = 0.5x^3$, $y = x^3$, and $y = 2x^3$.
9. Compare and contrast the three equations you graphed in Exercise 8 to the equations you graphed in Exercise 4.

Math Online **Other Calculator Keystrokes at** ca.gr7math.com

10-5 Multiplying Monomials

Main IDEA

Multiply monomials.

Standard 7NS2.3 **Multiply,** divide, **and simplify rational numbers by using exponent rules.**
Standard 7AF2.1 **Interpret positive whole-number powers as repeated multiplication** and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. **Simplify and evaluate expressions that include exponents.**
Standard 7AF2.2 **Multiply** and divide **monomials;** extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.

NEW Vocabulary

monomial

GET READY for the Lesson

SCIENCE The pH of a solution describes its acidity. Neutral water has a pH of 7. Lemon juice has a pH of 2. Each one-unit decrease in the pH means that the solution is 10 times more acidic. So, a pH of 8 is 10 times more acidic than a pH of 9.

pH	Times More Acidic Than a pH of 9	Written Using Powers
8	10	10^1
7	$10 \times 10 = 100$	$10^1 \times 10^1 = 10^2$
6	$10 \times 10 \times 10 = 1{,}000$	$10^1 \times 10^2 = 10^3$
5	$10 \times 10 \times 10 \times 10 = 10{,}000$	$10^1 \times 10^3 = 10^4$
4	$10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$	$10^1 \times 10^4 = 10^5$

1. Examine the exponents of the factors and the exponents of the products in the last column. What do you observe?

A **monomial** is a number, a variable, or a product of a number and one or more variables. Exponents are used to show repeated multiplication. You can use this fact to find a rule for multiplying monomials.

$$3^2 \cdot 3^4 = \underbrace{\overbrace{(3 \cdot 3)}^{\text{2 factors}} \cdot \overbrace{(3 \cdot 3 \cdot 3 \cdot 3)}^{\text{4 factors}}}_{\text{6 factors}} \text{ or } 3^6$$

Notice that the sum of the original exponents is the exponent in the final product. This relationship is stated in the following rule.

KEY CONCEPT — Product of Powers

Words To multiply powers with the same base, add their exponents.

Examples

Numbers	Algebra
$2^4 \cdot 2^3 = 2^{4+3}$ or 2^7	$a^m \cdot a^n = a^{m+n}$

EXAMPLES Multiply Powers

STUDY TIP

Common Error When multiplying powers, do not multiply the bases. $4^5 \cdot 4^2 = 4^7$, not 16^7.

1 **Find $5^2 \cdot 5$. Express using exponents.**

$5^2 \cdot 5 = 5^2 \cdot 5^1$ — $5 = 5^1$
$= 5^{2+1}$ — The common base is 5.
$= 5^3$ — Add the exponents.

Check $5^2 \cdot 5 = (5 \cdot 5) \cdot 5$
$= 5 \cdot 5 \cdot 5$
$= 5^3$ ✓

CORBIS

Real-World Link

A census is taken every ten years by the U.S. Census Bureau to determine population. The government uses the data from the census to make many decisions.

Source: census.gov

Personal Tutor

2 **Find $-3x^2(4x^5)$. Express using exponents.**

$$-3x^2(4x^5) = (-3 \cdot 4)(x^2 \cdot x^5) \quad \text{Commutative and Associative Properties}$$
$$= (-12)(x^{2+5}) \quad \text{The common base is } x.$$
$$= -12x^7 \quad \text{Add the exponents.}$$

CHECK Your Progress **Multiply. Express using exponents.**

a. $9^3 \cdot 9^2$ **b.** $\left(\frac{3}{5}\right)^2\left(\frac{3}{5}\right)^9$ **c.** $-2m(-8m^5)$

Real-World EXAMPLE

3 **The population of Groveton is 6^5. The population of Putnam is 6^3 times as great. How many people are in Putnam?**

To find out the number of people, multiply 6^5 by 6^3.

$6^5 \cdot 6^3 = 6^{5+3}$ or 6^8 Product of Powers

The population of Putnam is 6^8 or 1,679,616 people.

CHECK Your Progress

d. **RIVERS** The Guadalupe River is 2^8 miles long. The Amazon River is almost 2^4 times as long. Find the length of the Amazon River.

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In Lesson 2-9, you learned to evaluate negative exponents. Remember that any nonzero number to the negative n power is the multiplicative inverse of that number to the n^{th} power. The Product of Powers rule can be used to multiply powers with negative exponents.

EXAMPLE Multiply Negative Powers

4 **Find $x^4 \cdot x^{-2}$. Express using exponents.**

METHOD 1

$$x^4 \cdot x^{-2} = x^{4+(-2)} \quad \text{The common base is } x.$$
$$= x^2 \quad \text{Add the exponents.}$$

METHOD 2

$$x^4 \cdot x^{-2}$$
$$= x \cdot x \cdot \cancel{x} \cdot \cancel{x} \cdot \frac{1}{\cancel{x}} \cdot \frac{1}{\cancel{x}} \quad x^{-2} = \frac{1}{x^2}$$
$$= x^2 \quad \text{Simplify.}$$

CHECK Your Progress

Simplify. Express using positive exponents.

d. $3^8 \cdot 3^{-2}$ **e.** $n^9 \cdot n^{-4}$ **f.** $5^{-1} \cdot 5^{-2}$

Check Your Understanding

Examples 1–4 (pp. 539–540)

Simplify. Express using exponents.

1. $4^5 \cdot 4^3$
2. $n^2 \cdot n^9$
3. $-2a(3a^4)$
4. $5^2x^2y^4 \cdot 5^3xy^3$
5. $r^7 \cdot r^{-3}$
6. $6m \cdot 4m^2$

Example 3 (p. 540)

7. **AGE** Angelina is 2^3 years old. Her grandfather is 2^3 times her age. How old is her grandfather?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
8–25	1, 2, 4
26–28	3

Simplify. Express using exponents.

8. $6^8 \cdot 6^5$
9. $2^9 \cdot 2$
10. $n \cdot n^7$
11. $b^{13} \cdot b$
12. $2g \cdot 7g^6$
13. $(3x^8)(5x)$
14. $-4a^5(6a^5)$
15. $(8w^4)(-w^7)$
16. $(-p)(-9p^2)$
17. $-5y^3(-8y^6)$
18. $4m^{-2}n^5(3m^4n^{-2})$
19. $(-7a^4bc^3)(5ab^4c^2)$
20. $x^6 \cdot x^{-3}$
21. $y^{-1} \cdot y^4$
22. $z^{-2} \cdot z^{-3}$
23. $m^2n^{-1} \cdot m^{-3}n^3$
24. $3f^{-4} \cdot 5f^2$
25. $-3ab \cdot 4a^{-3}b^3$

26. **INSECTS** The number of ants in a nest was 5^3. After the eggs hatched, the number of ants increased 5^2 times. How many ants are there after the eggs hatch?

27. **COMPUTERS** The processing speed of a certain computer is 10^{11} insructions per second. Another computer has a processing speed that is 10^3 times as fast. How many instructions per second can the faster computer process?

28. **LIFE SCIENCE** A cell culture contains 2^6 cells. By the end of the day, there are 2^{10} times as many cells in the culture. How many cells are there in the culture by the end of the day?

Simplify. Express using exponents.

EXTRA PRACTICE
See pages 703, 717.
Math Online
Self-Check Quiz at ca.gr7math.com

29. $xy^2(x^3y)$
30. $2^6 \cdot 2 \cdot 2^3$
31. $4a^2b^3(7ab^2)$
32. $\left(\frac{2}{3}\right)^4\left(\frac{2}{3}\right)^3$
33. $\left(\frac{7}{8}\right)^{-5}\left(\frac{7}{8}\right)^{13}$
34. $\left(\frac{2}{5}\right)^4\left(\frac{2}{5}\right)^{-7}\left(\frac{2}{5}\right)^6$
35. $\left(\frac{1}{4}\right)^{-4}\left(\frac{1}{4}\right)$
36. $\left(\frac{2}{5}\right)^3\left(\frac{2}{5}\right)^{-2}$
37. $\left(\frac{2}{7}\right)^{-2}\left(\frac{7}{2}\right)^{-3}$

H.O.T. Problems

38. **CHALLENGE** What is twice 2^{30}? Write using exponents.

39. **OPEN ENDED** Write a multiplication expression whose product is 4^{15}.

40. **WRITING IN MATH** Determine whether the following statement is *true* or *false*. Explain your reasoning or give a counterexample.

If you change the order in which you multiply two monomials, the product will be different.

STANDARDS PRACTICE

41. Which expression is equivalent to $8x^2y \cdot 8yz^2$?

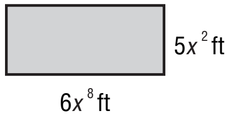
A $64x^2y^2z^2$

B $64x^2yz^2$

C $16x^2y^2z^2$

D $384x^2y^2z^2$

42. Which expression describes the area in square feet of the rectangle below?



F $11x^{10}$

G $30x^{10}$

H $11x^{16}$

J $30x^{16}$

Spiral Review

Graph each function. (Lessons 10-2 and 10-4)

43. $y = -x^3$

44. $y = 0.5x^3$

45. $y = x^3 - 2$

46. $y = 5x^2$

47. $y = x^2 + 5$

48. $y = x^2 - 4$

49. **BIOLOGY** The table shows how long it took for the first 400 bacteria cells to grow in a petri dish. Is the growth of the bacteria a linear function of time? Explain. (Lesson 10-1)

Time (min)	46	53	57	60
Number of cells	100	200	300	400

Express each number in scientific notation. (Lesson 2-10)

50. The flow rate of some Antarctic glaciers is 0.00031 mile per hour.

51. A human blinks about 6.25 million times a year.

ALGEBRA **Solve each equation. Check your solution.** (Lesson 2-7)

52. $k - 4.1 = -9.38$

53. $1\frac{3}{4} + p = -6\frac{1}{2}$

54. $\frac{c}{10} = 0.845$

Find each sum or difference. Write in simplest form. (Lesson 2-6)

55. $\frac{7}{8} - \frac{3}{10}$

56. $-\frac{1}{5} + \frac{5}{12}$

57. $9\frac{2}{3} + \frac{1}{6}$

58. $-2\frac{3}{4} - 1\frac{1}{8}$

GET READY for the Next Lesson

PREREQUISITE SKILL **Write each expression using exponents.** (Lesson 2-9)

59. $3 \cdot 3 \cdot 3 \cdot 3$

60. $5 \cdot 4 \cdot 5 \cdot 5 \cdot 4$

61. $7 \cdot (7 \cdot 7)$

62. $(2 \cdot 2) \cdot (2 \cdot 2 \cdot 2)$

CHAPTER 10

Mid-Chapter Quiz

Lessons 10-1 through 10-5

Determine whether each equation or table represents a *linear* or *nonlinear* function. Explain. (Lesson 10-1)

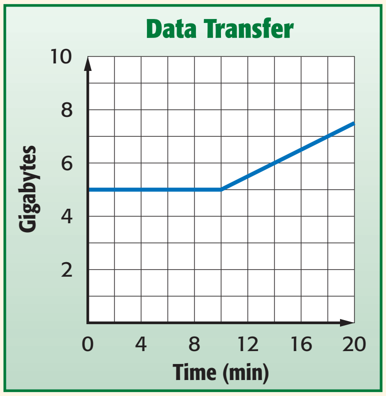
1. $3y = x$
2. $y = 5x^3 + 2$
3.

x	1	3	5	7
y	−5	−6	−7	−8

4.

x	−1	0	1	2
y	1	0	1	4

5. **LONG DISTANCE** The graph shows the amount of data transferred as a function of time. Is this a linear or nonlinear function? Explain your reasoning. (Lesson 10-1)

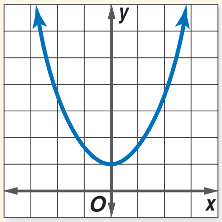


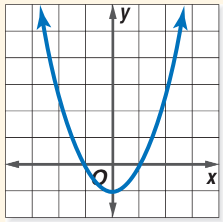
Graph each function. (Lesson 10-2)

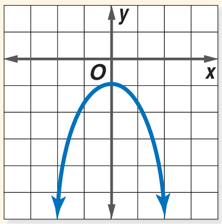
6. $y = 2x^2$
7. $y = -x^2 + 3$
8. $y = 4x^2 - 1$
9. $y = -3x^2 + 1$

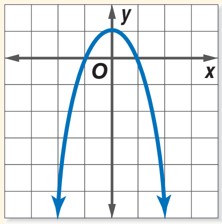
10. **AMUSEMENT PARK RIDES** Your height h feet above the ground t seconds after being released at the top of a free-fall ride is given by the function $h = -16t^2 + 200$. Graph this function. After about how many seconds will the ride be 60 feet above the ground? (Lesson 10-2)

11. **STANDARDS PRACTICE** Which graph shows $y = x^2 + 1$? (Lesson 10-2)

A


C


B


D


12. **MEASUREMENT** Brenda has a photograph that is 10 inches by 13 inches. She decides to frame it, using a frame that is $2\frac{1}{4}$ inches wide on each side. Find the total area of the framed photograph. Use the *make a model* strategy. (Lesson 10-3)

Graph each function. (Lesson 10-4)

13. $y = -2x^3$
14. $y = 3x^3$
15. $y = 2x^3$
16. $y = 0.1x^3$

Simplify. Express using exponents. (Lesson 10-4)

17. $10^4 \cdot 10^7$
18. $3^{-3} \cdot 3^5 \cdot 3^2$
19. $2^3a^7 \cdot 2a^{-3}$
20. $(3^2xy^4z^2)(3^5x^3y^{-2}z^3)$

21. **STANDARDS PRACTICE** Which expression below has the same value as $5m^2$? (Lesson 10-5)

F $5m$

G $5 \cdot m \cdot m$

H $5 \cdot 5 \cdot m \cdot m$

J $5 \cdot m \cdot m \cdot m$

10-6 Dividing Monomials

Main IDEA

Divide monomials.

Standard 7NS2.3 Multiply, **divide, and simplify rational numbers by using exponent rules.**
Standard 7AF2.1 **Interpret** positive whole-number powers as repeated multiplication and **negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.**
Standard 7AF2.2 Multiply and **divide monomials;** extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.

GET READY for the Lesson

NUMBER SENSE Refer to the table shown that relates division sentences using the numbers 2, 4, 8, and 16, and the same sentences written using powers of 2.

Division Sentence	Written Using Powers of 2
$4 \div 2 = 2$	$2^2 \div 2^1 = 2^1$
$8 \div 2 = 4$	$2^3 \div 2^1 = 2^2$
$8 \div 4 = 2$	$2^3 \div 2^2 = 2^1$
$16 \div 2 = 8$	$2^4 \div 2^1 = 2^3$
$16 \div 4 = 4$	$2^4 \div 2^2 = 2^2$
$16 \div 8 = 2$	$2^4 \div 2^3 = 2^1$

1. Examine the exponents of the divisors and dividends. Compare them to the exponents of the quotients. What do you notice?
2. **MAKE A CONJECTURE** Write the quotient of 2^5 and 2^2 using powers of 2.

As you learned in Lesson 10-5, exponents are used to show repeated multiplication. You can use this fact to find a rule for dividing powers with the same base.

Notice that the difference of the original exponents is the exponent in the final quotient. This relationship is stated in the following rule.

$$\frac{5^7}{5^4} = \frac{\overbrace{5 \cdot 5 \cdot 5 \cdot \cancel{5} \cdot \cancel{5} \cdot \cancel{5} \cdot \cancel{5}}^{\text{7 factors}}}{\underbrace{\cancel{5} \cdot \cancel{5} \cdot \cancel{5} \cdot \cancel{5}}_{\text{4 factors}}} \text{ or } 5^3$$

KEY CONCEPT — Quotient of Powers

Words To divide powers with the same base, subtract their exponents.

Examples

Numbers	Algebra
$\frac{3^7}{3^3} = 3^{7-3}$ or 3^4	$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

EXAMPLES Divide Powers

Simplify. Express using exponents.

1 $\frac{4^8}{4^2}$

$\frac{4^8}{4^2} = 4^{8-2}$ The common base is 4.

$= 4^6$ Simplify.

2 $\frac{n^9}{n^4}$

$\frac{n^9}{n^4} = n^{9-4}$ The common base is n.

$= n^5$ Simplify.

STUDY TIP

Common Error When dividing powers, do not divide the bases. $\frac{4^8}{4^2} = 4^6$, not 1^6.

CHECK Your Progress

Simplify. Express using exponents.

a. $\frac{5^7}{5^4}$

b. $\frac{x^{10}}{x^3}$

c. $\frac{12w^5}{2w}$

The Quotient of Powers rule can also be used to divide powers with negative exponents. It is customary to write final answers using positive exponents.

STUDY TIP

Look Back To review **adding and subtracting integers,** see Lessons 1-4 and 1-5.

EXAMPLES Use Negative Exponents

Simplify. Express using positive exponents.

3 $\dfrac{6^9}{6^{-3}}$

$\dfrac{6^9}{6^{-3}} = 6^{9-(-3)}$ Quotient of Powers

$= 6^{9+3}$ or 6^{12} Simplify.

4 $\dfrac{w^{-1}}{w^{-4}}$

$\dfrac{w^{-1}}{w^{-4}} = w^{-1-(-4)}$ Quotient of Powers

$= w^{-1+4}$ or w^3 Simplify.

CHECK Your Progress

Simplify. Express using positive exponents.

d. $\dfrac{11^{-8}}{11^2}$ **e.** $\dfrac{b^{-4}}{b^{-7}}$ **f.** $\dfrac{6h^5}{3h^{-5}}$

STANDARDS EXAMPLE

5 $\dfrac{2^2 \cdot 4^5 \cdot 5^2}{2^5 \cdot 4^4 \cdot 5^2} =$

A 2 **B** 1 **C** $\dfrac{1}{2}$ **D** 0

Test-Taking Tip

Remember that the Quotient of Powers Rule allows you to simplify $\frac{5^2}{5^2}$.

$\frac{5^2}{5^2} = 5^{2-2} = 5^0 = 1.$

Read the Item

You are asked to divide one monomial by another.

Solve the Item

$\dfrac{2^2 \cdot 4^5 \cdot 5^2}{2^5 \cdot 4^4 \cdot 5^2} = \left(\dfrac{2^2}{2^5}\right)\left(\dfrac{4^5}{4^4}\right)\left(\dfrac{5^2}{5^2}\right)$ Group by common base.

$= 2^{-3} \cdot 4^1 \cdot 5^0$ Subtract the exponents.

$= \dfrac{1}{2^3} \cdot 4 \cdot 1$ $2^{-3} = \dfrac{1}{2^3}$

$= \dfrac{4}{8}$ or $\dfrac{1}{2}$ Simplify.

The answer is C.

CHECK Your Progress

g. Simplify $\dfrac{\left(\frac{1}{6}\right)^4 \times \left(\frac{1}{6}\right)^{-12}}{\left(\frac{1}{6}\right)^{-3}}$.

F $\left(\frac{1}{6}\right)^5$ G $\frac{1}{6}$ H 6^4 J 6^5

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Real-World Link The decibel measure of the loudness of a sound is the exponent of its relative intensity multiplied by 10. A jet engine has a loudness of 120 decibels.

Real-World EXAMPLE

6 SOUND The loudness of a conversation is 10^6 times as intense as the loudness of a pin dropping, while the loudness of a jet engine is 10^{12} times as intense. How many times more intense is the loudness of a jet engine than the loudness of a conversation?

To find how many times more intense, divide 10^{12} by 10^6.

$\dfrac{10^{12}}{10^6} = 10^{12-6}$ or 10^6 Quotient of Powers

The loudness of a jet engine is 10^6 or 1,000,000 times as intense as the loudness of a conversation.

CHECK Your Progress

h. **SOUND** The loudness of a vacuum cleaner is 10^4 times as intense as the loudness of a mosquito buzzing, while the loudness of a jack hammer is 10^9 times as intense. How many times more intense is the loudness of a jack hammer than that of a vacuum cleaner?

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CHECK Your Understanding

Examples 1–4 (pp. 544-545)

Simplify. Express using positive exponents.

1. $\dfrac{7^6}{7}$
2. $\dfrac{2^9}{2^{13}}$
3. $\dfrac{y^8}{y^5}$
4. $\dfrac{z}{z^2}$
5. $\dfrac{9c^7}{3c^2}$
6. $\dfrac{24k^9}{6k^6}$
7. $\dfrac{15^{-6}}{15^2}$
8. $\dfrac{35p^1}{5p^{-4}}$

Example 5 (p. 545)

9. Simplify $\dfrac{2^2 \cdot 3^3 \cdot 4^4}{2 \cdot 3^3 \cdot 4^5}$.

A 2^2 B 2 C $\frac{1}{2}$ D $\left(\frac{1}{2}\right)^2$

Example 6 (p. 546)

10. **ASTRONOMY** Venus is approximately 10^8 kilometers from the Sun. Saturn is more than 10^9 kilometers from the Sun. About how many times farther away from the Sun is Saturn than Venus?

Exercises

HOMEWORK HELP

For Exercises	See Examples
11–26	1–4
27–30	5
31–34	6

Simplify. Express using positive exponents.

11. $\dfrac{8^{15}}{8^4}$

12. $\dfrac{2^9}{2}$

13. $\dfrac{4^3}{4^7}$

14. $\dfrac{13^2}{13^5}$

15. $\dfrac{h^7}{h^6}$

16. $\dfrac{g^{18}}{g^6}$

17. $\dfrac{x^8}{x^{11}}$

18. $\dfrac{n}{n^8}$

19. $\dfrac{36d^{10}}{6d^5}$

20. $\dfrac{16t^4}{8t}$

21. $\dfrac{20m^7}{5m^5}$

22. $\dfrac{75r^6}{25r^5}$

23. $\dfrac{22^{-9}}{22^4}$

24. $\dfrac{3^{-1}}{3^{-5}}$

25. $\dfrac{42w^{-6}}{7w^{-2}}$

26. $\dfrac{12y^{-6}}{2y^{-10}}$

27. $\dfrac{x^6y^{14}}{x^4y^9}$

28. $\dfrac{6^3 \cdot 6^6 \cdot 6^4}{6^2 \cdot 6^3 \cdot 6^4} =$

29. $\dfrac{\left(\frac{1}{5}\right)^2 \times \left(\frac{1}{5}\right)^{-6}}{\left(\frac{1}{5}\right)^2}$

30. $\dfrac{3x^4}{3^4x^{-2}}$

31. **POPULATION** The continent of North America contains approximately 10^7 square miles of land. If the population doubles, there will be about 10^9 people on the continent. At that point, on average, how many people will occupy each square mile of land?

32. **FOOD** An apple is 10^3 times as acidic as milk, while a lemon is 10^4 times as acidic. How many times more acidic is a lemon than an apple?

33. **ANIMALS** A common flea 2^{-4} inch long can jump about 2^3 inches high. About how many times its body size can a flea jump?

34. **MEDICINE** The mass of a molecule of penicillin is 10^{-18} kilograms and the mass of a molecule of insulin is 10^{-23} kilograms. How many times greater is a molecule of penicillin than a molecule of insulin?

Find each missing exponent.

35. $\dfrac{17^{\bullet}}{17^4} = 17^8$

36. $\dfrac{k^6}{k^{\bullet}} = k^2$

37. $\dfrac{5^{\bullet}}{5^{-9}} = 5^3$

38. $\dfrac{p^{-1}}{p^{\bullet}} = p^{10}$

ANALYZE TABLES **For Exercises 39 and 40, use the information below and in the table.**

For each increase of one on the Richter scale, an earthquake's vibrations, or seismic waves, are 10 times greater.

Earthquake	Richter Scale Magnitude
San Francisco, 1906	8.3
Adana, Turkey, 1998	6.3

Source: usgs.gov

39. How many times greater are the seismic waves of an earthquake with a magnitude of 6 than an aftershock with a magnitude of 3?

40. How many times greater were the seismic waves of the 1906 San Francisco earthquake than the 1998 Adana earthquake?

EXTRA PRACTICE

See pages 703, 717.

Math Online

Self-Check Quiz at ca.gr7math.com

H.O.T. Problems

41. **NUMBER SENSE** Is $\frac{3^{100}}{3^{99}}$ *greater than*, *less than*, or *equal to* 3? Explain your reasoning.

42. **OPEN ENDED** Write a division expression with a quotient of 4^{15}.

43. **CHALLENGE** What is half of 2^{30}? Write using exponents.

44. **WRITING IN MATH** Explain why the Quotient of Powers Rule cannot be used to simplify the expression $\frac{x^5}{y^2}$.

STANDARDS PRACTICE

45. Which expression below is equivalent to $\frac{9m^8}{3m^2}$?

A $6m^4$ **C** $3m^4$

B $6m^6$ **D** $3m^6$

46. The area of a rectangle is 2^6 square feet. If the length is 2^3 feet, find the width of the rectangle.

F 2 feet **H** 2^3 feet

G 2^2 feet **J** 2^9 feet

47. One meter is 10^3 times longer than one millimeter. One kilometer is 10^6 times longer than one millimeter. How many times longer is one kilometer than one meter?

A 10^9

B 10^6

C 10^3

D 10

Spiral Review

Simplify. Express using positive exponents. (Lesson 10-5)

48. $6^4 \cdot 6^7$ 49. $18^3 \cdot 18^{-5}$ 50. $(-3x^{11})(-6x^3)$ 51. $(-9a^4)(2a^{-7})$ –

Graph each function. (Lesson 10-4)

52. $y = x^3 + 2$ 53. $y = \frac{1}{3}x^3$ 54. $y = -2x^3$ 55. $y = -0.1x^3$

State the slope and the *y*-intercept for the graph of each equation. (Lesson 9-5)

56. $y = x - 3$ 57. $y = \frac{2}{3}x + 7$ 58. $3x + 4y = 12$ 59. $x + 2y = 10$

60. **COIN COLLECTING** Jada has 156 coins in her collection. This is 12 more than 8 times the number of nickels in the collection. How many nickels does Jada have in her collection? (Lesson 8-3)

GET READY for the Next Lesson

Simplify. Express using positive exponents. (Lesson 10-5)

61. $5n \cdot 3n^4$ 62. $(-x)(-8x^3)$ 63. $(-5b^7)(-2b^4)$ 64. $(-4w)(6w^{-2})$

10-7 Powers of Monomials

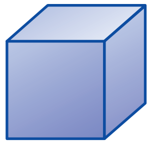
Main IDEA

Find powers of monomials.

Standard 7AF2.2 Multiply and divide monomials; **extend the process of taking powers** and extracting roots **to monomials** when the latter results in a monomial with an integer exponent.

GET READY for the Lesson

MEASUREMENT Suppose the side length of a cube is 2^2 centimeters.



2^2 cm

1. Write a multiplication expression for the volume of the cube.
2. Simplify the expression. Write as a single power of 2.
3. Using 2^2 as the base, write the multiplication expression $2^2 \cdot 2^2 \cdot 2^2$ using an exponent.
4. Explain why $(2^2)^3 = 2^6$.

You can use the rule for finding the *product* of a power to discover the rule for finding the *power* of a power.

$$(6^4)^5 = \overbrace{(6^4)\,(6^4)\,(6^4)\,(6^4)\,(6^4)}^{\text{5 factors}}$$

$$= 6^{4+4+4+4+4} \leftarrow \text{Apply the rule for the product of powers.}$$

$$= 6^{20}$$

Notice that the product of the original exponents, 4 and 5, is the final power 20. This relationship is stated in the following rule.

KEY CONCEPT — Power of a Power

Words To find the power of a power, multiply the exponents.

Examples	Numbers	Algebra
	$(5^2)^3 = 5^{2 \cdot 3}$ or 5^6	$(a^m)^n = a^{m \cdot n}$

STUDY TIP

Common Error When finding the power of a power, do not add the exponents.
$(8^4)^3 = 8^{12}$, not 8^7.

EXAMPLES Find the Power of a Power

1 Simplify $(8^4)^3$.

$(8^4)^3 = 8^{4 \cdot 3}$ Power of a Power

$= 8^{12}$ Simplify.

2 Simplify $(k^7)^5$.

$(k^7)^5 = k^{7 \cdot 5}$ Power of a Power

$= k^{35}$ Simplify.

CHECK Your Progress Simplify. Express using exponents.

a. $(2^5)^2$ b. $(w^4)^6$ c. $[(3^2)^3]^2$

Extend the power of a *power* rule to find the power of a *product*.

$$(3a^4)^5 = \underbrace{(3a^4)\,(3a^4)\,(3a^4)\,(3a^4)\,(3a^4)}_{\text{5 factors}}$$

$= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot a^4 \cdot a^4 \cdot a^4 \cdot a^4 \cdot a^4$ Associative Property of Multiplication

$= 3^5 \cdot (a^4)^5$ Write using powers.

$= 243 \cdot a^{20}$ or $243a^{20}$ Apply the rule for power of a power.

This example suggests the following rule.

KEY CONCEPT — Power of a Product

Words To find the power of a product, find the power of each factor and multiply.

Examples

Numbers	Algebra
$(6x^2)^3 = (6)^3 \cdot (x^2)^3$ or $216x^6$	$(ab)^m = a^m b^m$

EXAMPLES Power of a Product

3 **Simplify $(4p^3)^4$.**

$(4p^3)^4 = 4^4 \cdot p^{3 \cdot 4}$

$= 256p^{12}$ Simplify.

4 **Simplify $(-2m^7n^6)^5$.**

$(-2m^7n^6)^5 = (-2)^5 m^{7 \cdot 5} n^{6 \cdot 5}$

$= -32m^{35}n^{30}$ Simplify.

STUDY TIP

Alternative Method
$(4p^3)^4$ can also be expressed as
$(4p^3)(4p^3)(4p^3)(4p^3)$
or $(4 \cdot 4 \cdot 4 \cdot 4)$
$(p \cdot p \cdot p)(p \cdot p \cdot p)$
$(p \cdot p \cdot p)(p \cdot p \cdot p)$
which is $256p^{12}$.

CHECK Your Progress Simplify.

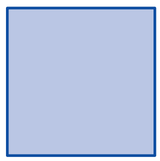
d. $(8b^9)^2$

e. $(6x^5y^{11})^4$

f. $(-5w^2z^8)^3$

Real-World EXAMPLE

5 **GEOMETRY** **Express the area of the square as a monomial.**



$7a^4b$

$A = s^2$ Area of a square

$A = (7a^4b)^2$ Replace s with $7a^4b$.

$A = 7^2(a^4)^2(b^1)^2$ Power of a Product

$A = 49a^8b^2$ Simplify.

The area of the square is $49a^8b^2$ square units.

CHECK Your Progress

g. **GEOMETRY** Find the volume of a cube with sides of length $8x^3y^5$. Express as a monomial.

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CHECK Your Understanding

Examples 1–4 (pp. 549-550)

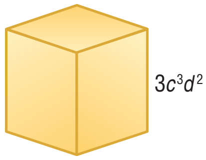
Simplify.

1. $(3^2)^5$ **2.** $(h^6)^4$ **3.** $[(2^3)^2]^3$

Example 5 (p. 550)

4. $(7w^7)^3$ **5.** $(5g^8k^{12})^4$ **6.** $(-6r^5s^9)^2$

7. MEASUREMENT Express the volume of the cube at the right as a monomial.



Exercises

HOMEWORK HELP	
For Exercises	**See Examples**
8–27	1–4
28–31	3

Simplify.

8. $(4^2)^3$ **9.** $(2^2)^7$ **10.** $(5^3)^3$ **11.** $(3^4)^2$

12. $(d^7)^6$ **13.** $(m^8)^5$ **14.** $(h^4)^9$ **15.** $(z^{11})^5$

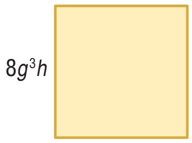
16. $[(3^2)^2]^2$ **17.** $[(4^3)^2]^2$ **18.** $[(5^2)^2]^2$ **19.** $[(2^3)^3]^2$

20. $(5j^6)^4$ **21.** $(8v^9)^5$ **22.** $(11c^4)^3$ **23.** $(14y)^4$

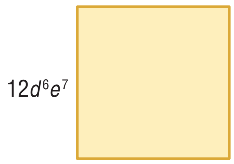
24. $(6a^2b^6)^3$ **25.** $(2m^5n^{11})^6$ **26.** $(-3w^3z^8)^5$ **27.** $(-5r^4s^{12})^4$

GEOMETRY Express the area of each square below as a monomial.

28.

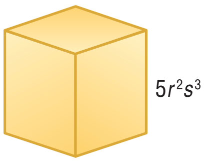


29.

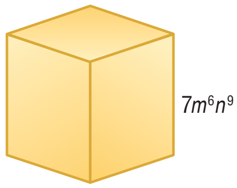


GEOMETRY Express the volume of each cube below as a monomial.

30.



31.



Simplify.

32. $(0.5k^5)^2$ **33.** $(0.3p^7)^3$ **34.** $(\frac{1}{4}w^5z^3)^2$

35. $(\frac{3}{5}a^{-6}b^9)^2$ **36.** $(3x^{-2})^4(5x^6)^2$ **37.** $(-2v^7)^3(-4v^{-2})^4$

38. PHYSICS A ball is dropped from the top of a building. The expression $4.9x^2$ gives the distance in meters the ball has fallen after x seconds. Write and simplify an expression that gives the distance in meters the ball has fallen after x^2 seconds. Then write and simplify an expression that gives the distance the ball has fallen after x^3 seconds.

39. BACTERIA A certain culture of bacteria doubles in population every hour. At 1 P.M., there are 5 cells. The expression $5(2^x)$ gives the number of bacteria that are present x hours after 1 P.M. Simplify the expressions $[5(2^x)]^2$ and $[5(2^x)]^3$ and describe what they each represent.

EXTRA PRACTICE

See pages 703, 717.

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MEASUREMENT For Exercises 40-42, use the table that gives the area and volume of a square and cube, respectively, with side lengths shown.

Side Length (units)	Area of Square (units^2)	Volume of Cube (units^3)
x	x^2	x^3
$2x$		
$3x$		
x^2		
x^3		

40. Copy and complete the table.

41. Describe how the area and volume are each affected if the side length is doubled. Then describe how they are each affected if the side length is tripled.

42. Describe how the area and volume are each affected if the side length is squared. Describe how they are each affected if the side length is cubed.

H.O.T. Problems

43. **OPEN ENDED** A *googol* is 10^{100}. Use the Power of a Power rule to write three different expressions that are equivalent to a googol where each expression uses exponents.

CHALLENGE Solve each equation for x.

44. $(7^x)^3 = 7^{15}$ 5

45. $(-2m^3n^4)^x = -8m^9n^{12}$ 3

46. **WRITING IN MATH** Compare and contrast how you would correctly simplify the expressions $(2a^3)(4a^6)$ and $(2a^3)^6$.

STANDARDS PRACTICE

47. Which expression is equivalent to $(10^4)^8$?

A 10^2 C 10^{12}

B 10^4 D 10^{32}

48. Which expression has the same value as $81h^8k^6$?

F $(9h^6k^4)^2$ H $(6h^5k^3)^3$

G $(9h^4k^3)^2$ J $(3h^2k)^6$

49. Which of the following has the same value as $64m^6$?

A the area in square units of a square whose side length is $8m^2$

B the expression $(32m^3)^2$

C the expression $(8m^3)^2$

D the volume in cubic units of a cube whose side length is $4m^3$

Spiral Review

Simplify. Express using positive exponents. (Lesson 10-6)

50. $\frac{15^7}{15^4}$

51. $\frac{y^{10}}{y^2}$

52. $\frac{18m^9}{6m^4}$

53. $\frac{24g^3}{3g^8}$

54. **MEASUREMENT** Find the area of a rectangle with a length of $9xy^2$ and a width of $4x^2y$. (Lesson 10-5)

GET READY for the Next Lesson

Find each square root. (Lesson 3-1)

55. $\sqrt{49}$

56. $\sqrt{121}$

57. $\sqrt{225}$

58. $\sqrt{400}$

10-8 Roots of Monomials

Main IDEA

Find roots of monomials.

Standard 7AF2.2 Multiply and divide monomials; **extend the process of** taking powers and **extracting roots to monomials when the latter results in a monomial with an integer exponent.**

NEW Vocabulary

cube root

REVIEW Vocabulary

square root: the opposite of squaring a number (Lesson 3-1)

GET READY for the Lesson

NUMBER THEORY The square root of a number is one of the two equal factors of the number. Some perfect squares can be factored into the product of two other perfect squares.

1. Find two factors of 100 that are also perfect squares.
2. Find the square root of 4 and 25. Then find their product.
3. How does the product relate to 100?
4. Repeat Questions 1–3 using 144.

The pattern you discovered about the factors of a perfect square is true for any number.

KEY CONCEPT — Product Property of Square Roots

Words For any numbers a and b, where $a \geq 0$ and $b \geq 0$, the square root of the product ab is equal to the product of each square root.

Examples

Numbers

$$\sqrt{9 \cdot 16} = \sqrt{9} \cdot \sqrt{16} = 3 \cdot 4 \text{ or } 12$$

Algebra

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

The square root of a monomial is also one of its two equal factors. You can use the product property of square roots to find the square roots of monomials.

$$\sqrt{x^2} = \sqrt{x \cdot x} = |x|$$

Since x represents an unknown value, absolute value is used to indicate the positive value of x.

$$\sqrt{x^4} = \sqrt{x^2 \cdot x^2} = x^2$$

Absolute value is not necessary since the value of x^2 will never be negative.

EXAMPLES Simplify Square Roots

1 Simplify $\sqrt{4y^2}$.

$$\sqrt{4y^2} = \sqrt{4} \cdot \sqrt{y^2}$$
$$= \sqrt{2 \cdot 2} \cdot \sqrt{y \cdot y}$$
$$= 2|y|$$

2 Simplify $\sqrt{36q^6}$.

$$\sqrt{36q^6} = \sqrt{36} \cdot \sqrt{q^6}$$
$$= \sqrt{6 \cdot 6} \cdot \sqrt{q^3 \cdot q^3}$$
$$= 6|q^3|$$

STUDY TIP

Absolute Value Use absolute value to indicate the positive value of y and q^3.

CHECK Your Progress Simplify.

a. $\sqrt{v^2}$ b. $\sqrt{c^6d^8}$ c. $\sqrt{121x^4z^{10}}$

READING Math

Cube Root Symbol The cube root of a is shown by the symbol $\sqrt[3]{a}$.

The process of simplifying expressions involving square roots can be extended to cube roots. The **cube root** of a monomial is one of the *three* equal factors of the monomial.

$$\sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2} = 2 \qquad \sqrt[3]{a^3} = \sqrt[3]{a \cdot a \cdot a} = a$$

KEY CONCEPT — Product Property of Cube Roots

Words For any numbers a and b, the cube root of the product ab is equal to the product of each cube root.

Examples

Numbers	Algebra
$\sqrt[3]{216} = \sqrt[3]{8} \cdot \sqrt[3]{27}$ $= 2 \cdot 3$ or 6	$\sqrt[3]{ab} = \sqrt[3]{a} \cdot \sqrt[3]{b}$

EXAMPLES Simplify Cube Roots

3 **Simplify $\sqrt[3]{c^3}$.**

$\sqrt[3]{c^3} = c$ $\qquad (c)^3 = c^3$

4 **Simplify $\sqrt[3]{64g^6}$.**

$$\sqrt[3]{64g^6} = \sqrt[3]{64} \cdot \sqrt[3]{g^6} \qquad \text{Product Property of Cube Roots}$$
$$= \sqrt[3]{4 \cdot 4 \cdot 4} \cdot \sqrt[3]{g^2 \cdot g^2 \cdot g^2}$$
$$= 4 \cdot g^2 \text{ or } 4g^2 \qquad \text{Simplify.}$$

STUDY TIP

Absolute Value Because a cube root can be negative, absolute value is not necessary when simplifying cube roots.

CHECK Your Progress **Simplify.**

d. $\sqrt[3]{s^3}$ **e.** $\sqrt[3]{27y^3}$ **f.** $\sqrt[3]{216k^{12}}$

Real-World EXAMPLE

5 **GEOMETRY** **Express the length of one side of the square whose area is $81y^2z^6$ square units as a monomial.**

$$A = s^2 \qquad \text{Area of a square}$$
$$81y^2z^6 = s^2 \qquad \text{Replace } A \text{ with } 81y^2z^6.$$
$$\sqrt{81y^2z^6} = s \qquad \text{Definition of square root.}$$
$$\sqrt{81} \cdot \sqrt{y^2} \cdot \sqrt{z^6} = s \qquad \text{Product Property of Square Roots.}$$
$$9\left|yz^3\right| = s \qquad \text{Simplify. Add absolute value.}$$

The length of one side of the square is $9\left|yz^3\right|$ units.

CHECK Your Progress

g. **GEOMETRY** Find the length of one side of a square whose volume is is $125a^{15}$ cubic units.

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CHECK Your Understanding

Examples 1–2 (p. 553)

Simplify.

1. $\sqrt{d^2}$
2. $\sqrt{25a^2}$
3. $\sqrt{49x^6y^2}$
4. $\sqrt{121h^8k^{10}}$

Example 3–4 (p. 554)

5. $\sqrt[3]{m^3}$
6. $\sqrt[3]{8p^3}$
7. $\sqrt[3]{125r^6s^9}$
8. $\sqrt[3]{64\,x^{12}y^3}$

Example 5 (p. 554)

9. **GEOMETRY** Express the length of one side of the square whose area is $256u^2v^6$ square units as a monomial.

10. **GEOMETRY** Express the length of one side of a cube whose volume is $27b^3c^{12}$ cubic units as a monomial.

Exercises

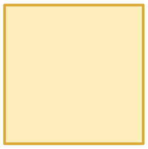
HOMEWORK HELP

For Exercises	See Examples
11–18	1–2
14–26	3
27–34	5

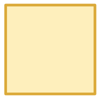
Simplify.

11. $\sqrt{n^2}$
12. $\sqrt{y^4}$
13. $\sqrt{g^8k^{14}}$
14. $\sqrt{64a^2}$
15. $\sqrt{36z^{12}}$
16. $\sqrt{144k^4m^6}$
17. $\sqrt{9p^8q^4}$
18. $\sqrt{225x^4y^6}$
19. $\sqrt[3]{h^3}$
20. $\sqrt[3]{v^3}$
21. $\sqrt[3]{27b^3}$
22. $\sqrt[3]{64k^3}$
23. $\sqrt[3]{125d^9e^3}$
24. $\sqrt[3]{8q^9r^{18}}$
25. $\sqrt[3]{343m^3n^{21}}$
26. $\sqrt[3]{216x^{12}w^{15}}$

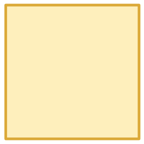
GEOMETRY **Express the length of one side of each square whose area is given as a monomial.**

27. 

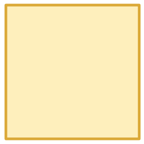
$A = 121a^2b^2$

28. 

$A = 36m^6n^8$

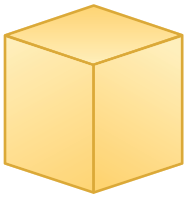
29. 

$A = 400x^2y^{10}$

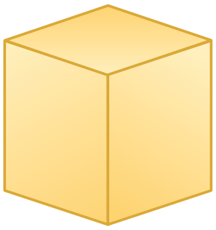
30. 

$A = 49p^4q^6$

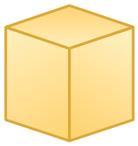
GEOMETRY **Express the length of one side of each cube whose volume is given as a monomial.**

31. 

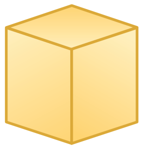
$V = 64w^3z^3$

32. 

$V = 343c^6d^{12}$

33. 

$V = 27g^{24}h^3$

34. 

$V = 125k^9m^{18}$

Simplify.

35. $\sqrt{0.25x^2}$
36. $\sqrt[3]{0.08p^9}$
37. $\sqrt[3]{\frac{8}{27}w^3x^6}$

Simplify each expression if $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

38. $\sqrt{\frac{x^2}{16}}$
39. $\sqrt{\frac{81}{m^4}}$
40. $\sqrt{\frac{121}{h^8k^6}}$

EXTRA PRACTICE

See pages 704, 717.

Math Online
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H.O.T. Problems

41. **OPEN ENDED** Write a monomial and its square root.

CHALLENGE Solve each equation for x.

42. $\sqrt{25a^x} = 5|a^3|$

43. $\sqrt[3]{64a^3b^x} = 4ab^7$

44. $\sqrt{81a^4b^x} = 9a^4|b^5|$

45. **WRITING IN MATH** Explain why absolute value is necessary when simplifying the expression $\sqrt{y^2}$ and not necessary when simplifying $\sqrt{y^4}$.

STANDARDS PRACTICE

46. Which expression is equivalent to $\sqrt{144g^2}$?

 A $12g$ C $12g^2$
 B $12|g|$ D $12|g^2|$

47. Which expression has the same value as $\sqrt{400h^2k^4}$?

 F $20hk^2$ H $20h^2k^4$
 G $20|h|k^2$ J $200|h|k^2$

48. Which of the following has the same value as $\sqrt[3]{27m^3n^6}$?

 A the length of the side of a square whose area is $27m^3n^6$
 B the expression $9mn^3$
 C the expression $3mn^2$
 D the length of the side of a cube whose volume is $3mn^2$

Spiral Review

Simplify. (Lesson 10-7)

49. $(6^3)^5$

50. $(n^7)^2$

51. $(2a^3b^2)^4$

52. $(-4p^{11}q)^3$

Simplify. Express using positive exponents. (Lesson 10-6)

53. $\dfrac{9^5}{9^3}$

54. $\dfrac{k^{15}}{k^6}$

55. $\dfrac{24y^4}{4y^2}$

56. $\dfrac{45g^3}{3g^7}$

57. **RETAIL** Find the discount to the nearest cent for a flat-screen television that costs $999 and is on sale at 15% off. (Lesson 5-8)

Cross-Curricular Project

Math and Economics

Getting Down to Business It's time to complete your project. Use the information and data you have gathered about the cost of materials and the feedback from your peers to prepare a video or brochure. Be sure to include a scatter plot with your project.

Math Online **Cross-Curricular Project at** ca.gr7math.com

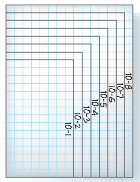
CHAPTER 10 Study Guide and Review

Download Vocabulary Review from ca.gr7math.com

FOLDABLES Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Functions (Lessons 10-1, 10-2, and 10-3)

- Linear functions have constant rates of change.
- Nonlinear functions do not have constant rates of change.
- Quadratic functions are functions in which the greatest power of the variable is 2.
- Cubic functions are functions in which the greatest power of the variable is 3.

Monomials (Lessons 10-5 through 10-8)

- To multiply powers with the same base, add their exponents.
- To divide powers with the same base, subtract their exponents.
- To find the power of a power, multiply the exponents.
- To find the power of a product, find the power of each factor and multiply.

Key Vocabulary

cube root (p. 554)
monomial (p. 539)
nonlinear function (p. 522)
quadratic function (p. 528)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. The expression $y = x^2 - 3x$ is an example of a monomial.
2. A nonlinear function has a constant rate of change.
3. A quadratic function is a function whose greatest power is 2.
4. The product of $3x$ and $x^2 + 3x$ will have 3 terms.
5. A quadratic function is a nonlinear function.
6. The graph of a linear function is a curve.
7. To divide powers with the same base, subtract the exponents.
8. The Quotient of Powers states when dividing powers with the same base, subtract their exponents.
9. The graph of a cubic function is a straight line.

CHAPTER 10

Study Guide and Review

Lesson-by-Lesson Review

10-1 Linear and Nonlinear Functions (pp. 522–527)

Determine whether each equation or table represents a *linear* or *nonlinear* function. Explain.

10. $y - 4x = 1$

11. $y = x^2 + 3$

12.

Time (h)	2	3	4	5
Number of Pages	98	147	199	248

Example 1 **Determine whether the table represents a *linear* or *nonlinear* function.**

x	y
−2	−3
−1	−1
0	1
1	3

As x increases by 1, y increases by 2. The rate of change is constant, so this function is linear.

10-2 Graphing Quadratic Functions (pp. 528–531)

Graph each function.

13. $-4x^2$

14. $y = x^2 + 4$

15. $y = -2x^2 + 1$

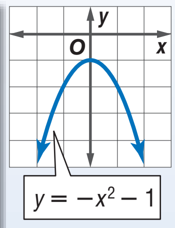
16. $y = 3x^2 - 1$

17. **SCIENCE** A ball is dropped from the top of a 36-foot tall building. The quadratic equation $d = -16t^2 + 36$ models the distance d in feet the ball is from the ground at time t seconds. Graph the function. Then use your graph to find how long it takes for the ball to reach the ground.

Example 2 **Graph $y = -x^2 - 1$.**

Make a table of values. Then plot and connect the ordered pairs with a smooth curve.

x	$y = -x^2 - 1$	(x, y)
−2	$-(-2)^2 - 1$	(−2, −5)
−1	$-(-1)^2 - 1$	(−1, −2)
0	$-(0)^2 - 1$	(0, −1)
1	$-(1)^2 - 1$	(1, −2)
2	$-(2)^2 - 1$	(2, −5)



10-3 PSI: Make a Model (pp. 532-533)

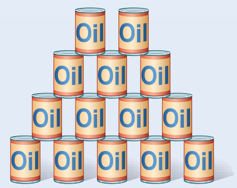
Solve the problem by using the make a model strategy.

18. **MEASUREMENT** Sydney has a postcard that measures 5 inches by 3 inches. She decides to frame it, using a frame that is $1\frac{3}{4}$ inches wide. What is the perimeter of the framed postcard?

19. **MAGAZINES** A book store arranges it best-seller magazines in the front window. In how many different ways can five best-seller magazines be arranged in a row?

Example 3

DISPLAYS **Cans of oil are displayed in the shape of a pyramid. The top layer has 2 cans in it. One more can is added to each layer, and there are 4 layers in the pyramid. How many cans are there in the display?**



So, based on the model there are 14 cans.

Mixed Problem Solving
For mixed problem-solving practice, see page 717.

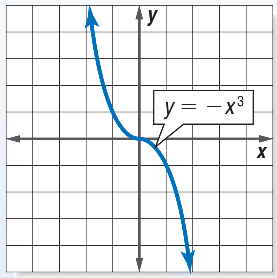
10-4 Graphing Cubic Functions (pp. 534-537)

Graph each function.

20. $y = 2x^3 - 4$
21. $y = 0.25x^3 - 2$
22. $y = 2x^3 + 4$
23. $y = 0.25x^3 + 2$
24. **MEASUREMENT** A rectangular prism with a square base of side length x inches has a height of $(x - 1)$ inches. Write the function for the volume V of the prism. Graph the function. Then estimate the dimensions of the box that would give a volume of approximately 18 cubic inches.

Example 4 Graph $y = -x^3$.

x	$y = -x^3$	(x, y)
-2	$-(-2)^3$	$(-2, 8)$
-1	$-(-1)^3$	$(-1, 1)$
0	$-(0)^3$	$(0, 0)$
1	$-(1)^3$	$(1, -1)$
2	$-(2)^3$	$(2, -8)$



10-5 Multiplying Monomials (pp. 539-542)

Simplify. Express using exponents.

25. $4 \cdot 4^5$
26. $x^6 \cdot x^2$
27. $-9y^2(-4y^9)$
28. $\left(\frac{3}{7}\right)^{-4} \cdot \left(\frac{3}{7}\right)^2$
29. **LIFE SCIENCE** The number of bacteria after t cycles of reproduction is 2^t. Suppose a bacteria reproduces every 30 minutes. If there are 1,000 bacteria in a dish now, how many will there be in 1 hour?

Example 5 Find $4 \cdot 4^3$. Express using exponents.

$4 \cdot 4^3 = 4^1 \cdot 4^3$ $\quad$ $4 = 4^1$

$= 4^{1+3}$ $\quad$ The common base is 4.

$= 4^4$ $\quad$ Add the exponents.

Example 6 Find $3a^3 \cdot 4a^7$.

$3a^3 \cdot 4a^7 = (3 \cdot 4)a^{3+7}$ $\quad$ Commutative and Associative Properties

$= 12a^{10}$

10-6 Dividing Monomials (pp. 544-548)

Simplify. Express using exponents.

30. $\frac{5^9}{5^2}$
31. $\frac{n^5}{n}$
32. $\frac{21c^{11}}{-7c^8}$
33. $\dfrac{\left(\frac{4}{7}\right)^3 \times \left(\frac{4}{7}\right)^{-1}}{\frac{4}{7}}$
34. **MEASUREMENT** The area of the family room is 3^4 square feet. The area of the kitchen is 4^3 square feet. What is the difference in area between the two rooms?

Example 7

Simplify $\frac{6^8}{6^3}$. Express using exponents.

$\frac{6^8}{6^3} = 6^{8-3}$ $\quad$ The common base is 6.

$= 6^5$ $\quad$ Simplify.

Example 8

Simplify $\frac{s^{-8}}{s^{-4}}$. Express using exponents.

$= \frac{s^{-8}}{s^{-4}} = s^{-8-(-4)}$ $\quad$ Quotient of Powers

$= s^{-8+4}$ or s^{-4} $\quad$ Simplify.

Chapter 10 Study Guide and Review

10-7 Powers of Monomials (pp. 549-552)

Simplify.

35. $(9^2)^3$
36. $(d^6f^3)^4$
37. $(5y^5)^4$
38. $(6z^4x^3)^5$
39. $(\frac{3}{4}n^{-1})^2$
40. $[(p^2)^3]^2$
41. $(5^{-1})^2$
42. $(-3k^2)^2(4k^{-3})^2$
43. **GEOMETRY** Find the volume of a cube with sides of length $5s^2t^4$ as a monomial.
44. **GEOMETRY** Find the area of a square with sides of length $6a^3b^5$ as a monomial.

Example 9

Simplify $(7^3)^5$.

$(7^3)^5 = 7^{3 \cdot 5}$ Power of a Power

$= 7^{15}$ Simplify.

Example 10

Simplify $(2x^2y^3)^3$.

$(2x^2y^3)^3 = 2^3 \cdot x^{2 \cdot 3}\, y^{3 \cdot 3}$ Power of a Product

$= 8x^6y^9$ Simplify.

10-8 Roots of Monomials (pp. 553-556)

Simplify.

45. $\sqrt{a^2}$
46. $\sqrt{49n^4}$
47. $\sqrt{36x^2y^6}$
48. $\sqrt{81q^{14}}$
49. $\sqrt[3]{p^6}$
50. $\sqrt[3]{8m^{18}}$
51. $\sqrt[3]{64c^6d^{21}}$
52. $\sqrt[3]{125r^9s^{15}}$
53. **GEOMETRY** Express the length of one side of the square whose area is $64b^{16}$ square units as a monomial.
54. **GEOMETRY** Express the length of one side of a cube whose volume is $216a^9c^3$ cubic units as a monomial.

Example 11

Simplify $\sqrt{16f^8g^6}$.

$\sqrt{16f^8g^6} = \sqrt{16} \cdot \sqrt{f^8} \cdot \sqrt{g^6}$ Product Property of Square Roots

$= 4 \cdot f^4 \cdot |g^3|$ or $4f^4|g^3|$ Use absolute value to indicate the positive value of g^3.

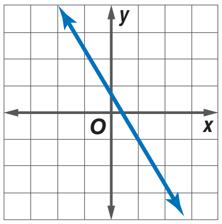
Example 12

Simplify $\sqrt[3]{x^9}$.

$\sqrt[3]{x^9} = x^3$ $(x)^9 = x^3$

CHAPTER 10 Practice Test

Determine whether each graph, equation, or table represents a *linear* or *nonlinear* function. Explain.

1. 

2. $2x = y$

3.

x	−3	−1	1	3
y	2	10	18	26

Graph each function.

4. $y = \frac{1}{2}x^2$

5. $y = -2x^2 + 3$

6. **BUSINESS** The function $p = 60 + 2d^2$ models the profit made by a manufacturer of digital audio players. Graph this function. Then use your graph to estimate the profit earned after making 20 players.

7. **STANDARDS PRACTICE** Simplify the algebraic expression $(3x^3y^2)(7x^3y)$.

   A $21x^9y^2$
   B $21x^6y^2$
   C $21x^6y^3$
   D $21x^6y^6$

Graph each function.

8. $y = x^3 + 4$

9. $y = x^3 - 4$

10. $y = \frac{1}{3}x^3$

11. **MEASUREMENT** A neighborhood group would like Jacob to fertilize their lawns. The average area of each lawn is 6^4 square feet. If there are 6^2 lawns in this neighborhood, how many total square feet of lawn does Jacob need to fertilize?

12. **CRAFTS** Martina is making cube-shaped gift boxes from decorative cardboard. Each side of the cube is to be 6 inches long, and there is a $\frac{1}{2}$-inch overlap on each side. How much cardboard does Martina need to make each box?

Simplify. Express using exponents.

13. $15^3 \cdot 15^5$

14. $-5m^6(-9m^8)$

15. $\frac{3^{15}}{3^7}$

16. $\frac{-40w^8}{8w}$

Simplify.

17. $\sqrt{m^2}$

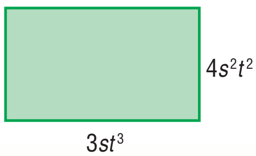
18. $\sqrt{144a^2b^6}$

19. $\sqrt[3]{64x^3y^{15}}$

20. **STANDARDS PRACTICE** Which expression is equivalent to $\frac{(12x^4)(4x^3)}{8x^5}$?

   F $12x^7$
   G $12x^2$
   H $6x^4$
   J $6x^2$

21. **MEASUREMENT** Find the area of the rectangle at the right.



Simplify.

22. $[(x^2)^4]^3$

23. $(-2b^3)^2(4b^2)^2$

24. $(3^{-3})^2$

25. **GEOMETRY** Express the length of one side of a square with an area of $121x^4y^{10}$ square units as a monomial.

CHAPTER 10

California Standards Practice

Cumulative, Chapters 1–10

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1 A car used 4.2 gallons of gasoline to travel 126 miles. How many gallons of gasoline would it need to travel 195 miles?

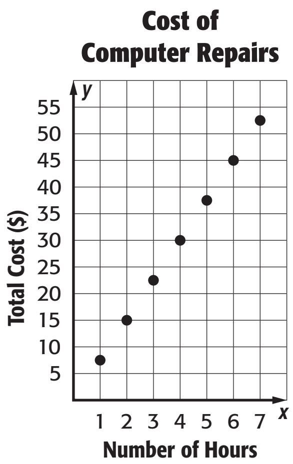
A 2.7

B 5.0

C 6.5

D 7.6

2 The scatter plot below shows the cost of computer repairs in relation to the number of hours the repair takes. Based on the information in the scatter plot, which statement is a valid conclusion?



F As the length of time increases, the cost of the repair increases.

G As the length of time increases, the cost of the repair stays the same.

H As the length of time decreases, the cost of the repair increases.

J As the length of time increases, the cost of the repair decreases.

3 The equation $c = 0.8t$ represents c, the cost of t tickets on a ferry. Which table contains values that satisfy this equation?

A

Cost of Ferry Tickets				
t	1	2	3	4
c	$0.80	$1.00	$1.20	$1.40

B

Cost of Ferry Tickets				
t	1	2	3	4
c	$0.80	$1.60	$2.40	$3.20

C

Cost of Ferry Tickets				
t	1	2	3	4
c	$0.75	$1.50	$2.25	$3.00

D

Cost of Ferry Tickets				
t	1	2	3	4
c	$1.80	$2.60	$3.40	$4.20

4 Shanelle purchased a new computer for $1,099 and a computer desk for $699 including tax. She plans to pay the total amount in 24 equal monthly payments. What is a reasonable amount for each monthly payment?

F $50

G $75

H $150

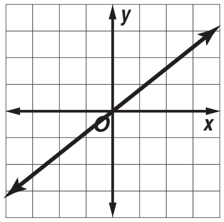
J $1,800

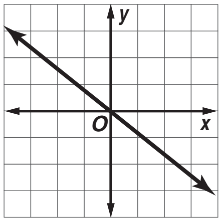
TEST-TAKING TIP

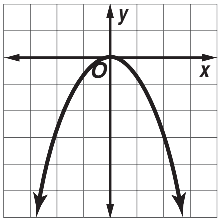
Question 4 You can often use estimation to eliminate incorrect answers. In this question, Shanelle's total spent can be estimated by adding $1,100 and $700, then dividing by 24. The sum of $1,100 and $700 is $1,800 before dividing by 24, so choice J can be eliminated.

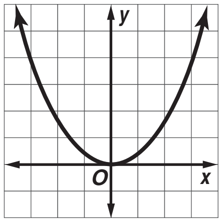
More California Standards Practice
For practice by standard, see pages CA1–CA39.

5 Which of the following is the graph of $y = \frac{2}{3}x^2$?

A


C


B


D


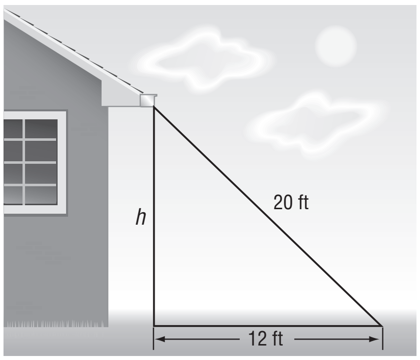
6 Simplify the expression shown below.

$$(3m^3n^2)(6m^4n)$$

F $18m^{12}n^2$ **H** $18\,m^7n^3$

G $18\,m^7n^2$ **J** $18\,m^7n^6$

7 What is the height h of the gutter in the figure below?



A 10 ft **C** 16 ft

B 14 ft **D** 18 ft

8 The area of a rectangle is $30m^{11}$ square feet. If the length of the rectangle is $6m^4$ feet, what is the width of the rectangle?

F $5m^7$ ft **H** $36m^{15}$ ft

G $24m^7$ ft **J** $180m^{15}$ ft

9 Which expression is equivalent to $5^4 \times 5^6$?

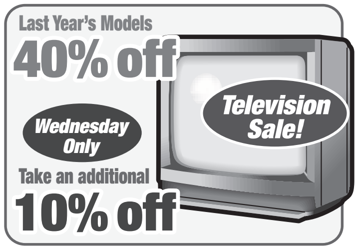
A 5^{10} **C** 25^{10}

B 5^{24} **D** 25^{24}

Pre-AP

Record your answers on a sheet of paper. Show your work.

10 An electronics store is having a sale on certain models of televisions. Mr. Castillo would like to buy a television that is on sale. This television normally costs \$679.



a. What price, not including tax, will Mr. Castillo pay if he buys the television on Saturday?

b. What price, not including tax, will Mr. Castillo pay if he buys the television on Wednesday?

c. How much money will Mr. Castillo save if he buys the television on Saturday?

NEED EXTRA HELP?										
If You Missed Question...	1	2	3	4	5	6	7	8	9	10
Go to Lesson...	4-3	9-8	9-1	5-5	10-2	10-4	10-4	10-5	10-5	5-8
For Help with Standard...	AF4.2	PS1.2	MR2.5	MR3.1	AF3.1	AF2.2	MG3.3	NS2.3	NS2.3	NS1.7