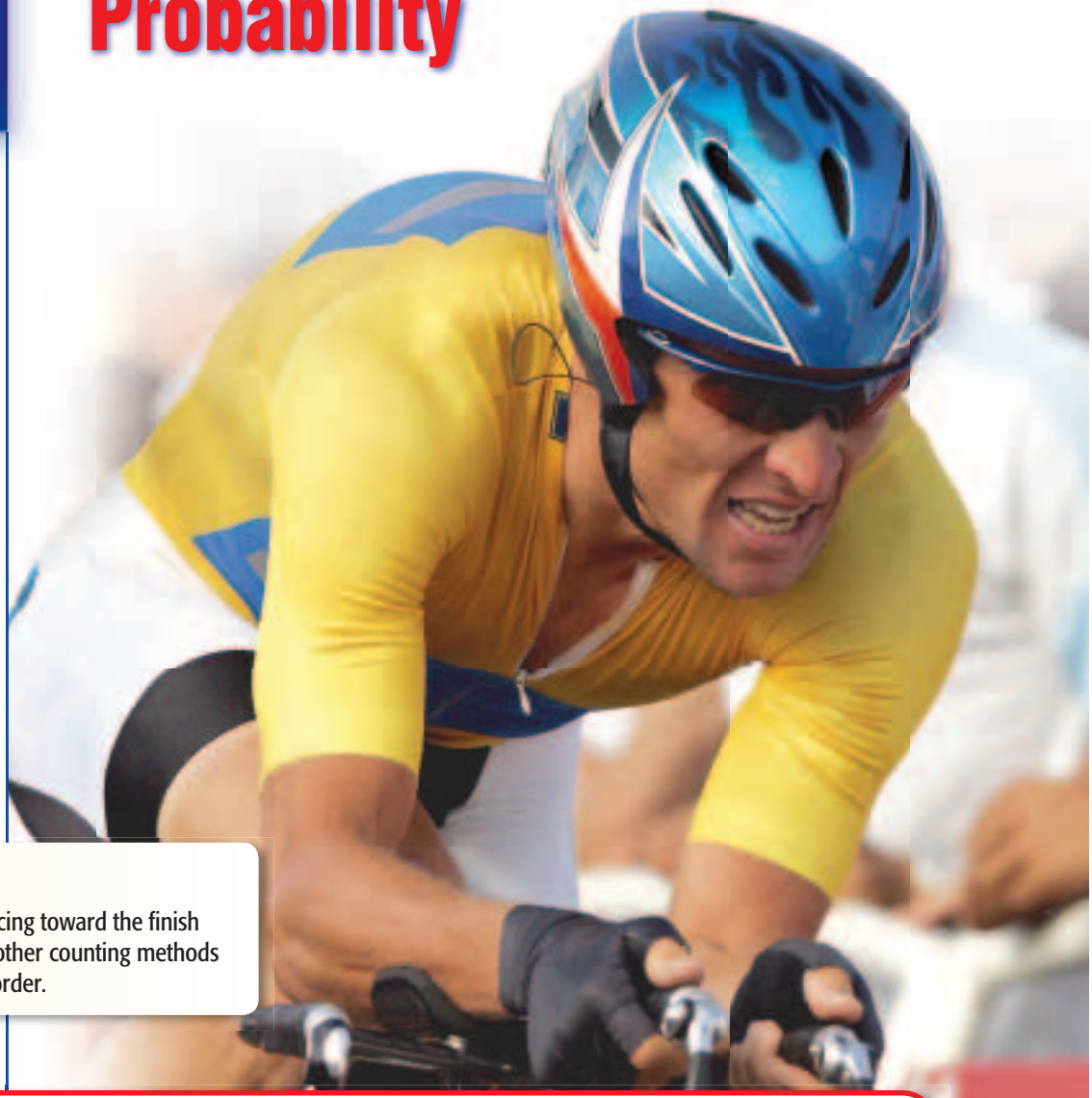
CHAPTER 12

Looking Ahead to Grade 8: Probability



BIG Idea

- **Standard 6SDAP3.0** Determine theoretical and experimental probabilities and use these to make predictions about events. (CAHSEE)

Key Vocabulary

dependent events (p. 632)
independent events (p. 631)
outcome (p. 626)

Real-World Link

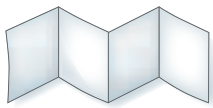
Bicycling If several bicyclists are racing toward the finish line, you can use a tree diagram or other counting methods to determine the possible finishing order.

FOLDABLES Study Organizer

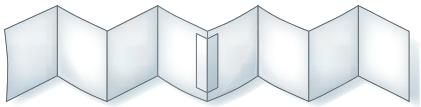
Probability Make this Foldable to help you organize your notes. Begin with a plain sheet of 11″ × 17″ paper.

1. **Fold** the sheet in half lengthwise. Cut along the fold.

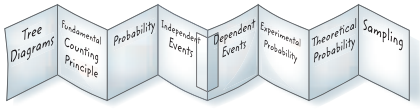
2. **Fold** each half in quarters along the width.



3. **Unfold** each piece and tape to form one long piece.



4. **Label** each page with a key topic as shown. Refold to form a booklet.



GET READY for Chapter 12

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

MathOnline Take the Online Readiness Quiz at ca.gr7math.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

Write each fraction in simplest form. (Prior Grade)

1. $\frac{48}{72}$
2. $\frac{35}{60}$
3. $\frac{21}{99}$

4. **TRAVEL** On a family trip to San José, California, Dustin drove 4 hours out of 18 hours. Write this portion of time spent driving as a fraction in simplest form. (Prior Grade)

Multiply. Write in simplest form. (Lesson 2-3)

5. $\frac{3}{4} \cdot \frac{8}{9}$
6. $\frac{5}{6} \cdot \frac{1}{2}$
7. $\frac{2}{4} \cdot \frac{2}{7}$
8. $\frac{7}{8} \cdot \frac{4}{6}$

Solve each problem. (Lesson 5-7)

9. Find 35% of 90.
10. Find 42% of 340.
10. What is 60% of 220?
11. What is 5% of 72?
13. **SURVEY** Anna surveyed 144 students in her school. She found that 82% of the students said pizza is their favorite lunch. How many students surveyed said their favorite lunch is pizza? (Lesson 5-7)

QUICK Review

Example 1

Write $\frac{45}{51}$ in simplest form.

$\frac{45}{51} = \frac{15}{17}$ (÷ 3 above and below)

Divide the numerator and denominator by their GCF, 3.

Example 2

Find $\frac{3}{7} \cdot \frac{1}{6}$. Write in simplest form.

$\frac{3}{7} \cdot \frac{1}{6} = \frac{\overset{1}{\cancel{3}}}{7} \cdot \frac{1}{\underset{2}{\cancel{6}}}$ Divide 3 and 6 by their GCF, 3.

$= \frac{1 \cdot 1}{7 \cdot 2}$ or $\frac{1}{14}$

Example 3

Find 20% of 170.

$\frac{a}{b} = \frac{p}{100}$ Use the percent proportion.

$\frac{a}{170} = \frac{20}{100}$ Replace b with 170 and p with 20.

$a \cdot 100 = 170 \cdot 20$ Find the cross products.

$100a = 3{,}400$ Multiply.

$\frac{100a}{100} = \frac{3{,}400}{100}$ Divide each side by 100.

$a = 34$

34 is 20% of 170.

12-1 Counting Outcomes

Main IDEA

Count outcomes by using a tree diagram or the Fundamental Counting Principle.

Reinforcement of Standard 6SDAP3.1 Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome. (CAHSEE)

NEW Vocabulary

outcome
event
sample space
tree diagram
Fundamental Counting Principle
probability
random

GET READY for the Lesson

BICYCLES Antonio wants to buy a Dynamo bicycle.

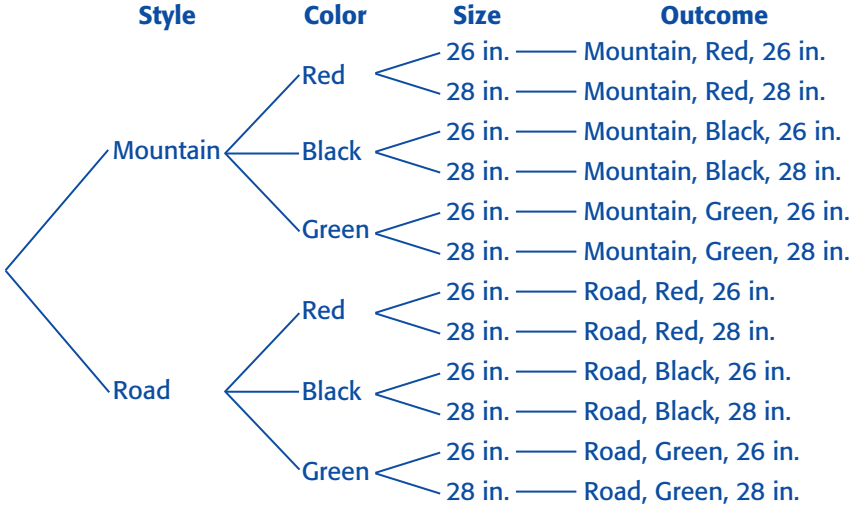
1. How many different styles are available? colors? sizes?
2. Make a list showing all of the different bicycles that are available.

An **outcome** is any one of the possible results of an action. For selecting a specific type, color, and size of bicycle, there are 12 total outcomes. An **event** is an outcome or a collection of all the outcomes.

An organized list of outcomes, called a **sample space**, can help you determine the total number of possible outcomes for an event. One type of organized list is a **tree diagram**.

EXAMPLE 1 Use a Tree Diagram

BICYCLES Draw a tree diagram to determine the number of different bicycles described in the real-world example above.



There are 12 different Dynamo bicycles.

CHECK Your Progress

a. A dime and a penny are tossed. Draw a tree diagram to determine the number of outcomes.

You can also find the total number of outcomes by multiplying. This principle is known as the **Fundamental Counting Principle**.

KEY CONCEPT Fundamental Counting Principle

If event M has m possible outcomes and event N has n possible outcomes, then event M followed by event N has $m \cdot n$ possible outcomes.

Real-World EXAMPLE

2 **COMMUNICATIONS** **In the United States, radio and television stations use call letters that start with K or W. How many different station call letters are possible when four letters are used?**

There are 2 choices for the first letter and 26 for each of the others. Use the Fundamental Counting Principle.

$$2 \times 26 \times 26 \times 26 = 35{,}152$$

There are 35,152 possible call letters.

Real-World Link

In 1940, plans were made for a new radio studio in San Francisco to be suspended on springs. The springs were meant to absorb the sound and vibrations from the outside so that they could not get into the studio.

Source: *San Francisco Chronicle*

CHECK Your Progress

b. **DINING** A restaurant offers a choice of 3 types of pasta with 5 types of sauce. Each pasta entrée comes with or without a meatball. How many different entrées are available?

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The **probability** of an event is the ratio of the number of outcomes in that event to the total number of outcomes. Outcomes occur at **random** if each outcome is equally likely to occur.

Real-World EXAMPLE Find Probability

3 **GAMES** **In a lottery game, you pick a 3-digit number. One of these numbers is the winning number. What is the probability of winning?**

First, find the number of possible outcomes.

$$10 \times 10 \times 10 = 1{,}000$$

There are 1,000 possible outcomes. There is 1 winning number.

$$P(\text{win}) = \frac{1}{1{,}000}$$ There is 1 winning number out of 1,000.

This can also be written as a decimal, 0.001, or a percent, 0.1%.

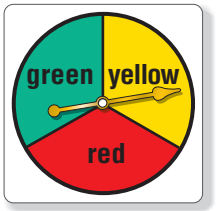
CHECK Your Progress

c. Two number cubes are rolled. What is the probability that the sum of the numbers on the cubes is 12?

CHECK Your Understanding

Example 1 (p. 626)

1. The spinner is spun two times. Draw a tree diagram to determine the number of possible outcomes.



Example 2 (p. 627)

2. **FOOD** A pizza shop has regular, deep-dish, and thin crusts; 2 different cheeses; and 4 toppings. How many different one-cheese and one-topping pizzas can be ordered?

Example 3 (p. 627)

3. **GOVERNMENT** The first three digits of a social security number are a geographic code. The next two digits are determined by the year and the state where the number is issued. The final four digits are random numbers. What is the probability of the last four digits being the current year?

Exercises

HOMEWORK HELP

For Exercises	See Examples
4–7	1
8–13	2
14–15	3

Draw a tree diagram to determine the number of possible outcomes.

4. A penny, a nickel, and a dime are tossed.
5. A number cube is rolled and a penny is tossed.
6. A white or red ball cap comes in small, medium, large, or extra large.
7. The Sweet Treats Shoppe offers single-scoop ice cream in chocolate, vanilla, or strawberry, and two types of cones, regular or sugar.

Use the Fundamental Counting Principle to find the number of possible outcomes.

8. The day of the week is picked at random and a number cube is rolled.
9. A number cube is rolled 3 times.
10. There are 5 true-false questions on a history quiz.
11. There are 4 choices for each of 5 multiple-choice questions on a science quiz.
12. **SCHOOL** Doli can take 4 different classes first period, 3 different classes second period, and 5 different classes third period. How many different schedules can she have?
13. **VEHICLES** A state's license plates are issued with 2 letters, followed by 2 numbers and a letter. How many different license plates could the state issue?
14. **CLOTHES** Felisa has a red and a white sweatshirt. Courtney has a black, a green, a red, and a white sweatshirt. Each girl picks a sweatshirt at random to wear to the picnic. What is the probability the girls will wear the same color sweatshirt?

15. **GAMES** The winning number in a lottery game is made up of five digits from 0 to 9 chosen at random. If the digits can repeat, what is the probability of winning the lottery?

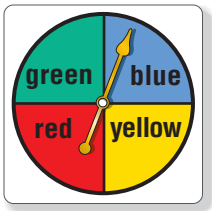
ELECTRONICS For Exercises 16 and 17, use the table that shows various options for a digital music player.

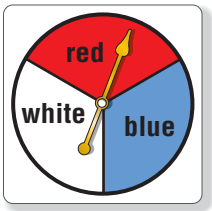
Storage Capacity	Colors	
256 megabytes	blue	purple
512 megabytes	red	pink
1 gigabyte	green	silver
2.5 gigabytes	white	black

16. How many different players are available, based on storage capacity and color?

17. If an FM radio tuner is also available as an option, how many players are available?

For Exercises 18 and 19, each spinner at the right is spun once. Use a tree diagram to answer each question.





18. What is the probability that at least one spinner lands lands on blue?

19. What is the probability that at least one spinner lands on yellow?

LUNCHES For Exercises 20–24, use the following information.
Parent volunteers made lunches for an 8th-grade field trip. Each lunch had a peanut butter and jelly or a deli-meat sandwich; a bag of potato chips or pretzels; an apple, an orange, or a banana; and juice, water, or soda. One of each possible lunch combinations was made.

20. How many different lunch combinations were made?

21. How many of these combinations contained an apple?

22. If the lunches are handed out randomly, what is the probability that a student receives a lunch containing a banana?

23. What is the probability of a student receiving a lunch with potato chips and soda?

24. Suppose 4 types of meat were used for the deli-meat sandwiches. What is the probability that a student receives one specific type of sandwich?

EXTRA PRACTICE
See pages 706, 719.
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H.O.T. Problems

25. **OPEN ENDED** Give an example of a situation that has 15 possible outcomes.

26. **NUMBER SENSE** Whitney has a choice of a floral, plaid, or striped blouse to wear with a choice of a tan, black, navy, or white skirt. Without calculating the number of possible outcomes, how many more outfits can she make if she buys a print blouse?

27. **CHALLENGE** If x coins are tossed, write an algebraic expression for the number of possible outcomes.

28. **WRITING IN MATH** Describe a possible advantage for using a tree diagram rather than the Fundamental Counting Principle.

STANDARDS PRACTICE

29. A school cafeteria offers sandwiches with three types of meat and two types of bread. Which table shows all possible sandwich combinations available?

A

Bread	Meat
White	Ham
Wheat	Turkey
White	Ham
Wheat	Turkey

B

Bread	Meat
White	Ham
White	Turkey
White	Beef
Wheat	Ham
Wheat	Turkey
Wheat	Beef
Rye	Ham
Rye	Turkey
Rye	Beef

C

Bread	Meat
White	Ham
White	Turkey
White	Beef
Wheat	Ham
Wheat	Turkey
Wheat	Beef

D

Bread	Meat
White	Ham
White	Turkey
White	Beef
White	Bologna
Wheat	Ham
Wheat	Turkey
Wheat	Beef
Wheat	Bologna

Spiral Review

Choose an appropriate type of display for each situation. (Lesson 11-8)

30. the amount of each flavor of ice cream sold relative to the total sales

31. the number of people attending a fair for specific intervals of ages

32. **STATISTICS** Display the data set {$12, $15, $18, $21, $14, $37, $27, $9} in a stem-and-leaf plot. (Lesson 11-7)

33. **GRADES** Mr. Francis has told his students that he will remove the lowest exam score for each student at the end of the grading period. Seki received grades of 43, 78, 84, 85, 88, and 90 on her exams. What will be the difference between the mean of her original grades and the mean of her five grades after Mr. Francis removes one grade? (Lesson 11-4)

34. What is 35% of 130? (Lesson 5-3)

GET READY for the Next Lesson

PREREQUISITE SKILL **Multiply. Write in simplest form.** (Lesson 2-3)

35. $\frac{4}{5} \cdot \frac{3}{8}$

36. $\frac{3}{10} \cdot \frac{5}{6}$

37. $\frac{7}{12} \cdot \frac{3}{14}$

38. $\frac{2}{3} \cdot \frac{9}{10}$

12-2 Probability of Compound Events

Main IDEA

Find the probability of independent and dependent events.

Reinforcement of Standard 6SDAP3.1 Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and **express the theoretical probability of each outcome.** (CAHSEE)

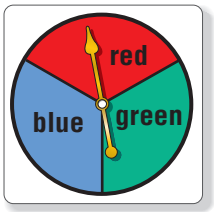
NEW Vocabulary

compound event
independent events
dependent events

Vocabulary Link Independent
Everyday Use not under the control of others
Math Use not relying on another quantity or action

GET READY for the Lesson

GAMES A game uses a number cube and the spinner shown.



1. A player rolls the number cube. What is P(odd number)?
2. The player spins the spinner. What is P(red)?
3. What is the product of the probabilities in Exercises 1 and 2?
4. Draw a tree diagram to determine the probability that the player will roll an odd number and spin red.

The combined action of rolling a number cube and spinning a spinner is a compound event. In general, a **compound event** consists of two or more simple events.

The outcome of the spinner does not depend on the outcome of the number cube. These events are independent. For **independent events**, the outcome of one event does not affect the other event.

KEY CONCEPT Probability of Independent Events

Words	The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event.
Symbols	$P(A \text{ and } B) = P(A) \cdot P(B)$

EXAMPLE 1 Probability of Independent Events

The two spinners are spun. What is the probability that both spinners will show an even number?

$P(\text{first spinner is even}) = \frac{3}{7}$

$P(\text{second spinner is even}) = \frac{1}{2}$

$P(\text{both spinners are even}) = \frac{3}{7} \cdot \frac{1}{2}$ or $\frac{3}{14}$

CHECK Your Progress

Use the above spinners to find each probability.

a. P(both show a 2)

b. P(both are less than 4)

Test-Taking Tip

Mental Math You may wish to simplify individual probabilities before multiplying them.

STANDARDS EXAMPLE

2 A spinner and a number cube are used in a game. The spinner has an equal chance of landing on one of five colors: red, yellow, blue, green, and purple. The faces of the cube are labeled 1 through 6. What is the probability of a player spinning blue and then rolling a 3 or 4?

A $\frac{3}{11}$ **B** $\frac{1}{4}$ **C** $\frac{1}{15}$ **D** $\frac{1}{30}$

Read the Item

You are asked to find the probability of the spinner landing on blue and rolling a 3 or 4 on a number cube. The events are independent because spinning the spinner does not affect the outcome of rolling a number cube.

Solve the Item

First, find the probability of each event.

$P(\text{blue}) = \frac{1}{5}$ $\quad \frac{\text{number of ways to spin blue}}{\text{number of possible outcomes}}$

$P(3 \text{ or } 4) = \frac{2}{6} \text{ or } \frac{1}{3}$ $\quad \frac{\text{number of ways to roll 3 or 4}}{\text{number of possible outcomes}}$

Then, find the probability of both events occurring.

$P(\text{blue and 3 or 4}) = \frac{1}{5} \cdot \frac{1}{3}$ $\quad P(A \text{ and } B) = P(A) \cdot P(B)$

$= \frac{1}{15}$ $\quad$ Multiply.

The probability is $\frac{1}{15}$, which is answer C.

CHECK Your Progress

c. A game requires players to roll two fair number cubes to move the game pieces. The faces of the cubes are labeled 1 through 6. What is the probability of rolling a 2 or 4 on the first number cube and then rolling a 5 on the second?

F $\frac{1}{3}$ **G** $\frac{1}{2}$ **H** $\frac{1}{12}$ **J** $\frac{1}{18}$

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If the outcome of one event affects the outcome of another event, the events are called **dependent events**.

Vocabulary Link
Dependent
Everyday Use under the control of others
Math Use relying on another quantity or action

KEY CONCEPT Probability of Dependent Events

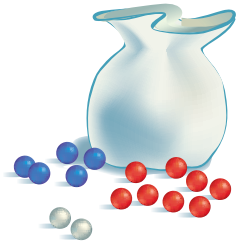
Words If two events, A and B, are dependent, then the probability of both events occurring is the product of the probability of A and the probability of B after A occurs.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

EXAMPLE 3 Probability of Dependent Events

Concepts in Motion
BrainPOP® ca.gr7math.com

There are 2 white, 8 red, and 5 blue marbles in a bag. Once a marble is selected, it is not replaced. Find the probability that two red marbles are chosen.



Since the first marble is not replaced, the first event affects the second event. These are dependent events.

$P(\text{first marble is red}) = \frac{8}{15}$ ← number of red marbles / ← total number of marbles

$P(\text{second marble is red}) = \frac{7}{14}$ ← number of red marbles after one red marble is removed / ← total number of marbles after one red marble is removed

$P(\text{two red marbles}) = \frac{\cancel{8}^{4}}{15} \cdot \frac{\cancel{7}^{1}}{\cancel{14}_{1}} \text{ or } \frac{4}{15}$

CHECK Your Progress

Refer to the situation above. Find each probability.

d. P(two blue marbles)

e. P(a white marble and then a blue marble)

f. P(a red marble and then a white marble)

g. P(two white marbles)

CHECK Your Understanding

Example 1 (p. 631)

A penny is tossed and a number cube is rolled. Find each probability.

1. P(tails and 3)
2. P(heads and odd)

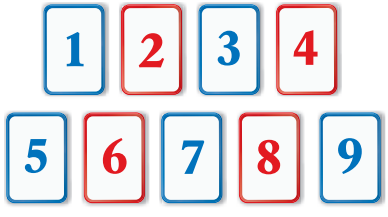
Example 2 (p. 632)

3. **STANDARDS PRACTICE** A spinner and a number cube are used in a game. The spinner has an equal chance of landing on 1 of 3 colors: red, yellow, and blue. The faces of the cube are labeled 1 through 6. What is the probability of a player spinning red and then rolling an even number?

A $\frac{2}{5}$ **B** $\frac{1}{3}$ **C** $\frac{1}{6}$ **D** $\frac{1}{12}$

Example 3 (p. 633)

A card is drawn from the cards shown and not replaced. Then, a second card is drawn. Find each probability.



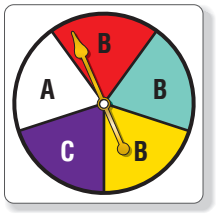
4. P(two even numbers)
5. P(a number less than 4 and then a number greater than 4)

Exercises

HOMEWORK HELP

For Exercises	See Examples
6–11	1
12, 13	2
14–19	3

A number cube is rolled, and the spinner at the right is spun. Find each probability.



6. P(1 and A)
7. P(3 and B)
8. P(even and C)
9. P(odd and B)
10. P(greater than 2 and A)
11. P(less than 3 and B)

12. **LAUNDRY** A laundry basket contains 18 blue socks and 24 black socks. What is the probability of randomly picking 2 black socks from the basket?

13. **GAMES** Beth is playing a board game that requires rolling two number cubes to move a game piece. She needs to roll a sum of 6 on her next turn and then a sum of 10 to land on the next two bonus spaces. What is the probability that Beth will roll a sum of 6 and then a sum of 10 on her next two turns?

A jar contains 3 yellow, 5 red, 4 blue, and 8 green candies. After a candy is selected, it is not replaced. Find each probability.

14. P(two red candies)
15. P(two blue candies)
16. P(a yellow candy and then a blue candy)
17. P(a green candy and then a red candy)
18. P(two candies that are not green)
19. P(two candies that are neither blue nor green)

20. **MARKETING** A discount supermarket has found that 60% of their customers spend more than \$75 each visit. What is the probability that the next two customers will each spend more than \$75?

SCHOOL For Exercises 21 and 22, use the information below and in the table.

At Clearview Middle School, 56% of the students are girls and 44% are boys.

Clearview Middle School	
Favorite Subject	
Art	16%
Language Arts	13%
Math	28%
Music	7%
Science	21%
Social Studies	15%

21. If two students are chosen at random, what is the probability that the first student is a girl and that the second student's favorite subject is science?
22. What is the probability that of two randomly selected students, one is a boy and the other is a student whose favorite subject is *not* art or math?

23. **MOVIES** You and a friend plan to see 2 movies over the weekend. You can choose from 6 comedy, 2 drama, 4 romance, 1 science fiction, or 3 action movies. You write the movie titles on pieces of paper and place them in a bag, and you each randomly select a movie. What is the probability that neither of you selects a comedy? Is this a dependent or independent event? Explain.

EXTRA PRACTICE

See pages 707, 719.

Math Online

Self-Check Quiz at ca.gr7math.com

24. **MONEY** Donoma had 8 dimes and 6 pennies in her pocket. If she took out 1 coin and then a second coin without replacing the first, what is the probability that both coins were dimes? Is this a dependent or independent event? Explain.

POPULATION For Exercises 25 and 26, use the information in the table.

Assume that age is *not* dependent on the region.

Lewburg County Population	
Demographic Group	**Fraction of the Population**
Under age 18	$\frac{3}{10}$
18 to 64 years old	$\frac{3}{5}$
65 years or older	$\frac{1}{10}$
Rural Area	$\frac{4}{5}$
Urban Area	$\frac{1}{5}$

25. A resident of Lewburg County is picked at random. What is the probability that the person is under 18 years old or 18 to 64 years old and from an urban area?

26. What is the probability that the person is less than 18 years old or 65 years or older and from a rural area?

Real-World Link

The game of dominoes is believed to have originated in 12th century China.

Source: infoplease.com

27. **CONTESTS** A car dealer is giving away a new car to one of 10 contestants. Each contestant randomly selects a key from 10 keys, with only 1 winning key. What is the probability that none of the first three contestants selects the winning key?

28. **DOMINOES** A standard set of dominoes contains 28 tiles, with each tile having two sides of dots from 0 to 6. Of these tiles, 7 have the same number of dots on each side. If four players each randomly choose a tile, what is the probability that each chooses a tile with the same number of dots on each side?

29. **WEATHER** A weather forecaster states that there is an 80% chance of rain on Monday and a 30% chance of rain on Tuesday. What is the probability of it raining on Monday and Tuesday? Assume these are independent events.

30. **FIND THE DATA** Refer to the California Data File on pages 16–19. Choose some data and write a real-world problem in which you would find a compound probability.

H.O.T. Problems

31. **OPEN ENDED** There are 9 marbles representing 3 different colors. Write a problem where 2 marbles are selected at random without replacement and the probability is $\frac{1}{6}$.

32. **FIND THE ERROR** The spinner at the right is spun twice. Evita and Tia are finding the probability that both spins will result in an odd number. Who is correct? Explain.

Evita: $\frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}$

Tia: $\frac{3}{5} \cdot \frac{2}{4} = \frac{6}{10}$

33. **CHALLENGE** Determine whether the following statement is *true* or *false*. If the statement is false, provide a counterexample.
If two events are independent, then the probability of both events is less than 1.

34. **WRITING IN MATH** Compare and contrast independent events and dependent events.

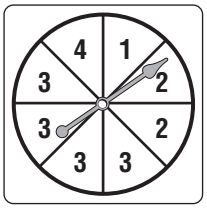
STANDARDS PRACTICE

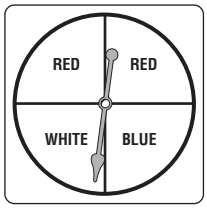
35. Mr. Fernandez is holding four straws of different lengths. He has asked four students to each randomly pick a straw to see who goes first in a game. John picks first, gets the second longest straw, and keeps it. What is the probability that Jeremy will get the longest straw if he picks second?

A $\frac{1}{4}$ C $\frac{1}{3}$

B $\frac{1}{2}$ D $\frac{1}{5}$

36. The spinners below are each spun once.





What is the probability of spinning 2 and white?

F $\frac{1}{16}$ H $\frac{2}{5}$

G $\frac{1}{4}$ J $\frac{3}{5}$

Spiral Review

37. **SPORTS** The Silvercreek Ski Resort has 4 ski lifts up the mountain and 11 trails down the mountain. How many different ways can a skier take a ski lift up the mountain and then ski down? (Lesson 12-1)

38. **RADIO LISTENING** Choose an appropriate display for the data at the right. Then make a display. Justify your reasoning. (Lesson 11-8)

Adult Audience of Oldies Radio					
Age	18 to 24	25 to 34	35 to 44	45 to 54	55 or older
Percent of Audience	10%	14%	29%	33%	14%

Source: Interep Research Division

MEASUREMENT **Find the volume of each solid described. Round to the nearest tenth if necessary.** (Lessons 7-5 and 7-6)

39. rectangular pyramid: length, 14 m; width, 12 m; height 7 m

40. cone: diameter, 22 cm; height, 24 cm

GET READY for the Next Lesson

PREREQUISITE SKILL **Write each fraction in simplest form.**

41. $\frac{52}{120}$ 42. $\frac{33}{90}$ 43. $\frac{49}{70}$ 44. $\frac{24}{88}$

12-3 Experimental and Theoretical Probability

In Motion

Main IDEA

Find experimental and theoretical probabilities and use them to make predictions.

Reinforcement of Standard 6SDAP3.3 Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if P is the probability of an event, $1 - P$ is the probability of an event *not* occurring. (CAHSEE)

NEW Vocabulary

experimental probability
theoretical probability

MINI Lab Concepts in Motion Interactive Lab ca.gr7math.com

In Motion for MAC

Draw one marble from a bag containing 10 different-colored marbles. Record its color, and replace it in the bag. Repeat 50 times.

1. Find the ratio $\frac{\text{number of times color was drawn}}{\text{total number of draws}}$ for each color.
2. Is it possible to have a certain color marble in the bag and never draw that color?
3. Open the bag and count the marbles. Find the ratio $\frac{\text{number of each color marble}}{\text{total number of marbles}}$ for each color of marble.
4. Are the ratios in Exercises 1 and 3 the same? Explain.

In the Mini Lab above, you determined a probability by conducting an experiment. Probabilities that are based on the outcomes obtained by conducting an experiment are called **experimental probabilities**.

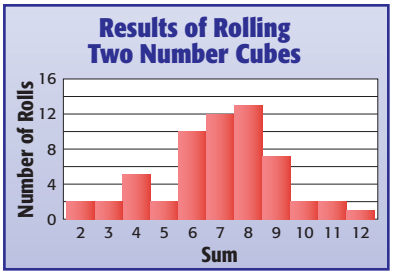
Probabilities based on known characteristics or facts are called **theoretical probabilities**. For example, you can compute the theoretical probability of picking a certain color marble from a bag. Theoretical probability tells you what *should* happen in an experiment.

EXAMPLES Theoretical and Experimental Probability

1 **What is the theoretical probability of rolling a double 6 using two number cubes?**

The theoretical probability is $\frac{1}{6} \cdot \frac{1}{6}$ or $\frac{1}{36}$.

2 **The graph shows the results of an experiment in which two number cubes were rolled. According to the experimental probability, is a sum of 12 likely to occur?**



Only 1 of the 58 sums is 12. So, the experimental probability of rolling a sum of 12 is $\frac{1}{58}$. It is not likely that a sum of 12 will occur.

STUDY TIP

Experimental Probability Experimental probabilities usually vary depending on the number of trials performed or when the experiment is repeated.

CHECK Your Progress

a. Refer to the graph above. According to the experimental probability, which sum is most likely to occur?

Real-World Career . . .
How Does a Marketing Manager Use Math?
A marketing manager uses information from surveys and experimental probability to help make decisions about changes in products and advertising.

MathOnline
For more information, go to ca.gr7math.com.

Real-World EXAMPLE

3 **MARKETING** **Two hundred teenagers were asked whether they purchased certain items in the past year. What is the experimental probability that a teenager bought a photo frame in the last year?**

Item	Number Who Purchased the Item
candle	110
photo frame	95

There were 200 teenagers surveyed and 95 purchased a photo frame in the last year. The experimental probability is $\frac{95}{200}$ or $\frac{19}{40}$.

CHECK Your Progress

b. What is the experimental probability that a teenager bought a candle in the last year?

Online **Personal Tutor at** ca.gr7math.com Personal Tutor

You can use past performance to predict future events.

EXAMPLES Use Probability to Predict

4 **FARMING** **Over the last 10 years, the probability that soybean seeds planted by Ms. Diaz produced soybeans is $\frac{10}{13}$.**
Is this probability experimental or theoretical? Explain.

This is an experimental probability since it is based on what happened in the past.

If Ms. Diaz wants to have 10,000 soybean-bearing plants, how many seeds should she plant?

This problem can be solved using a proportion.

10 out of 13 seeds should produce soybeans. → $\frac{10}{13} = \frac{10{,}000}{x}$ ← 10,000 out of x seeds should produce soybeans.

Solve the proportion.

$\frac{10}{13} = \frac{10{,}000}{x}$ Write the proportion.

$10 \cdot x = 13 \cdot 10{,}000$ Find the cross products.

$10x = 130{,}000$ Multiply.

$\frac{10x}{10} = \frac{130{,}000}{10}$ Divide each side by 10.

$x = 13{,}000$ Ms. Diaz should plant 13,000 seeds.

STUDY TIP
Mental Math
For every 10 soybean-bearing plants, Ms. Diaz must plant 3 extra seeds. Think:
10,000 ÷ 10 = 1,000
Ms. Diaz must plant 3 × 1,000 or 3,000 extra seeds. She must plant a total of 10,000 + 3,000 or 13,000 seeds.

CHECK Your Progress

c. **SURVEYS** In a recent survey of 150 people, 18 responded that they were left-handed. If an additional 2,500 people are surveyed, how many would be expected to be left-handed?

CHECK Your Understanding

Example 1 (p. 637)

For Exercises 1–3, use the table that shows the results of tossing three coins, one at a time, 50 times.

Result	Frequency	Result	Frequency
HHH	6	TTT	3
HHT	5	TTH	6
HTH	10	THT	5
HTT	5	THH	10

1. What is the theoretical probability of tossing exactly two heads?

Example 2 (p. 637)

2. Find the experimental probability of tossing exactly two heads.
3. How likely is it that a toss will have two heads? Explain.

For Exercises 4 and 5, use the table at the right showing the results of a survey of cars that passed the school.

Cars Passing the School	
Color	Number of Cars
white	35
red	23
green	12
other	20

Example 3 (p. 638)

4. What is the probability that the next car will be white?

Example 4 (p. 638)

5. Out of the next 180 cars, how many would you expect to be white?

Exercises

HOMEWORK HELP

For Exercises	See Examples
6, 9	1, 2
8, 11	3
7, 10	4

SCHOOL **For Exercises 6 and 7, use the following information.**

In keyboarding class, 4 out of the 60 words Cleveland typed contained an error.

6. What is the probability that his next word will have an error?
7. In a 1,000-word essay, how many errors would you expect Cleveland to make?
8. **BASKETBALL** In practice, Crystal made 80 out of 100 free throws. What is the experimental probability that she will make a free throw?

FOOD **For Exercises 9 and 10, use the results of a survey of 150 people shown at the right.**

Favorite Fruit	
Fruit	Number
apples	55
bananas	40
oranges	35
grapes	15
other	5

9. What is the probability that a person's favorite fruit was bananas?
10. Out of 450 people, how many would you expect to state that bananas are their favorite fruit?
11. **SCHOOL** In the last 40 school days, Esteban's bus has been late 8 times. What is the experimental probability that the bus will be late tomorrow?
12. **SPORTS** In a survey of 90 students at Genoa Middle School, 42 liked to watch basketball and 24 liked to watch soccer. If there are 300 students in the middle school, how many would you expect to like to watch soccer?

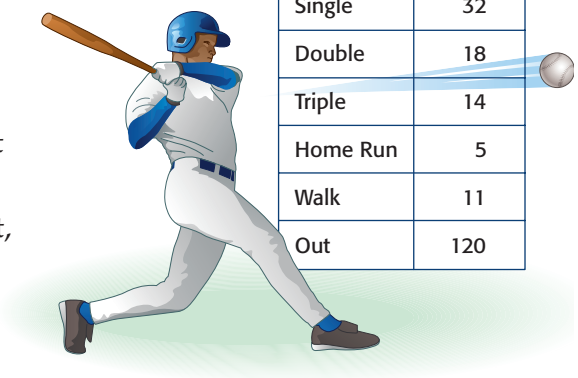
For Exercises 13–15, use the table that shows the results of spinning an equally divided 8-section spinner.

Number on Spinner	Frequency
1	8
2	5
3	9
4	4
5	10
6	6
7	5
8	3

13. Compare the theoretical and experimental probabilities of the spinner landing on 5.
14. Based on the experimental probability, how many times would you expect the spinner to land on 3 if the spinner is spun 200 times?
15. Jarred predicts that the spinner will land on 4 or 8 on the next spin. Is this a reasonable prediction? Explain.

BASEBALL For Exercises 16 and 17, use the table which shows the batting results of a baseball player for a season.

Result	Frequency
Single	32
Double	18
Triple	14
Home Run	5
Walk	11
Out	120



16. Based on the results, how likely is it that the player would be out after his next turn batting?
17. The next time the player is at bat, how likely is it for him to hit a single or a double?

FOOD For Exercises 18 and 19, use the following information.
The manager of a school cafeteria asked selected students to pick their favorite menu item. The results of the survey are shown in the table.

Menu Item	Students
Hot Dog	22
Hamburger	19
Pizza	30
Taco	16
Chicken Strips	13

18. If the cafeteria serves 350 lunches, and students can choose only one lunch, how many hamburgers could the manager expect to sell?
19. Is the next student more likely to buy a hot dog or a hamburger, or is the student more likely to buy pizza? Explain.

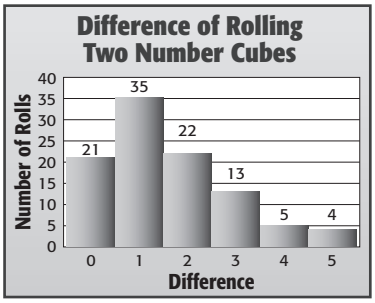
EXTRA PRACTICE
See pages 707, 719.
MathOnline
Self-Check Quiz at ca.gr7math.com

H.O.T. Problems

20. **OPEN ENDED** Two hundred fifty people are surveyed about their favorite color. Make a table of possible results if the experimental probability that the favorite color is blue is 40%.
21. **CHALLENGE** An inspector found that 15 out of 250 cars had a loose front door and that 10 out of 500 cars had headlight problems. What is the probability that a car has both a loose door and a headlight problem?
22. **WRITING IN MATH** Explain why you would *not* expect the theoretical probability of an event and the experimental probability of the same event to always be the same.

STANDARDS PRACTICE

23. Two number cubes are rolled and the difference is recorded. The graph shows the results of several rolls.

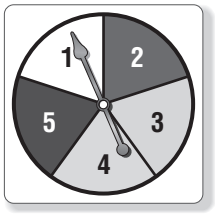


Based on past results, what is the probability that the difference is 2?

A $\frac{7}{20}$ **C** $\frac{11}{50}$

B $\frac{1}{20}$ **D** $\frac{1}{25}$

24. Shannon spun the spinner shown and recorded her results.



Number on Spinner	Frequency
1	20
2	10
3	2
4	40
5	8

What is the experimental probability of landing on the number five?

F 10% **H** 30%

G 20% **J** 40%

Spiral Review

A jar contains 3 red marbles, 4 green marbles, and 5 blue marbles. Once a marble is selected, it is not replaced. Find each probability. (Lesson 12-2)

25. 2 green marbles

26. a blue marble and then a red marble

27. **SCHOOL** At the school cafeteria, students can choose from 4 entreés and 3 beverages. How many different lunches of one entreé and one beverage can be purchased at the cafeteria? (Lesson 12-1)

28. **STATISTICS** Find the range, median, upper and lower quartiles, interquartile range, and any outliers of the set of data. (Lesson 11-5)

115, 117, 111, 121, 110, 127, 116, 126, 105,
115, 100, 103, 122, 130, 101, 100, 108, 130

ALGEBRA **Write an inequality for each sentence.** (Lesson 8-6)

29. **HEALTH** Your heart beats over 100,000 times a day.

30. **BIRDS** A peregrine falcon can spot a pigeon up to 8 kilometers away.

GET READY for the Next Lesson

31. **PREREQUISITE SKILL** Lawanda was assigned some math exercises for homework. She answered half of them in study period. After school, she answered 7 more exercises. If she still has 11 exercises to complete, how many exercises were assigned? Use the *work backward* strategy. (Lesson 1-8)

Extend 12-3

Probability Lab
Fair Games

Main IDEA

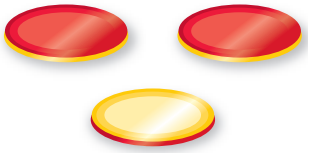
Use experimental and theoretical probabilities to decide whether a game is fair.

Reinforcement of Standard 6SDAP3.2 Use data to estimate the probability of future events (e.g., batting averages or number of accidents per mile driven).
Standard 7MR2.6 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

Mathematically speaking, a two-player game is fair if each player has an equally-likely chance of winning. In this lab, you will analyze two simple games and determine whether each game is fair.

ACTIVITY

1 **In a counter-toss game, players toss three two-color counters. The winner of each game is determined by how many counters land with either the red or yellow side facing up. Play this game with a partner.**



STEP 1 Player 1 tosses the counters. If 2 or 3 chips land red-side up, Player 1 wins. If 2 or 3 chips land yellow-side up, Player 2 wins. Record the results in a table like the one shown below. Place a check in the winner's column for each game.

Game	Player 1	Player 2
1		
2		

STEP 2 Player 2 then tosses the counters and the results are recorded.

STEP 3 Continue alternating the tosses until each player has tossed the counters 10 times.

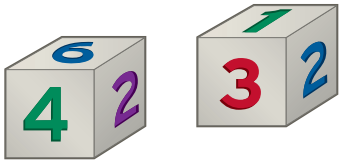
Analyze the Results

1. Make an organized list of all the possible outcomes resulting from one toss of the 3 counters. Explain your method.
2. Calculate the theoretical probability of each player winning. Write each probability as a fraction and as a percent.
3. **MAKE A CONJECTURE** Based on the theoretical probabilities of each player winning, is this a fair game? Explain your reasoning.
4. Calculate the experimental probability of each player winning. Write each probability as a fraction and as a percent.
5. Compare the probabilities in Exercises 2 and 4.
6. **GRAPH THE DATA** Make a graph of the experimental probabilities of Player 1 winning for 5, 10, 15, and 20 games. Graph the ordered pairs (games played, Player 1 wins) using a blue pencil, pen, or marker. Describe how the points appear on your graph.

7. Add to the graph you created in Exercise 6 the theoretical probabilities of Player 1 winning for 5, 10, 15, and 20 games. Graph the ordered pairs (games played, Player 1 wins) using a red pencil, pen, or marker. Connect these red points and describe how they appear on your graph.
8. As the number of games played increases, how does the experimental probability compare to the theoretical probability?
9. **MAKE A PREDICTION** Predict the number of times Player 1 would win if the game were played 100 times.

ACTIVITY

2 **In a number-cube game, players roll two number cubes. Play this game with a partner.**



STEP 1 Player 1 rolls the number cubes. Player 1 wins if the total of the numbers rolled is 5 or if a 5 is shown on one number cube. Otherwise, Player 2 wins. Record the results in a table like the one shown below.

Game	Player 1	Player 2
1		
2		

STEP 2 Player 2 then rolls the number cubes and the results are recorded.

STEP 3 Continue alternating the rolls until each player has rolled the number cubes 10 times.

Analyze the Results

10. Make an organized list of all the possible outcomes resulting from one roll. Explain your method.
11. Calculate the theoretical probability of each player winning and the experimental probability of each player winning. Write each probability as a fraction and as a percent. Then compare these probabilities.
12. **MAKE A CONJECTURE** Based on the theoretical and experimental probabilities of each player winning, is this a fair game? Explain your reasoning.
13. **WRITING IN MATH** If the game is fair, explain how you could change the game so that it is not fair. If the game is not fair, explain how you could change the game to make it fair. Explain.

12-4 Problem-Solving Investigation

MAIN IDEA: Solve problems by acting it out.

Standard 7MR2.5 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, **to explain mathematical reasoning. Reinforcement of Standard 6SDAP3.2 Use data to estimate the probability of future events** (e.g., batting averages or number of accidents per mile driven).

P.S.I. TEAM +

e-Mail: ACT IT OUT

YOUR MISSION: Act it out to solve the problem.

THE PROBLEM: Is tossing a coin a good way to answer a true-false quiz?

> Bonita: I wonder if tossing a coin would be a good way to answer a 5-question true-false quiz.

EXPLORE	You know there are five true-false questions on the quiz. You can carry out an experiment to test if tossing a coin would be a good way to answer the questions and get a good grade.
PLAN	Toss a coin 5 times. If the coin shows tails, the answer is T. If the coin shows heads, the answer is F. Do three trials.
SOLVE	Suppose the correct answers are T, F, F, T, F. Let's circle them in each trial.
CHECK	Check by doing several more trials.

Answers	T	F	F	T	F	Number Correct
Trial 1	(T)	T	(F)	F	T	2
Trial 2	F	(F)	T	(T)	(F)	3
Trial 3	(T)	(F)	T	F	T	2

Since the experiment produced 2–3 correct answers on a 5-question quiz, it shows that tossing a coin to answer a true-false quiz is not the way to get a good grade.

Analyze The Strategy

1. Explain an advantage of using the *act it out* strategy to solve a problem.
2. WRITING IN MATH Write a problem that could be solved by acting it out. Then use the strategy to solve the problem. Explain your reasoning.

Brent Turner

Mixed Problem Solving

For Exercises 3–5, solve using the *act it out* strategy.

3. **COINS** Nina wants to buy a granola bar from a vending machine. The granola bar costs $0.45. If Nina uses exact change, in how many different ways can she use nickels, dimes, and quarters?

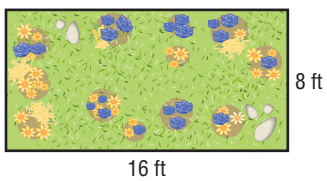
4. **FITNESS** The length of a basketball court is 84 feet long. Hector runs 20 feet forward and then 8 feet back. How many more times will he have to do this until he reaches the end of the basketball court?

5. **PHOTOGRAPHS** Omar is taking a picture of the French Club's five officers. The club secretary will always stand on the left and the treasurer will always stand on the right. How many different ways can he arrange the officers in a single row for the picture?

Use any strategy to solve Exercises 6–10. Some strategies are shown below.

PROBLEM-SOLVING STRATEGIES
- Use the four-step plan.
- Work backward.
- Look for a pattern.
- Logical reasoning.
- Act it out.

6. **MEASUREMENT** Mrs. Lopez is designing her garden in the shape of a rectangle. The perimeter of her garden is $2\frac{1}{4}$ times greater than the perimeter of the rectangle shown. Find the perimeter of Mrs. Lopez's garden.

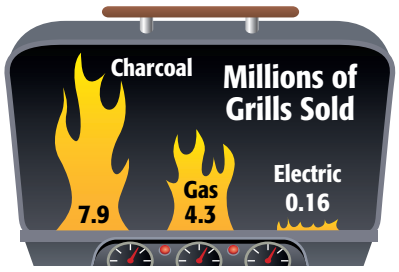


7. **ALGEBRA** Complete the pattern.
100, 98, 94, ■, 80, ■.

8. **MONEY** Carmen received money for a birthday gift. She loaned $5 to her sister Emily and spent half of the remaining money. The next day she received $10 from her uncle. After spending $9 at the movies, she still had $11 left. How much money did she receive for her birthday?

9. **UNIFORMS** Nick has to wear a uniform to school. He can wear either navy blue, black, or khaki pants with a green, white, or yellow shirt. How many uniform combinations can Nick wear?

10. **STATISTICS** The graphic shows the number of types of outdoor grills sold. How does the number of charcoal grills compare to the number of gas grills?



Source: Barbecue Industry Association

Select the Operation

For Exercises 11–13, select the appropriate operation(s) to solve the problem. Justify your selection(s) and solve the problem.

11. **SHOPPING** Rita is shopping for fishing equipment. She has $135 and has already selected items that total $98.50. If the sales tax is 8%, will she have enough money to purchase a fishing net that costs $23?

12. **TIME** At 2:00 P.M., Cody began writing the final draft of a report. At 3:30 P.M., he had written 5 pages. If he works at the same pace, when should he complete 8 pages?

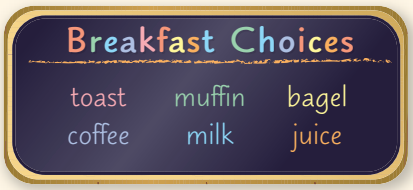
13. **MEASUREMENT** The length of a rectangle is 8 inches longer than its width. What are the length and width of the rectangle if the area is 84 square inches?

CHAPTER 12

Mid-Chapter Quiz

Lessons 12-1 through 12-4

1. **BREAKFAST** Draw a tree diagram to determine the number of one-bread and one-beverage outcomes using the breakfast choices listed below. (Lesson 12-1)



2. **FASHION** Reina has three necklaces, three pairs of earrings, and two bracelets. How many combinations of the three types of jewelry are possible? (Lesson 12-1)

3. **STANDARDS PRACTICE** Roman has ten cards numbered 1 to 10. What is the probability of picking two even-numbered cards one after the other, if the first card picked is replaced? (Lesson 12-2)

A $\frac{1}{5}$ C $\frac{1}{4}$

B $\frac{2}{9}$ D $\frac{3}{8}$

A box contains 3 purple, 2 yellow, 4 pink, 3 orange, and 2 blue markers. Once a marker is selected, it is *not* replaced. Find each probability. (Lesson 12-2)

4. P(two purple markers)
5. P(two orange markers)
6. P(a pink marker then an orange marker)
7. P(two markers that are *not* blue)
8. P(two markers that are neither yellow nor pink)
9. P(two markers that are neither purple nor pink)

10. **STANDARDS PRACTICE** A bag contains 4 red, 20 blue, and 6 green marbles. Seth picks one at random and keeps it. Then Amy picks a marble. What is the probability that they each select a red marble? (Lesson 12-2)

F $\frac{1}{150}$ H $\frac{2}{145}$

G $\frac{1}{15}$ J $\frac{1}{870}$

11. **FOOD** Two hundred twenty-five high school freshman were asked to name their favorite hot lunch. One hundred thirty-five students named tacos as their favorite. If an additional 80 freshman are asked, how many would be expected to choose tacos? (Lesson 12-3)

MUSIC A survey asked 500 teenagers what formats of music they had purchased in the past two months. Use the table at the right to answer Exercises 12 and 13. (Lesson 12-3)

Format	Number Purchased
CD	380
Download	415

12. What is the experimental probability that a teenager purchased a CD in the past two months?
13. What is the experimental probability that a teenager purchased a music download in the past two months?
14. A coin is tossed three times, and it landed heads up all three times. What is the theoretical probability that the next toss will land tails up? (Lesson 12-3)

15. **BOOKS** Jackie has two math books and two English books that she wants to place on a shelf. Use the *act it out* strategy to determine how many different ways she can organize the books. (Lesson 12-4)

12-5 Using Sampling to Predict

Main IDEA

Predict the actions of a larger group by using a sample.

Reinforcement of Standard 6SDAP2.5 Identify claims based on statistical data and, in simple cases, evaluate the validity of the claims. (CAHSEE)

NEW Vocabulary

sample
population
unbiased sample
simple random sample
stratified random sample
systematic random sample
biased sample
convenience sample
voluntary response sample

READING in the Content Area

For strategies in reading this lesson, visit ca.gr7math.com.

GET READY for the Lesson

ENTERTAINMENT The manager of a television station wants to conduct a survey to determine what type of sports people like to watch.

What Type of Sports Do You Like to Watch?
Baseball
Basketball
Football
Lacrosse
Soccer

1. Suppose she decides to survey a group of people at a basketball game. Do you think the results would represent all of the people in the viewing area? Explain.
2. Suppose she decides to survey students at your middle school. Do you think the results would represent all of the people in the viewing area? Explain.
3. Suppose she decides to call every 100th household in the telephone book. Do you think the results would represent all of the people in the viewing area? Explain.

The manager of the radio station cannot survey everyone in the listening area. A smaller group called a **sample** must be chosen. A sample is used to represent a larger group called a **population**.

To get valid results, a sample must be chosen very carefully. An **unbiased sample** is selected so that it accurately represents the entire population. Three ways to pick an unbiased sample are listed below.

CONCEPT Summary — Unbiased Samples

Type	Description	Example
Simple Random Sample	Each item or person in the population is as likely to be chosen as any other.	Each student's name is written on a piece of paper. The names are placed in a bowl, and names are picked without looking.
Stratified Random Sample	The population is divided into similar, non-overlapping groups. A simple random sample is then selected from each group.	Students are picked at random from each grade level at a school.
Systematic Random Sample	The items or people are selected according to a specific time or item interval.	Every 20th person is chosen from an alphabetical list of all students attending a school.

In a **biased sample**, one or more parts of the population are favored over others. Two ways to pick a biased sample are listed below.

Vocabulary Link
Bias
Everyday Use a tendency or prejudice.
Math Use error introduced by selecting or encouraging a specific outcome.

CONCEPT Summary — Biased Samples

Type	Description	Example
Convenience Sample	A convenience sample consists of members of a population that are easily accessed.	To represent all the students attending a school, the principal surveys the students in one math class.
Voluntary Response Sample	A voluntary response sample involves only those who want to participate in the sampling.	Students at a school who wish to express their opinions complete an online survey.

EXAMPLES Determine Validity of Conclusions

Determine whether each conclusion is valid. Justify your answer.

1. **To determine what videos their customers like, every tenth person to walk into the video store is surveyed. Out of 150 customers, 70 stated that they prefer comedies. The manager concludes that about half of all customers prefer comedies.**

The conclusion is valid. Since the population is the customers of the video store, the sample is a systematic random sample. It is an unbiased sample.

2. **To determine what people like to do in their leisure time, the customers of a video store are surveyed. Of these, 85% said that they like to watch movies, so the store manager concludes that most people like to watch movies in their leisure time.**

The conclusion is not valid. The customers of a video store probably like to watch videos in their leisure time. This is a biased sample. The sample is a convenience sample since all of the people surveyed are in one specific location.

CHECK Your Progress

Determine whether each conclusion is valid. Justify your answer.

a. A radio station asks its listeners to call one of two numbers to indicate their preference for two candidates for mayor in an upcoming election. Seventy-two percent of the listeners who responded preferred candidate A, so the radio station announced that candidate A would win the election.

b. To award prizes at a sold-out hockey game, four seat numbers are picked from a barrel containing individual papers representing each seat number. Tyler concludes that he has as good a chance as everyone else to win a prize.

A valid sampling method uses unbiased samples. If a sampling method is valid, you can use the results to make predictions.

Real-World EXAMPLE Using Sampling to Predict

STUDY TIP

Misleading Probabilities
Probabilities based on biased samples can be misleading. If the students surveyed were all boys, the probabilities generated by the survey would not be valid, since both girls and boys purchase sweatshirts at the store.

3 **SCHOOL** **The school bookstore sells sweatshirts in 4 different colors; red, black, white, and gold. The students who run the store survey 50 students at random. The colors they prefer are indicated at the right. If 450 sweatshirts are to be ordered to sell in the store, how many should be white?**

Color	Number
red	25
black	10
white	13
gold	2

First, determine whether the sample method is valid. The sample is a simple random sample since students were randomly selected. Thus, the sample method is valid.

$\frac{13}{50}$ or 26% of the students prefer white sweatshirts. So, find 26% of 450.

$0.26 \times 450 = 117$ About 117 sweatshirts should be white.

CHECK Your Progress

c. **RECREATION** A swimming instructor at a community pool asked her students if they would be interested in an advanced swimming course, and 60% stated that they would. If there are 870 pool members, how many people can the instructor expect to take the course?

Personal Tutor

Personal Tutor at ca.gr7math.com

CHECK Your Understanding

Examples 1, 2 (p. 648)

Determine whether each conclusion is valid. Justify your answer.

1. To determine how much money the average family in the United States spends to cool their home, a survey of 100 households from Alaska are picked at random. Of the households, 85 said that they spend less than $75 a month on cooling. The researcher concluded that the average household in the United States spends less than $75 on cooling per month.

2. To determine the benefits that employees consider most important, one person from each department of the company is chosen at random. Medical insurance was listed as the most important benefit by 67% of the employees. The company managers conclude that medical insurance should be provided to all employees.

Example 3 (p. 649)

3. **ELECTIONS** Three students are running for class president. Jonathan randomly surveyed some of his classmates and recorded the results at the right. If there are 180 students in the class, how many do you think will vote for Della?

Candidate	Number
Luke	7
Della	12
Ryan	6

Exercises

HOMEWORK HELP

For Exercises	See Examples
4–9	1, 2
10, 11	3

Determine whether each conclusion is valid. Justify your answer.

4. To evaluate the quality of their product, a manufacturer of cell phones pulls every 50th phone off the assembly line to check for defects. Out of 200 phones tested, 4 are defective. The manager concludes that about 2% of the cell phones produced will be defective.

5. To determine whether the students will attend a spring music concert at the school, Rico surveys his friends in the chorale. All of his friends plan to attend, so Rico assumes that all the students at his school will also attend.

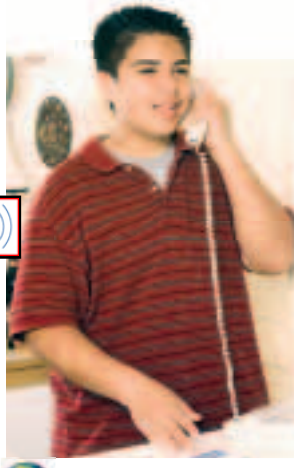
6. To determine the most popular television stars, a magazine asks its readers to complete a questionnaire and send it back to the magazine. The majority of those who replied liked one actor the most, so the magazine decides to write more articles about that actor.

7. To determine what people in California think about a proposed law, 2 people from each county in the state are surveyed at random. Of those surveyed, 42% said that they do not support the proposal. The legislature concludes that the law should not be passed.

Do You Support Proposed Law?	
Yes	30%
No	42%
Not sure	28%

8. Two students need to be chosen to represent the 28 students in a science class. The teacher decides to use a computer program to randomly pick 2 numbers from 1 to 28. The students whose names are next to those numbers in his grade book will represent the class.

9. To determine if the oranges in 20 crates are fresh, the produce manager at a grocery store takes 5 oranges from the top of the first crate off the delivery truck. None of the oranges are bad, so the manager concludes that all of the oranges are fresh.



Real-World Link . . . . .
63% of teens prefer to use a telephone to talk to their friends.
Source: *Pew Internet & American Life Project*

10. **COMMUNICATION** The Student Council advisor asked every tenth student in the lunch line how they preferred to be contacted with school news. The results are shown in the table. If there are 680 students at the school, how many can be expected to prefer e-mail?

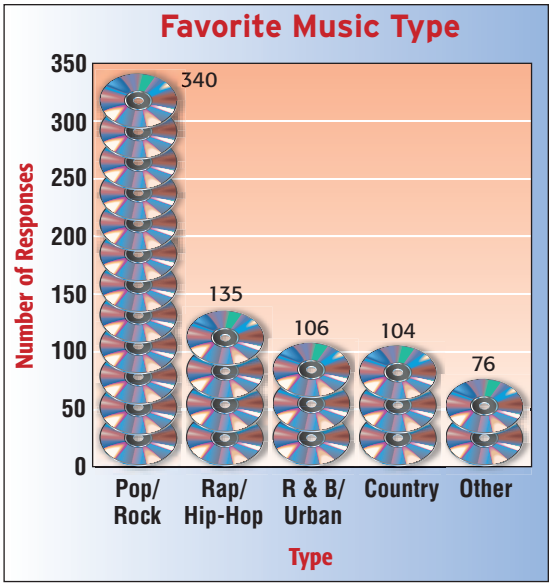
Method	Number
Announcement	5
Newsletter	12
E-mail	16
Telephone	3

11. **SALES** A random survey of shoppers at a grocery store shows that 19 prefer whole milk, 44 prefer low-fat milk, and 27 prefer skim milk. If 800 containers of milk are ordered, how many should be skim milk?

12. **MARKETING** A grocery store is considering adding a world foods area. They survey 500 random customers, and 350 customers agree the world foods area is a good idea. Should the store add this area? Explain your reasoning.

13. **ACTIVITIES** Brett wants to conduct a survey about who stays for after-school activities. Describe a valid sampling method he could use.

MUSIC **For Exercises 14 and 15, use the following information.**
The manager of a music store sent out 1,000 survey forms to households near her store. The results of the survey are shown in the graph at the right.



14. Based on this survey, if the manager orders 2,500 CDs, how many pop/rock CDs should be ordered?

15. Based on the survey results, the manager concludes that 25% of customers will buy either rap/hip-hop or R&B/urban CDs. Is this a valid conclusion? Explain.

HOMEWORK **A survey is to be conducted to find out how many hours students at a school spend on homework each weekday. Describe the sample and explain why each sampling method might not be valid.**

16. A questionnaire is handed out to all students taking a world language.

17. The students from one homeroom from each grade level are asked to keep a log for one week.

18. Students in a randomly selected Language Arts class are asked to discuss their study habits in an essay.

19. Randomly selected parents are sent a questionnaire and asked to return it.

COLLECT THE DATA **For Exercises 20–23, conduct a survey of the students in your math class to determine whether they prefer hamburgers or pizza.**

20. What percent prefer hamburgers?

21. Use your survey to predict how many students in your school prefer hamburgers.

22. Is your survey a good way to determine the preferences of the students in your school? Explain.

23. How could you improve your survey?

EXTRA PRACTICE
See pages 707, 719.
Math Online
Self-Check Quiz at ca.gr7math.com

24. **FIND THE DATA** Refer to the California Data File on pages 16–19. Choose some data and write a real-world problem in which you would make a prediction based on samples.

H.O.T. Problems

25. **CHALLENGE** How could the wording of a question or the tone of voice of the interviewer affect a survey? Give at least two examples.

26. **WRITING IN MATH** Compare taking a survey and finding an experimental probability.

STANDARDS PRACTICE

27. Maci surveyed all the members of her softball team about their favorite sport.

Sport	Number of Members
Softball	12
Basketball	5
Soccer	3
Volleyball	8

From these results, Maci concluded that softball was the favorite sport among all her classmates. Which is the best explanation for why her conclusion might *not* be valid?

A The softball team meets only on weekdays.

B She should have asked only people who do not play sports.

C The survey should have been done daily for a week.

D The sample was not representative of all of her classmates.

28. Ms. Hernandez determined that 60% of the students in her classes brought an umbrella to school when the weather forecast predicted rain. If she has a total of 150 students, which statement does *not* represent Ms. Hernandez's data?

F On days when rain is forecast, less than $\frac{2}{5}$ of her students bring an umbrella to school.

G On days when rain is forecast, 90 of her students bring an umbrella to school.

H On days when rain is forecast, more than $\frac{1}{2}$ of her students bring an umbrella to school.

J On days when rain is forecast, 60 of her students do not bring an umbrella to school.

Spiral Review

29. **PIZZA** A pizza parlor has thin crust and thick crust, 2 different cheeses, and 4 toppings. Use the *act it out* strategy to determine how many different one-cheese and one-topping pizzas can be ordered. (Lesson 12-4)

30. **MANUFACTURING** An inspector finds that 3 out of the 250 DVD players he checks are defective. What is the experimental probability that a DVD player is defective? (Lesson 12-3)

31. **CAR RENTAL** You can rent a car for either \$35 a day plus \$0.40 per mile or for \$20 a day plus \$0.55 per mile. Write and solve an equation to find the number of miles that result in the same cost for one day. (Lesson 8-4)

Cross-Curricular Project

Math and Science

It's all in the Genes It's time to complete your project. Use the information and data you have gathered about genetics and the traits of your classmates to prepare a Web page or poster. Be sure to include a chart displaying your data with your project.

MathOnline **Cross-Curricular Project at** ca.gr7math.com

CHAPTER 12

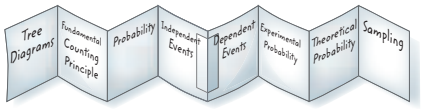
Study Guide and Review

Download Vocabulary Review from ca.g7math.com.

FOLDABLES Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Counting Outcomes (Lesson 12-1)

- If event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by the event N can occur in $m \cdot n$ ways.

Probability (Lessons 12-2 and 12-3)

- The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event.
- If two events, A and B, are dependent, then the probability of both events occurring is the product of the probability of A and the probability of B after A occurs.

Statistics (Lesson 12-5)

- An unbiased sample is representative of an entire population.
- A biased sample favors one or more parts of a population over others.

Key Vocabulary

biased sample (p. 648)
compound events (p. 631)
convenience sample (p. 648)
dependent events (p. 632)
event (p. 626)
experimental probability (p. 637)
Fundamental Counting Principle (p. 627)
independent events (p. 631)
outcome (p. 626)
population (p. 647)
probability (p. 627)
random (p. 627)
sample (p. 647)
sample space (p. 626)
simple random sample (p. 647)
stratified random sample (p. 647)
systematic random sample (p. 647)
theoretical probability (p. 637)
tree diagram (p. 626)
unbiased sample (p. 647)
voluntary response sample (p. 648)

Vocabulary Check

Choose the correct term to complete each sentence.

1. A list of all possible outcomes is called the (sample space, event).
2. The (population, probability) of an event is the ratio of a specific outcome to the total number of outcomes.
3. A (combination, compound event) consists of two or more simple events.
4. For (independent, dependent) events, the outcome of one does not affect the other.
5. (Theoretical, Experimental) probability is based on known characteristics or facts.
6. A (simple random sample, convenience sample) is a biased sample.

CHAPTER 12 Study Guide and Review

Lesson-by-Lesson Review

12-1 Counting Outcomes (pp. 626–630)

For Exercises 7–9, use the following informaton.

A penny is tossed and a 4-sided number pyramid with sides labeled 1, 2, 3, and 4 is rolled.

7. Draw a tree diagram to show the possible outcomes.
8. Find the probability of getting a head and a 3.
9. Find the probability of getting a tail and an odd number.

10. **FOOD** A restaurant offers 15 main menu items, 5 salads, and 8 desserts. How many meals of a main menu item, a salad, and a dessert are there?

Example 1 **A car manufacturer makes 8 different models in 12 different colors. They also offer manual or automatic transmission. How many choices does a customer have?**

$$\underset{\text{number of models}}{8} \times \underset{\text{number of colors}}{12} \times \underset{\text{number of transmissions}}{2} = \underset{\text{total number of cars}}{192}$$

The customer has 192 choices.

12-2 Probability of Compound Events (pp. 631–636)

A number cube is rolled and a penny is tossed. Find each probability.

11. P(2 and heads)
12. P(even and heads)
13. P(1 or 2 and tails)
14. P(odd and tails)

15. **TIES** Mr. Dominguez has 4 black ties, 3 gray ties, 2 maroon ties, and 1 brown tie. If he selects two ties without looking, what is the probability that he will pick two black ties?

Example 2 **A bag of marbles contains 7 white and 3 blue marbles. Once selected, the marble is not replaced. What is the probability of choosing 2 blue marbles?**

$P(\text{first marble is blue}) = \frac{3}{10}$

$P(\text{second marble is blue}) = \frac{2}{9}$

$P(\text{two blue marbles}) = \frac{3}{10} \cdot \frac{2}{9}$

$= \frac{6}{90}$ or $\frac{1}{15}$

Mixed Problem Solving
For mixed problem-solving practice, see page 719.

12-3 Experimental and Theoretical Probability (pp. 637–641)

A spinner has four equal-sized sections. Each section is a different color. In the last 30 spins, the pointer landed on red 5 times, blue 10 times, green 8 times, and yellow 7 times. Find each experimental probability.

16. P(red)
17. P(green)
18. P(red or blue)
19. Compare the theoretical and experimental probabilities of the spinner landing on red.

SPELLING For Exercises 20 and 21, use the following information.
On a spelling test, Angie misspells 2 out of the first 10 words.

20. What is the probability that she will misspell the next spelling word?
21. If the spelling test has 25 words on it, how many words would you expect Angie to misspell?

For Exercises 22 and 23, use the following information.
A group of three coins are each tossed 20 times. The results are shown in the table.

Outcome	Frequency
0 heads, 3 tails	2
1 head, 2 tails	8
2 heads, 1 tail	6
3 heads, 0 tails	4

22. What is the experimental probability that there will be one head and two tails?
23. What is the experimental probability that there will be three heads and zero tails?

Example 3 **A nickel and a dime are tossed. What is theoretical probability of tossing two tails?**

The theoretical probability is $\frac{1}{2} \cdot \frac{1}{2}$ or $\frac{1}{4}$.

Example 4 **In an experiment, the same two coins are tossed 50 times. Ten of those times, tails were both showing. Find the experimental probability of tossing two tails.**

Since tails were showing 10 out of the 50 tries, the experimental probability is $\frac{10}{50}$ or $\frac{1}{5}$.

Example 5 **Compare the theoretical and experimental probabilities of tossing two tails.**

The theoretical probability $\frac{1}{4}$ is greater than the experimental probability $\frac{1}{5}$.

CHAPTER 12

Study Guide and Review

12-4 PSI: Act it Out (pp. 645–646)

Solve. Use the *act it out* strategy.

24. **READING** In English class, each student must select 4 short stories from a list of 5 short stories to read. How many different combinations of short stories could a student read?

25. **CARPENTRY** Jaime has $14\frac{1}{4}$ feet of lumber. She uses $2\frac{7}{8}$ feet for a bookshelf. Does Jaime have enough lumber for four more identical shelves? Explain.

Example 6 **The Spirit Club is making a banner using three sheets of paper. How many different banners can they make using their school colors of black, orange, and white?**

Use three index cards labeled black, orange, and white to model the different banners.

There are six different combinations they can make.

12-5 Using Sampling to Predict (pp. 647–652)

CONCERTS **For Exercises 26 and 27, use the following information.**
A radio station is taking a survey to determine how many people would attend a music festival.

26. Describe the sample if the station asks listeners to call in a response to the survey.

27. Suppose 12 out of 80 people surveyed said they would attend the festival. How many out of 800 people would be expected to attend the festival?

Example 7 **In a survey, 25 out of 40 students in the school cafeteria preferred chocolate milk rather than white milk. How much chocolate milk should the school order for 400 students each day?**

25 out of 40 or 62.5% of the students prefer chocolate milk.

Find 62.5% of 400.
$0.625 \times 400 = 250$

The school should order about 250 cartons of chocolate milk.

CHAPTER 12

Practice Test

1. **FOOD** Students at West Middle School can purchase a box lunch to take on their field trip. They choose one item from each category. How many lunches can be ordered?

Categories for Box Lunches
5 types of sandwiches
3 types of fresh fruits
2 types of cookies

2. **STANDARDS PRACTICE** Ms. Hawthorne randomly selects 2 students from 6 volunteers to be on the school activities committee. If Roberto and Joel volunteer, what is the probability that they will both be selected?

A $\frac{1}{3}$ C $\frac{1}{30}$

B $\frac{1}{15}$ D $\frac{1}{60}$

A jar contains 4 blue, 7 red, 6 yellow, 8 green, and 3 white tiles. Once a tile is selected, it is not replaced. Find each probability.

3. P(2 blue)
4. P(red, then white)
5. P(white, then green)
6. P(two tiles that are neither yellow nor red)

Two coins are tossed 20 times. No tails were tossed 4 times, one tail was tossed 11 times, and 2 tails were tossed 5 times.

7. What is the experimental probability of no tails?
8. What is the experimental probability of one tail?
9. Draw a tree diagram to show the outcomes of tossing two coins.
10. Compare the experimental probability with the theoretical probability of getting no tails when two coins are tossed.

11. **SHOES** A tennis shoe comes in men's and women's sizes; cross training, walking, and running styles; and blue, black, or white. What is the number of possible outcomes?

12. **SOFTBALL** Miranda had the opportunity to bat 15 times during the tournament. Of those at-bats, she made an out 6 times, hit a single 5 times, a double three times, and a home run once. What is the experimental probability that Miranda hit a double?

13. **VOLUNTEERING** Student Council surveyed four homerooms to find out how many hours students volunteer each year. The results are shown in the table. If there are 864 students at the school, how many can be expected to volunteer 21–40 hours?

Number of Hours	Number of Students
0–10	38
11–20	26
21–40	10
40 or more	6

14. **STANDARDS PRACTICE** The Centerville School Board wants to know if it has community support to build a new school. How should they conduct a valid survey?

F Ask parents at a school open house.
G Ask people at the Senior Center.
H Call every 50th number in the phone book.
J Ask people to call with their opinions.

15. **BASEBALL** To determine the favorite sport, a random survey is administered at a baseball game. Of those surveyed, 72% responded that baseball is their favorite sport. It is concluded that baseball is the favorite sport of most people. Is this conclusion valid?

CHAPTER 12

California Standards Practice

Cumulative, Chapters 1–12

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1 The table below shows all of the possible outcomes of a 3-panel light switch being turned on or off.

1st switch	2nd switch	3rd switch
ON	ON	ON
ON	ON	OFF
ON	OFF	ON
ON	OFF	OFF
OFF	ON	ON
OFF	ON	OFF
OFF	OFF	ON
OFF	OFF	OFF

Which of the following statements must be true if an outcome is chosen at random?

A The probability that all of the switches will be on is the same as the probability that all of the switches will be off.

B The probability that one light switch is on is higher than the probability that two light switches are on.

C The probability that exactly two switches have the same outcome is $\frac{1}{2}$.

D The probability of having at least one light switch on is higher than the probability of having at least one light switch off.

2 A drawer contained two blue, three black, and four white socks. Michael removed one blue sock from the drawer and did *not* put the sock back in the drawer. He then randomly removed another sock from the drawer. What is the probability that the second sock Michael removed was blue?

F $\frac{1}{18}$ **H** $\frac{1}{8}$

G $\frac{1}{9}$ **J** $\frac{1}{4}$

3 Of the 32 students surveyed in J.T.'s homeroom, 14 recycle at home. How many students would you expect to recycle at home if a total of 880 students were surveyed?

A 495 **C** 281

B 385 **D** 123

4 A car tire travels about 100 inches in 1 full rotation. What is the radius of the tire, to the nearest inch?

F 32 inches **H** 24 inches

G 28 inches **J** 16 inches

5 What is the volume of a rectangular prism with a length of 7 centimeters, a width of 14 centimeters, and a height of 10 centimeters?

A 31 cm^3 **C** 980 cm^3

B 108 cm^3 **D** 1,000 cm^3

6 An ice cream store surveyed 100 of its customers about their favorite flavor. The results are shown in the table. If the store uses only these data to order ice cream, what conclusion can be drawn from the data?

Favorite Flavor	
Flavor	Frequency
Chocolate Chip	40
Vanilla	15
Cookie Dough	20
Chocolate	15
Other	10

F More than half of each order should be chocolate chip and cookie dough ice cream.

G Half of the order should be vanilla and chocolate ice cream.

H Only chocolate, cookie dough, and vanilla ice cream should be ordered.

J About one third of the order should be vanilla and chocolate chip ice cream.

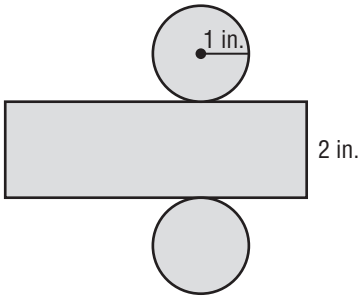
MathOnline **California Standards Practice at** ca.gr7math.com.

7 The probability that Maryanne gets a hit in softball is $\frac{3}{5}$. How many hits would you expect her to get in her next 60 at-bats?

A 50 **C** 30

B 36 **D** 24

8 The net below forms a cylinder when folded. What is the surface area of the cylinder?



F 6.3 in^2 **H** 21.3 in^2

G 18.8 in^2 **J** 42.6 in^2

9 If three coins are tossed, what is the probability that they all show tails?

A 6.25% **C** 25%

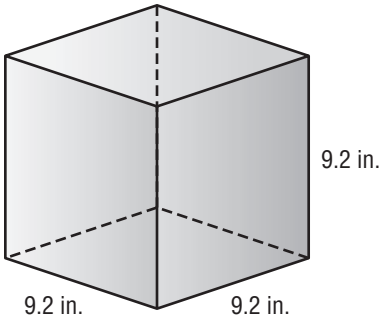
B 12.5% **D** 50%

10 What is the solution set of the inequality $4n - 8 \leq 40$?

F $\{n:n \leq 8\}$ **H** $\{n:n \geq 8\}$

G $\{n:n \leq 12\}$ **J** $\{n:n \geq 12\}$

11 A sporting goods company ships basketballs in cube-shaped boxes. Which of the following is closest to the surface area of the box?



A 85 in^2 **C** 475 in^2

B 320 in^2 **D** 510 in^2

Pre-AP

Record your answers on a sheet of paper. Show your work.

12 Tiffany has a bag of 10 yellow, 10 red, and 10 green marbles. Tiffany picks two marbles at random and gives them to her sister.

a. What is the probability of choosing 2 yellow marbles?

b. Of the marbles left, what is the probability of choosing a green marble next?

TEST-TAKING TIP

Question 12 Extended-response questions often involve several parts. When one part of the question involves the answer to a previous part of the question, make sure to check your answer to the first part before moving on. Also, remember to show all of your work. You may be able to get partial credit for your answers, even if they are not entirely correct.

NEED EXTRA HELP?												
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson...	12-2	12-2	12-5	7-1	7-5	12-5	12-3	7-7	12-2	8-8	7-7	12-2
For Help with Standard...	6SDAP3.1	6SDAP3.5	6SDAP2.5	MG2.1	MG2.3	6SDAP2.5	6SDAP3.2	MG3.5	6SDAP3.1	AF4.1	MG2.1	6SDAP3.1