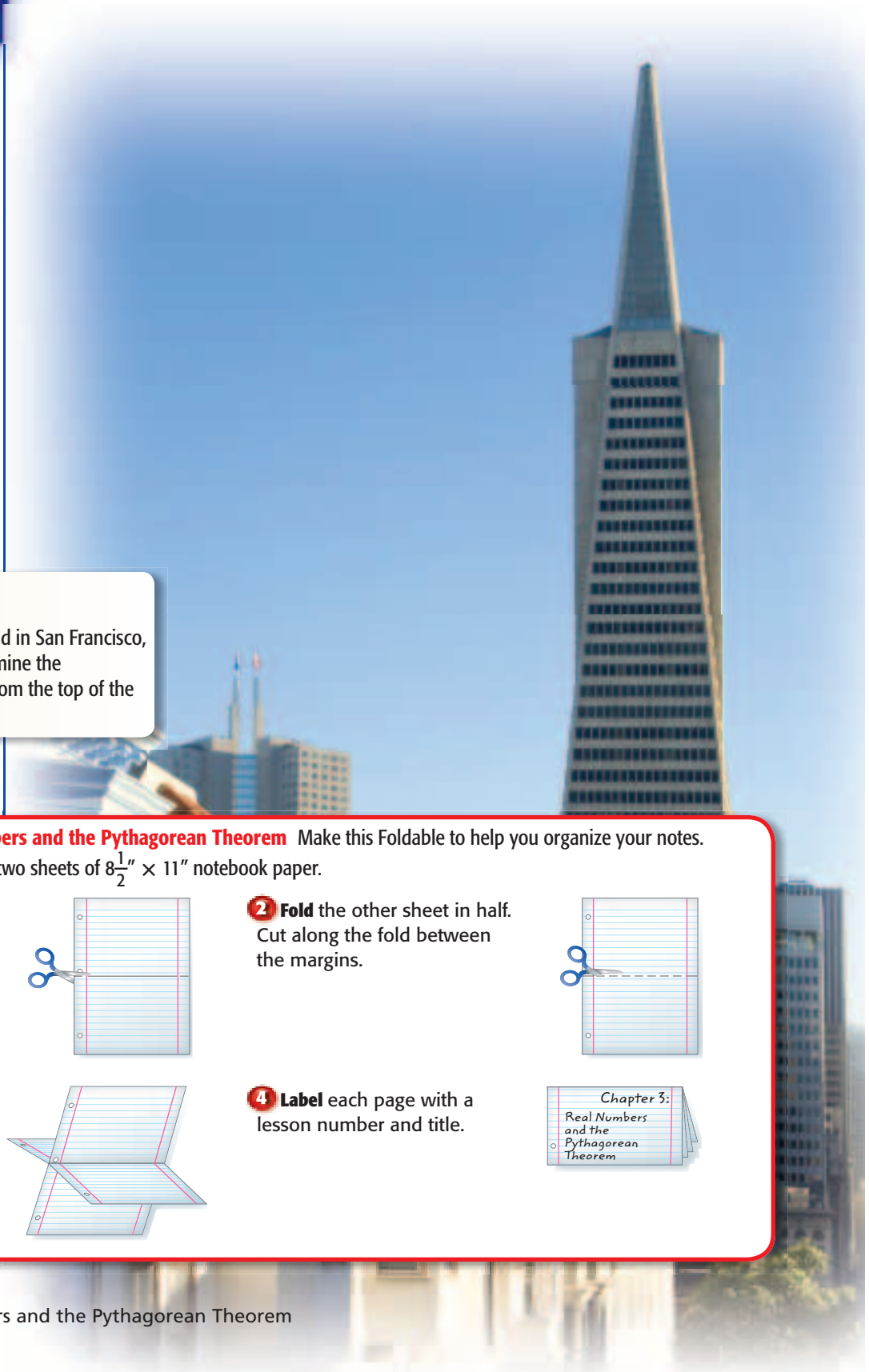
CHAPTER 3

Real Numbers and the Pythagorean Theorem



BIG Idea

- **Standard 7MG3.0** Know the Pythagorean theorem and understand plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures.

Key Vocabulary

ordered pair (p. 173)
Pythagorean Theorem (p. 162)
real number (p. 155)
square root (p. 144)

Real-World Link

Buildings The Transamerica Pyramid in San Francisco, California, is 853 feet high. To determine the approximate distance you can see from the top of the tower, multiply 1.23 by $\sqrt{853}$.

FOLDABLES Study Organizer

Real Numbers and the Pythagorean Theorem Make this Foldable to help you organize your notes. Begin with two sheets of $8\frac{1}{2}'' \times 11''$ notebook paper.

1. **Fold** one sheet in half from top to bottom. Cut along the fold from edges to margin.

2. **Fold** the other sheet in half. Cut along the fold between the margins.

3. **Insert** the first sheet through the second sheet and align the folds.

4. **Label** each page with a lesson number and title.

GET READY for Chapter 3

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

MathOnline Take the Online Readiness Quiz at ca.gr7math.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

Graph each point on a coordinate plane. (Prior Grade)

1. $A(-1, 3)$
2. $B(2, -4)$
3. $C(-2, -3)$
4. $D(-4, 0)$

Evaluate each expression. (Lesson 2-9)

5. $2^2 + 4^2$
6. $3^2 + 3^2$
7. $10^2 + 8^2$
8. $7^2 + 5^2$

9. **AGES** Find the sum of the squares of Tina's age and Warren's age if Tina is 13 years old and Warren is 15 years old. (Lesson 2-9)

Solve each equation. Check your solution. (Lesson 1-9)

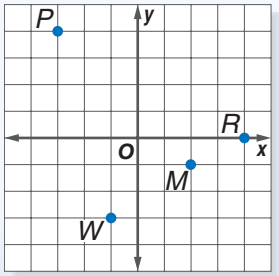
10. $x + 13 = 45$
11. $56 + d = 71$
12. $101 = 39 + a$
13. $62 = 45 + m$

14. **MARBLES** Barry has 18 more marbles than Heidi. If Barry has 92 marbles, how many marbles does Heidi have? (Lesson 1-9)

QUICK Review

Example 1

Graph the points $P(-3, 4)$, $M(2, -1)$, $R(4, 0)$, and $W(-1, -3)$.



The first number in an ordered pair tells you to move left or right from the origin. The second number tells you to move up or down.

Example 2

Find $6^2 + 4^2$.

$6^2 + 4^2 = 36 + 16$ Evaluate 6^2 and 4^2.

$= 52$ Simplify.

Example 3

Solve $49 + b = 72$.

$49 + b = 72$ Write the equation.

$-49 \quad -49$ Subtract 49 from each side.

$b = 23$

3-1 Square Roots

Main IDEA

Find square roots of perfect squares.

Standard 7NS2.4 **Use the inverse relationship between raising to a power and extracting the root of a perfect square;** for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.

NEW Vocabulary

perfect square
square root
radical sign

READING in the Content Area

For strategies in reading this lesson, visit ca.gr7math.com.

MINI Lab Concepts in Motion
Interactive Lab ca.gr7math.com

Continue the pattern of square tiles until you reach 5 tiles on each side.

1. Copy and complete the following table.

Tiles on a Side	1	2	3	4	5
Total Number of Tiles in the Square Arrangement	1	4			

2. Suppose a square arrangement has 36 tiles. How many tiles are on a side?
3. What is the relationship between the number of tiles on a side and the number of tiles in the arrangement?

Numbers such as 1, 4, 9, 16, and 25 are called **perfect squares** because they are squares of integers. Squaring a number and finding a square root are inverse operations. A **square root** of a number is one of its two equal factors. The symbol $\sqrt{\ }$, called a **radical sign**, is used to indicate a square root. Every positive number has *both* a positive and a negative square root.

EXAMPLES Find Square Roots

Find each square root.

1 $\sqrt{64}$

$\sqrt{64}$ indicates the *positive* square root. Since $8^2 = 64$, $\sqrt{64} = 8$.

2 $-\sqrt{\frac{25}{36}}$

$-\sqrt{\frac{25}{36}}$ indicates the *negative* square root of $\frac{25}{36}$.

Since $\left(\frac{5}{6}\right)^2 = \frac{25}{36}$, $-\sqrt{\frac{25}{36}} = -\frac{5}{6}$.

3 $\pm\sqrt{1.21}$

$\pm\sqrt{1.21}$ indicates *both* the positive and negative square roots of 1.21.

Since $1.1^2 = 1.21$ and $(-1.1)^2 = 1.21$, $\pm\sqrt{1.21} = \pm 1.1$, or 1.1 and -1.1.

CHECK Your Progress **Find each square root.**

a. $\sqrt{\frac{9}{16}}$ b. $-\sqrt{49}$ c. $\pm\sqrt{0.81}$

By the definition of square roots, if $n^2 = a$, then $n = \pm\sqrt{a}$. You can use this relationship to solve equations that involve squares.

EXAMPLE Use Square Roots to Solve an Equation

4 **ALGEBRA** **Solve $t^2 = 169$. Check your solution(s).**

$t^2 = 169$ Write the equation.

$t = \pm\sqrt{169}$ Definition of square root

$t = 13$ and -13 **Check** $13 \cdot 13 = 169$ and $(-13)(-13) = 169$ ✓

The equation has two solutions, 13 and -13.

CHECK Your Progress

Solve each equation. Check your solution(s).

d. $289 = a^2$ **e.** $m^2 = 0.09$ **f.** $y^2 = \frac{4}{25}$

In most real-world situations, a negative square root does not make sense. Only the positive or *principal* square root is considered.

Real-World EXAMPLE

5 **HISTORY** The base of the Great Pyramid covers an area of about 562,500 square feet. Determine the length of each side of the base.

Words	Area is equal to the square of the length of a side.
Variable	Let s represent the length of a side.
Equation	$562{,}500 = s^2$

$562{,}500 = s^2$ Write the equation.

$\pm\sqrt{562{,}500} = s$ Definition of square root

To find $\sqrt{562{,}500}$, find two equal factors of 562,500.

$562{,}500 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ Find the prime factors.

$= (2 \cdot 3 \cdot 5 \cdot 5 \cdot 5)(2 \cdot 3 \cdot 5 \cdot 5 \cdot 5)$ Regroup into two equal factors.

So, $s = 2 \cdot 3 \cdot 5 \cdot 5 \cdot 5$ or 750.

Since distance cannot be negative, the length of each side is 750 feet.

Real-World Link
The Great Pyramid of Khufu is the largest of the ancient pyramids.
Source: infoplease.com

CHECK Your Progress

g. **CONCERTS** A concert crew needs to set up 900 chairs on the floor level. If the chairs are placed in a square arrangement, how many should be in each row?

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CORBIS

CHECK Your Understanding

Examples 1–3 (p. 144) **Find each square root.**

1. $\sqrt{25}$
2. $\sqrt{0.64}$
3. $-\sqrt{1.69}$
4. $-\sqrt{\frac{16}{81}}$
5. $\pm\sqrt{100}$
6. $\pm\sqrt{\frac{49}{144}}$

Example 4 (p. 145) **ALGEBRA Solve each equation. Check your solution(s).**

7. $p^2 = 36$
8. $t^2 = \frac{1}{9}$
9. $6.25 = r^2$

Example 5 (p. 145)

10. **GAMES** A checkerboard is a large square that is made up of 32 small red squares and 32 small black squares. How many small squares are along one side of a checkerboard?

Exercises

HOMEWORK HELP

For Exercises	See Examples
11–14	1
15–18	2
19–22	3
23–30	4
31, 32	5

Find each square root.

11. $\sqrt{16}$
12. $-\sqrt{81}$
13. $-\sqrt{484}$
14. $\pm\sqrt{36}$
15. $\sqrt{\frac{121}{324}}$
16. $-\sqrt{\frac{64}{225}}$
17. $\pm\sqrt{\frac{9}{49}}$
18. $-\sqrt{\frac{16}{25}}$
19. $-\sqrt{2.56}$
20. $\pm\sqrt{1.44}$
21. $\sqrt{0.25}$
22. $\pm\sqrt{0.0196}$

ALGEBRA Solve each equation. Check your solution(s).

23. $v^2 = 81$
24. $b^2 = 100$
25. $144 = s^2$
26. $225 = y^2$
27. $w^2 = \frac{36}{100}$
28. $\frac{9}{64} = c^2$
29. $0.0169 = d^2$
30. $a^2 = 1.21$

31. **PHOTOGRAPHY** A group of 169 students needs to be seated in a square formation for a yearbook photo. How many students should be in each row?

32. **MARCHING BAND** A marching band wants to form a square in the middle of the field. If there are 81 members in the band, how many should be in each row?

ALGEBRA Solve each equation. Check your solution(s).

33. $\sqrt{x} = 5$
34. $\sqrt{y} = 20$
35. $\sqrt{z} = 10.5$

MEASUREMENT The formula for the perimeter of a square is $P = 4s$, where s is the length of a side. Find the perimeter of each square.

36. Area = 121 square inches
37. Area = 25 square feet
38. Area = 36 square meters

EXTRA PRACTICE

See pages 682, 710.

Math Online

Self-Check Quiz at ca.gr7math.com

H.O.T. Problems

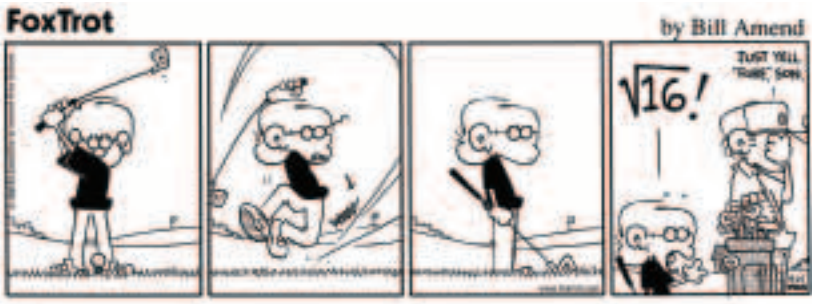
39. **OPEN ENDED** Create an equation that can be solved by finding the square root of a perfect square.

40. **CHALLENGE** Find each value.

a. $\left(\sqrt{36}\right)^2$ b. $\left(\sqrt{\frac{25}{81}}\right)^2$ c. $\left(\sqrt{1.99}\right)^2$ d. $\left(\sqrt{x}\right)^2$

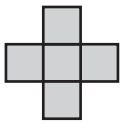
41. **NUMBER SENSE** Under what condition is $\sqrt{x} > \sqrt{25}$?

42. **WRITING IN MATH** Analyze the cartoon. Create a cartoon of your own that uses the square root of a perfect square.



STANDARDS PRACTICE

43. The area of each square is 4 square units.



Find the perimeter of the figure.

A 8 units **C** 20 units

B 16 units **D** 24 units

44. Mr Freeman's farm has a square cornfield. Find the area of the cornfield if the sides are measured in whole numbers.

F 164,000 ft^2

G 170,150 ft^2

H 170,586 ft^2

J 174,724 ft^2

Spiral Review

45. **SPACE** The Alpha Centuari stars are about 2.5×10^{13} miles from Earth. Write this distance in standard form. (Lesson 2-10)

Write each expression using exponents. (Lesson 2-9)

46. $6 \cdot 6 \cdot 6$ 47. $2 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2$ 48. $s \cdot t \cdot t \cdot s \cdot s \cdot t \cdot s$

49. What is the absolute value of -18? (Lesson 1-3)

GET READY for the Next Lesson

PREREQUISITE SKILL **Between which two perfect squares does each number lie?** (Lesson 2-2)

50. 57 51. 68 52. 33 53. 40

3-2 Estimating Square Roots

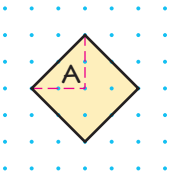
Main IDEA

Estimate square roots.

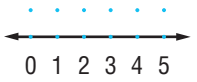
Standard 7NS2.4 Use the inverse relationship between raising to a power and extracting the root of a perfect square; **for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.**

MINI Lab

STEP 1 On dot paper, draw and cut out a square like the one at the right. The area of section A is $\frac{1}{2}(2 \cdot 2)$ or 2 square units. So, the shaded square has an area of 8 square units.



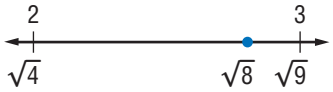
STEP 2 Draw a number line on your dot paper so that 1 unit equals the distance between dots.



1. Place your square on the number line. Between what two consecutive whole numbers is $\sqrt{8}$, the side length of the square, located?
2. Between what two perfect squares is 8 located?
3. Estimate the length of a side of the square. Verify your estimate by using a calculator to compute the value of $\sqrt{8}$.

In the Mini Lab, $\sqrt{8}$ is not a whole number since 8 is not a perfect square.

The number line shows that $\sqrt{8}$ is between 2 and 3. Since 8 is closer to 9 than 4, the best whole number estimate for $\sqrt{8}$ is 3.



READING Math

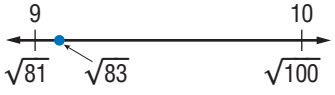
Inequalities
$81 < 83 < 100$ is read *81 is less than 83 is less than 100* or *83 is between 81 and 100.*

EXAMPLES Estimate Square Roots

1 **Estimate $\sqrt{83}$ to the nearest whole number.**

- The largest perfect square less than 83 is 81. $\sqrt{81} = 9$
- The smallest perfect square greater than 83 is 100. $\sqrt{100} = 10$

Plot each square root on a number line. Then estimate $\sqrt{83}$.



$81 < 83 < 100$ Write an inequality.

$9^2 < 83 < 10^2$ $81 = 9^2$ and $100 = 10^2$

$\sqrt{9^2} < \sqrt{83} < \sqrt{10^2}$ Find the square root of each number.

$9 < \sqrt{83} < 10$ Simplify.

So, $\sqrt{83}$ is between 9 and 10. Since $\sqrt{83}$ is closer to $\sqrt{81}$ than $\sqrt{100}$, the best whole number estimate for $\sqrt{83}$ is 9.

STUDY TIP

Mental Math It is important to memorize common perfect squares.

$1^2 = 1$	$2^2 = 4$
$3^2 = 9$	$4^2 = 16$
$5^2 = 25$	$6^2 = 36$
$7^2 = 49$	$8^2 = 64$
$9^2 = 81$	$10^2 = 100$
$11^2 = 121$	$12^2 = 144$

2 **Estimate $\sqrt{23.5}$ to the nearest whole number.**

- The largest perfect square less than 23.5 is 16. $\sqrt{16} = 4$
- The smallest perfect square greater than 23.5 is 25. $\sqrt{25} = 5$

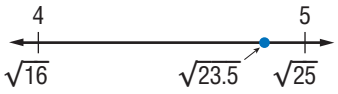
$16 < 23.5 < 25$ Write an inequality.

$4^2 < 23.5 < 5^2$ $16 = 4^2$ and $25 = 5^2$

$\sqrt{4^2} < \sqrt{23.5} < \sqrt{5^2}$ Find the square root of each number.

$4 < \sqrt{23.5} < 5$ Simplify.

So, $\sqrt{23.5}$ is between 4 and 5. Since 23.5 is closer to 25 than 16, the best whole number estimate for $\sqrt{23.5}$ is 5.



CHECK Your Progress **Estimate to the nearest whole number.**

a. $\sqrt{35}$ b. $\sqrt{44.8}$ c. $\sqrt{170}$

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Real-World EXAMPLE

3 **ART** **The Parthenon is an example of a *golden rectangle*. The length of the longer side divided by the length of the shorter side is equal to $\frac{1+\sqrt{5}}{2}$. Estimate this value.**

First estimate the value of $\sqrt{5}$.

$4 < 5 < 9$ 4 and 9 are the closest perfect squares.

$2^2 < 5 < 3^2$ $4 = 2^2$ and $9 = 3^2$

$\sqrt{2^2} < \sqrt{5} < \sqrt{3^2}$ Find the square root of each number.

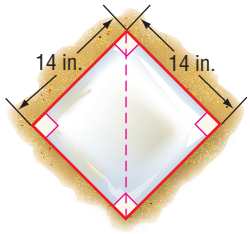
$2 < \sqrt{5} < 3$ Simplify.

Since 5 is closer to 4 than 9, the best whole number estimate for $\sqrt{5}$ is 2. Use this value to evaluate the expression.

$\frac{1+\sqrt{5}}{2} \approx \frac{1+2}{2}$ or 1.5

CHECK Your Progress

d. **BASEBALL** In Little League, the bases are squares with sides of 14 inches. The expression $\sqrt{(s^2 + s^2)}$ represents the distance *across* a square of side length s. Estimate the distance across a base to the nearest inch.



CHECK Your Understanding

Examples 1, 2 (pp. 148–149)

Estimate to the nearest whole number.

1. $\sqrt{28}$
2. $\sqrt{60}$
3. $\sqrt{135}$
4. $\sqrt{13.5}$
5. $\sqrt{38.7}$
6. $\sqrt{79.2}$

Example 3 (p. 149)

7. **SCIENCE** The number of swings back and forth of a pendulum of length L, in inches, each minute is $\frac{375}{\sqrt{L}}$. About how many swings will a 40-inch pendulum make each minute?

Exercises

HOMEWORK HELP

For Exercises	See Examples
8–15	1, 2
16, 17	3

Estimate to the nearest whole number.

8. $\sqrt{44}$
9. $\sqrt{23}$
10. $\sqrt{125}$
11. $\sqrt{197}$
12. $\sqrt{15.6}$
13. $\sqrt{23.5}$
14. $\sqrt{85.1}$
15. $\sqrt{38.4}$

16. **GEOMETRY** The radius of a circle with area A is approximately $\sqrt{\frac{A}{3}}$. If a pizza has an area of 78 square inches, estimate its radius.

17. **CAVES** The formula $t = \frac{\sqrt{h}}{4}$ represents the time t in seconds that it takes an object to fall from a height of h feet. Suppose a rock falls from a 200-feet high cave ceiling. Estimate how long will it take to reach the ground.

Estimate to the nearest whole number.

18. $\sqrt{5\frac{1}{5}}$
9. $\sqrt{21\frac{7}{10}}$
20. $\sqrt{17\frac{3}{4}}$

Order from least to greatest.

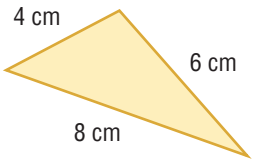
21. $7, 9, \sqrt{50}, \sqrt{85}$
22. $\sqrt{91}, 7, 5, \sqrt{38}$
23. $\sqrt{62}, 6, \sqrt{34}, 8$

ALGEBRA **Estimate the solution of each equation to the nearest integer.**

24. $y^2 = 55$
25. $d^2 = 95$
26. $p^2 = 6.8$

27. **FIND THE DATA** Refer to the California Data File on pages 16–19. Choose some data and write a real-world problem in which you would estimate a square root.

28. **GEOMETRY** Egyptian mathematician Heron created the formula $A = \sqrt{s(s-a)(s-b)(s-c)}$ to find the area A of a triangle. In this formula, a, b, and c are the measures of the sides, and s is one-half of the perimeter. Use this formula to estimate the area of the triangle at the right.



EXTRA PRACTICE

See pages 683, 710.

Math Online

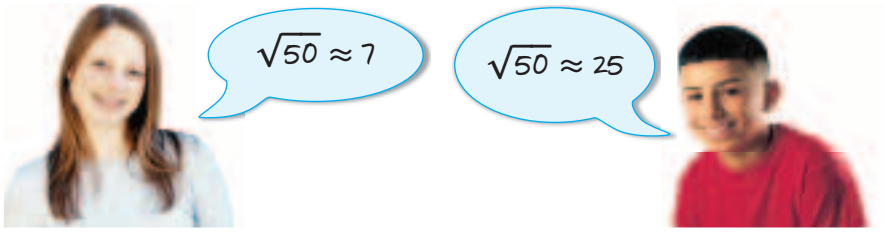
Self-Check Quiz at ca.gr7math.com

29. **NUMBER SENSE** Without a calculator, determine which is greater, $\sqrt{94}$ or 10. Explain your reasoning.

H.O.T. Problems

30. **OPEN ENDED** Find two numbers that have square roots between 7 and 8. One number should have a square root closer to 7, and the other number should have a square root closer to 8. Justify your answer.

31. **FIND THE ERROR** Josephina and Dario are estimating $\sqrt{50}$. Who is correct? Explain your reasoning.



32. **CHALLENGE** If $x^3 = y$, then x is the cube root of y. Explain how to estimate the cube root of 30. Find the cube root of 30 to the nearest whole number.

33. **WRITING IN MATH** Explain how to graph $\sqrt{78}$ on a number line.

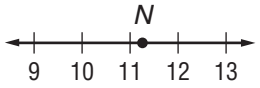
STANDARDS PRACTICE

34. A whole number is squared. The result is between 950 and 1,000. The number is between

A 26 and 28. **C** 30 and 32.

B 28 and 30. **D** 32 and 34.

35. Point N on the number line best represents which square root?



F $\sqrt{140}$ **H** $\sqrt{116}$

G $\sqrt{121}$ **J** $\sqrt{126}$

Spiral Review

36. **ALGEBRA** Find a number that, when squared, equals 8,100. (Lesson 3-1)

37. **GEOGRAPHY** The Great Lakes cover about 94,000 square miles. Write this number in scientific notation. (Lesson 2-10)

Multiply or divide. (Lesson 1-6)

38. $(-5)(-13)$ 39. $(-2)(5)(7)$ 40. $72 \div (-2)$ 41. $-80 \div (-16)$

GET READY for the Next Lesson

42. **PREREQUISITE SKILL** To attend a field trip to an art museum, each student will have to pay \$6.50 for transportation and \$10.00 for admission and lunch. Find the total amount of money to be collected for a class of 240 students. (Lesson 1-1)

3-3 Problem-Solving Investigation

MAIN IDEA: Use a Venn diagram to solve problems.

STANDARD 7MR2.5 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, **diagrams,** and models, **to explain mathematical reasoning. STANDARD 7NS1.2 Add, subtract, multiply, and divide rational numbers (integers,** fractions, and **terminating decimals**) and take positive rational numbers to whole-number powers.

P.S.I. TEAM +

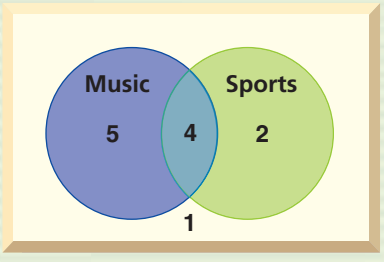
e-Mail: USE A VENN DIAGRAM

YOUR MISSION: Use a Venn diagram to solve the problem.

THE PROBLEM: How many students are not involved in either music or sports?

Amy: Of the 12 students who ate lunch with me today, 9 are involved in music activities and 6 play sports. Of these students, 4 are involved in both music and sports.

EXPLORE	You know how many students are in involved in each activity and how many are involved in both activities. You want to organize the information.
PLAN	Make a Venn diagram to organize the information.
SOLVE	Draw two overlapping circles to represent the two different activities. Since 4 students are involved in both activities, place a 4 in the section that is a part of both circles. Use subtraction to determine the number for each of the other sections. only music: $9 - 4 = 5$ only sports: $6 - 4 = 2$ neither music nor sports: $12 - 5 - 2 - 4 = 1$
CHECK	Check each circle to see if the appropriate number of students is represented.



Analyze The Strategy

1. Describe how to determine the number of students who are in either music or sports but not both using the above Venn diagram.
2. WRITING IN MATH Explain what each section of the Venn diagram above represents and the number of students that belong to that category.

Mixed Problem Solving

Use a Venn diagram to solve Exercises 3–5.

3. **MASCOTS** Nick conducted a survey of 85 students about a new school mascot. The results showed that 40 students liked Tigers, and 31 students liked Bears. Of those students, 12 liked both Tigers and Bears. How many students liked neither Tigers nor Bears?

4. **MARKETING** A survey showed that 70 customers bought white bread, 63 bought wheat bread, and 35 bought rye bread. Of those who bought exactly two types of bread, 12 bought wheat and white, 5 bought white and rye, and 7 bought wheat and rye. Two customers bought all three. How many customers bought only wheat bread?

5. **HEALTH** Dr. Bagentose is an allergist. Her patients had the following symptoms. How many patients had only watery eyes?

Symptom(s)	Number of Patients
runny nose	22
watery eyes	20
sneezing	28
runny nose and watery eyes	8
runny nose and sneezing	15
watery eyes and sneezing	12
runny nose, watery eyes, and sneezing	5

Use any strategy to solve Exercises 6–9. Some strategies are shown below.

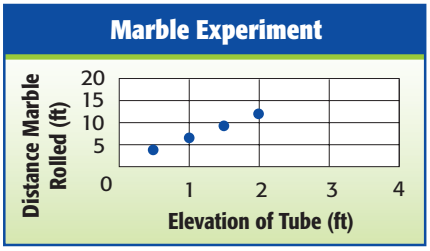
PROBLEM-SOLVING STRATEGIES
- Use the four-step plan.
- Look for a pattern.
- Use a Venn diagram.

6. **ALGEBRA** What are the next two numbers in the pattern?

864, 432, 216, 108, ■, ■

7. **MONEY** The soccer team sponsored a car wash to pay for their new uniforms. They charged \$3 for a car and \$5 for an SUV. During the first two hours they washed 19 vehicles and earned \$71. How many of each type of vehicle did they wash?

8. **ALGEBRA** Emilio created a graph of the data he collected for a science project. If the pattern continues, about how far will the marble roll if the end of the tube is raised to an elevation of $3\frac{1}{2}$ feet?



9. **SPORTS** Student Council surveyed a group of 24 students. The results showed that 14 students liked softball, and 18 liked basketball. Of these, 8 liked both. How many students liked just softball and how many liked just basketball?

Select the Operation

For Exercises 10 and 11, select the appropriate operation(s) to solve the problem. Justify your selection(s) and solve the problem.

10. **JOBS** Three after-school jobs are posted on the job board. The first job pays \$5.15 per hour for 15 hours of work each week. The second job pays \$10.95 per day for two hours of work, 5 days a week. The third job pays \$82.50 for 15 hours of work each week. If you want to apply for the best-paying job, which job should you choose? Explain your reasoning.

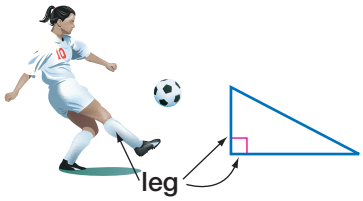
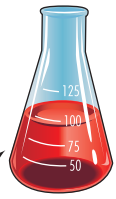
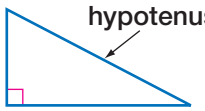
11. **ROLLER COASTERS** The Silver Streak roller coaster can accommodate 1,296 people in one hour. The coaster has 12 vehicles. If each vehicle carries 4 passengers, how many runs are made in one hour?

READING Word Problems

The Language of Mathematics

Standard 7AF1.4 **Use algebraic terminology** (e.g. **variable,** equation, term, coefficient, inequality, expression, constant) **correctly.**

The language of mathematics is very specific. But many of the words you use in mathematics are also used in everyday language as well as scientific language. Sometimes the everyday or scientific usage can give you clues to the mathematical meaning. Here are some examples.

Usage	Example
Some words are used in English and in mathematics, but have distinct meanings.	
Some words are used in science and in mathematics, but the meanings are different.	$x + 4 = -2$ $x = -6$ solution 
Some words are used only in mathematics.	

PRACTICE

Explain how the mathematical meaning of each word compares to its everyday meaning.

1. factor
2. leg
3. rational
4. root

Explain how the mathematical meaning of each word compares to its meaning in science.

5. radical
6. variable

Some words are used in English and in mathematics, but the mathematical meaning is more precise. Explain how the mathematical meaning of each word is more precise than the everday meaning.

7. similar
8. real

3-4 The Real Number System

Main IDEA

Identify and classify numbers in the real number system.

Standard 7NS1.4 Differentiate between rational and irrational numbers.

NEW Vocabulary

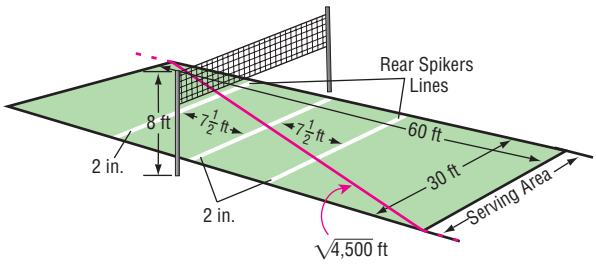
irrational number
real number

REVIEW Vocabulary

rational number any number that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$ (Lesson 2-1)

GET READY for the Lesson

SPORTS Most sports have rules for the size of the field or court where the sport is played. A diagram of a volleyball court is shown.



1. The length of the court is 60 feet. Is 60 a rational number? Explain.
2. The distance from the net to the rear spikers line is $7\frac{1}{2}$ feet. Is $7\frac{1}{2}$ a rational number? Explain.
3. The diagonal across the court is $\sqrt{4,500}$ feet. Can this square root be written as a rational number? Explain.

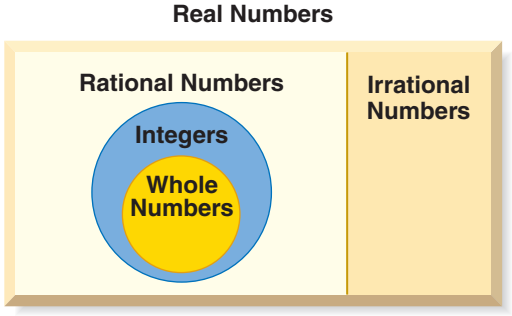
A calculator gives a decimal value of 67.08203932... for $\sqrt{4,500}$. Although this continues on and on, it does not repeat. Since the decimal does not terminate or repeat, $\sqrt{4,500}$ cannot be written as a fraction. Therefore, it is *not* a rational number. Numbers that are not rational are called **irrational numbers**. The square root of any number that is not a perfect square number is irrational.

KEY CONCEPT — Irrational Numbers

Words An irrational number is a number that cannot be expressed as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Examples $\sqrt{2} \approx 1.414213562\ldots$ $\quad -\sqrt{3} \approx -1.732050807\ldots$

The set of rational numbers and the set of irrational numbers together make up the set of **real numbers.** Study the diagram below.



STUDY TIP

Classifying Numbers
Always simplify numbers before classifying them.

EXAMPLES Classify Numbers

Name all sets of numbers to which each real number belongs.

1 **0.252525…** The decimal ends in a repeating pattern. It is a rational number because it is equivalent to $\frac{25}{99}$.

2 $\sqrt{36}$ Since $\sqrt{36} = 6$, it is a whole number, an integer, and a rational number.

3 $-\sqrt{7}$ $-\sqrt{7} \approx -2.645751311\ldots$ Since the decimal does not terminate or repeat, it is an irrational number.

CHECK Your Progress

a. $\sqrt{10}$ b. $-2\frac{2}{5}$ c. $\sqrt{100}$

Real numbers follow the properties that are true for whole numbers, integers, and rational numbers.

CONCEPT Summary Real Number Properties

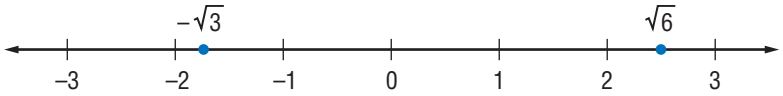
Property	Arithmetic	Algebra
Commutative	$3.2 + 2.5 = 2.5 + 3.2$ $5.1 \cdot 2.8 = 2.8 \cdot 5.1$	$a + b = b + a$ $a \cdot b = b \cdot a$
Associative	$(2 + 1) + 5 = 2 + (1 + 5)$ $(3 \cdot 4) \cdot 6 = 3 \cdot (4 \cdot 6)$	$(a + b) + c = a + (b + c)$ $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Distributive	$2(3 + 5) = 2 \cdot 3 + 2 \cdot 5$	$a(b + c) = a \cdot b + a \cdot c$
Identity	$\sqrt{8} + 0 = \sqrt{8}$ $\sqrt{7} \cdot 1 = \sqrt{7}$	$a + 0 = a$ $a \cdot 1 = a$
Additive Inverse	$4 + (-4) = 0$	$a + (-a) = 0$
Multiplicative Inverse	$\frac{2}{3} \cdot \frac{3}{2} = 1$	$\frac{a}{b} \cdot \frac{b}{a} = 1$, where $a, b \neq 0$

EXAMPLE Graph Real Numbers

4 **Estimate $\sqrt{6}$ and $-\sqrt{3}$ to the nearest tenth. Then graph $\sqrt{6}$ and $-\sqrt{3}$ on a number line.**

$\sqrt{6} \approx 2.449489742\ldots$ or about 2.4 Use a calculator.

$-\sqrt{3} \approx -1.7320508075\ldots$ or about -1.7 Use a calculator.



STUDY TIP

Real Numbers
The graph of all real numbers is the entire number line without any "holes."

CHECK Your Progress **Estimate each square root to the nearest tenth. Then graph the square root on a number line.**

d. $\sqrt{5}$ e. $-\sqrt{7}$ f. $\sqrt{22}$

EXAMPLES Compare Real Numbers

STUDY TIP

Mental Math
Remember that a negative number is always less than a positive number. Therefore, you can determine that $-\sqrt{3}$ is less than 1.7 without computation.

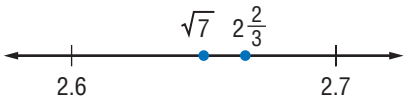
Replace each ● with <, >, or = to make a true sentence.

5 $\sqrt{7}$ ● $2\frac{2}{3}$

Write each number as a decimal.

$\sqrt{7} \approx 2.645751311\ldots$

$2\frac{2}{3} = 2.666666666\ldots$



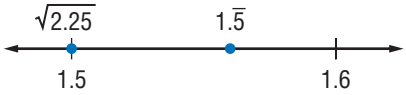
Since $2.645751311\ldots$ is less than $2.66666666\ldots$, $\sqrt{7} < 2\frac{2}{3}$.

6 $1.\overline{5}$ ● $\sqrt{2.25}$

Write $\sqrt{2.25}$ as a decimal.

$\sqrt{2.25} = 1.5$

$1.\overline{5} = 1.555555555\ldots$



Since $1.555555555\ldots$ is greater than 1.5, $1.\overline{5} > \sqrt{2.25}$.

CHECK Your Progress

g. $\sqrt{11}$ ● $3\frac{1}{3}$ h. $\sqrt{17}$ ● 4.03 i. $\sqrt{6.25}$ ● $2\frac{1}{2}$

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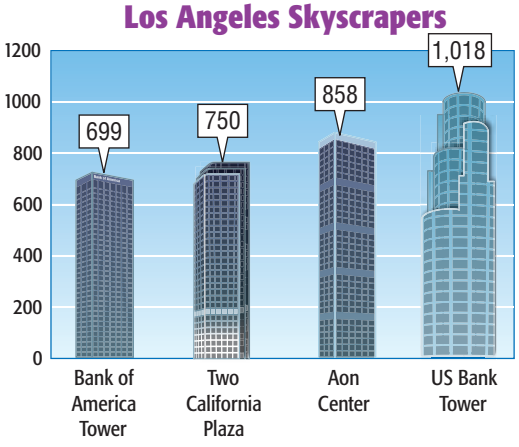
Real-World EXAMPLE

7 **SKYSCRAPERS** **On a clear day, the number of miles a person can see to the horizon is about 1.23 times the square root of his or her distance from the ground, in feet. Suppose Domingo is at the top of Bank of America Tower and Jewel is at the top of Two California Plaza. How much farther can Jewel see than Domingo?**



Source: National Park Service

Use a calculator to approximate the distance each person can see.

Domingo: $1.23\sqrt{699} \approx 32.52$ Jewel: $1.23\sqrt{750} \approx 33.68$

Jewel can see about $33.68 - 32.52$ or 1.16 miles farther than Domingo.

CHECK Your Progress

j. **MEASUREMENT** How much greater is the perimeter of a square with area 250 square meters than a square with area 125 square meters?

Personal Tutor

MathOnline **Personal Tutor at** ca.gr7math.com

Artiga Photo/CORBIS

CHECK Your Understanding

Examples 1–3 (p. 156) **Name all sets of numbers to which each real number belongs.**

1. $0.050505\ldots$
2. $-\sqrt{64}$
3. $\sqrt{17}$
4. $-3\frac{1}{4}$

Example 4 (p. 156) **Estimate each square root to the nearest tenth. Then graph the square root on a number line.**

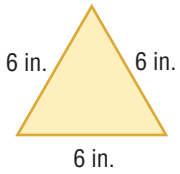
5. $\sqrt{2}$
6. $-\sqrt{18}$

Examples 5, 6 (p. 157) **Replace each ● with <, >, or = to make a true sentence.**

7. $\sqrt{15}$ ● 3.5
8. $\sqrt{2.25}$ ● $1\frac{1}{2}$
9. $2.\overline{21}$ ● $\sqrt{5.2}$

Example 7 (p. 157)

10. **MEASUREMENT** The area of a triangle with all three sides the same length is $\frac{s^2\sqrt{3}}{4}$, where s is the length of a side. Find the area of the triangle.



Exercises

HOMEWORK HELP

For Exercises	See Examples
11–18	1–3
19–22	4
23–28	5,6
29–30	7

Name all sets of numbers to which each real number belongs.

11. 14
12. $\frac{2}{3}$
13. $-\sqrt{16}$
14. $-\sqrt{20}$
15. 4.83
16. $7.\overline{2}$
17. $-\sqrt{90}$
18. $\frac{12}{4}$

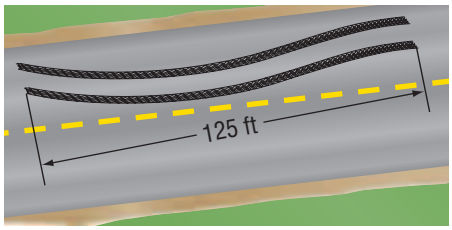
Estimate each square root to the nearest tenth. Then graph the square root on a number line.

19. $\sqrt{6}$
20. $\sqrt{8}$
21. $-\sqrt{22}$
22. $-\sqrt{27}$

Replace each ● with <, >, or = to make a true sentence.

23. $\sqrt{10}$ ● 3.2
24. $\sqrt{12}$ ● 3.5
25. $6\frac{1}{3}$ ● $\sqrt{40}$
26. $2\frac{2}{5}$ ● $\sqrt{5.76}$
27. $5\frac{1}{6}$ ● $5.1\overline{6}$
28. $\sqrt{6.2}$ ● $2.\overline{4}$

29. **LAW ENFORCEMENT** Police can use the formula $s = 5.5\sqrt{0.75d}$ to estimate the speed of a vehicle, where s is the speed of the vehicle in miles per hour, and d is the length of the skid marks in feet. How fast was the vehicle going for the skid marks at the right?



30. **FOOTBALL** The time t in seconds that a football remains in the air is $t = \sqrt{\frac{2y}{9.8}}$, where y is the initial height in meters of the football. Find the time to the nearest hundredth of a second that a football remains in the air if the initial height is 2 meters.

EXTRA PRACTICE

See pages 683, 710.

Math Online

Self-Check Quiz at ca.gr7math.com

31. **ALGEBRA** In the sequence 4, 12, ■, 108, 324, the missing number can be found by simplifying $\sqrt{ab}$ where a and b are the numbers on either side of the missing number. Find the missing number.

Order each set of numbers from least to greatest.

32. $\sqrt{5}, \sqrt{3}, 2.25, 2.\overline{2}$

33. $3.01, 3.\overline{1}, 3.\overline{01}, \sqrt{9}$

34. $-4.1, \sqrt{17}, -4.\overline{1}, 4.01$

35. $-\sqrt{5}, \sqrt{6}, -2.5, 2.5$

H.O.T. Problems

36. **OPEN ENDED** Give a counterexample for the statement *all square roots are irrational numbers*. Explain your reasoning.

CHALLENGE Tell whether the following statement is *always*, *sometimes*, or *never* true. If a statement is not always true, explain.

37. Integers are rational numbers.

38. Rational numbers are integers.

39. The product of a rational number and an irrational number is an irrational number.

40. **WRITING IN MATH** Write a real-world problem in which you would need to approximate a square root. Then, solve the problem.

STANDARDS PRACTICE

41. Which is an irrational number?

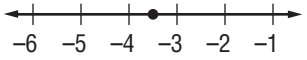
A -6

B $\frac{2}{3}$

C $\sqrt{9}$

D $\sqrt{3}$

42. Which number represents the point graphed on the number line?



F $-\sqrt{12}$

G $-\sqrt{10}$

H $-\sqrt{15}$

J $-\sqrt{8}$

Spiral Review

43. **SPORTS** Students were surveyed about the sports in which they participate. Thirty-five play baseball, 31 play basketball, and 28 play soccer. Of these, 7 play baseball and basketball, 9 play basketball and soccer, 6 play baseball and soccer, and 5 play all three sports. How many students were surveyed? Use a Venn diagram. (Lesson 3-3)

44. Order 7, $\sqrt{53}$, $\sqrt{32}$, and 6 from least to greatest. (Lesson 3-2)

ALGEBRA Solve each equation. (Lesson 3-1)

45. $t^2 = 25$

46. $y^2 = \frac{1}{49}$

47. $0.64 = a^2$

GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate each expression. (Lesson 2-9)

48. $3^2 + 5^2$

49. $6^2 + 4^2$

50. $9^2 + 11^2$

51. $4^2 + 7^2$

CHAPTER 3

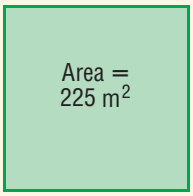
Mid-Chapter Quiz

Lessons 3-1 through 3-4

Find each square root. (Lesson 3-1)

1. $\sqrt{1}$
2. $\pm\sqrt{81}$
3. $\pm\sqrt{36}$
4. $-\sqrt{121}$
5. $-\sqrt{\frac{1}{25}}$
6. $\sqrt{0.09}$

7. **MEASUREMENT** What is the length of a side of the square? (Lesson 3-1)



8. **STANDARDS PRACTICE** The area of a square picture frame is 529 square centimeters. How long is each side of the frame? (Lesson 3-1)

   A 26 cm
   B 25 cm
   C 23 cm
   D 21 cm

9. **FOOTBALL** A group of 121 football players needs to be in a square formation for practice. How many players should be in each row? (Lesson 3-1)

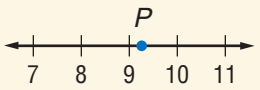
Estimate to the nearest whole number. (Lesson 3-2)

10. $\sqrt{90}$
11. $\sqrt{28}$
12. $\sqrt{226}$
13. $\sqrt{17}$
14. $\sqrt{21}$
15. $\sqrt{75}$

16. **ALGEBRA** Estimate the solution(s) of $x^2 = 50$ to the nearest integer. (Lesson 3-2)

17. **MEASUREMENT** The radius of a circle with area A is approximately $\sqrt{\frac{A}{3}}$. If a pie has an area of 42 square inches, estimate its radius. (Lesson 3-2)

18. **STANDARDS PRACTICE** Point P on the number line best represents which square root? (Lesson 3-2)



   F $\sqrt{85}$
   G $\sqrt{81}$
   H $\sqrt{98}$
   J $\sqrt{79}$

19. **MARKETING** A survey showed 83 customers bought wheat cereal, 83 bought rice cereal, and 20 bought corn cereal. Of those who bought exactly two boxes of cereal, 6 bought corn and wheat, 10 bought rice and corn, and 12 bought rice and wheat. Four customers bought all three. How many customers bought only rice cereal? (Lesson 3-3)

20. **FOOD** Napoli's pizza conducted a survey of 75 customers. The results showed that 35 customers liked mushroom pizza, 41 liked pepperoni pizza, and 11 liked both mushroom and pepperoni pizza. How many liked neither mushroom nor pepperoni pizza? Use a Venn diagram. (Lesson 3-3)

Name all sets of numbers to which each real number belongs. (Lesson 3-4)

21. $\frac{2}{3}$
22. $\sqrt{25}$
23. $-\sqrt{15}$
24. $\sqrt{3}$
25. 10
26. $-\sqrt{4}$

Replace each ● with <, >, or = to make a true sentence. (Lesson 3-4)

27. $\sqrt{15}$ ● 4.1
28. 6.5 ● $\sqrt{45}$
29. $\sqrt{35}$ ● 5.75
30. $3.\overline{3}$ ● $\sqrt{10}$

Explore 3-5

Geometry Lab
The Pythagorean Theorem

Main IDEA

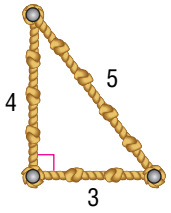
Find the relationship among the sides of a right triangle.

Standard 7MG3.3 **Know and understand the Pythagorean theorem** and its converse **and use it to find the length of the missing side of a right triangle** and the lengths of other line segments **and, in some situations, empirically verify the Pythagorean theorem by direct measurement.**
Standard 7MR2.4 **Make and test conjectures by using** both **inductive** and deductive **reasoning.**

REVIEW Vocabulary

right triangle a triangle with one angle that measures 90°

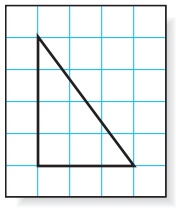
Four thousand years ago, the ancient Egyptians used mathematics to lay out their fields with square corners. They took a piece of rope and knotted it into 12 equal spaces. Taking three stakes, they stretched the rope around the stakes to form a right triangle. The sides of the triangle had lengths of 3, 4, and 5 units.



ACTIVITY

Concepts in Motion
BrainPOP® ca.gr7math.com

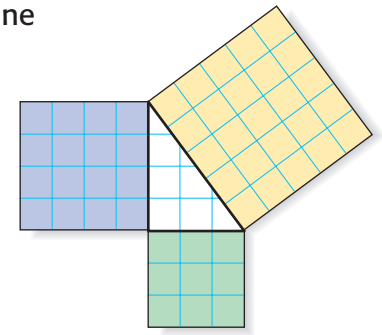
STEP 1 On centimeter grid paper, draw a triangle as shown at the right. Cut out the triangle.



STEP 2 Measure the length of the longest side in centimeters. In this case, it is 5 centimeters.

STEP 3 Cut out three squares: one with 3 centimeters on a side, one with 4 centimeters on a side, and one with 5 centimeters on a side.

STEP 4 Place the edges of the squares against the corresponding sides of the right triangle.



STEP 5 Find the area of each square.

Analyze the Results

1. What relationship exists among the areas of the three squares?

Repeat the activity for each right triangle whose two shorter sides have the following measures. Write an equation to show your findings. Use a ruler to verify your measures.

2. 6 cm, 8 cm
3. 5 cm, 12 cm
4. Write a sentence or two summarizing your findings.
5. **MAKE A CONJECTURE** Determine the length of the third side of a right triangle if the legs of the triangle are 9 inches and 12 inches long.

3-5 The Pythagorean Theorem

Main IDEA

Use the Pythagorean Theorem.

Standard 7MG3.3 Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.
Standard 7MR3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

NEW Vocabulary

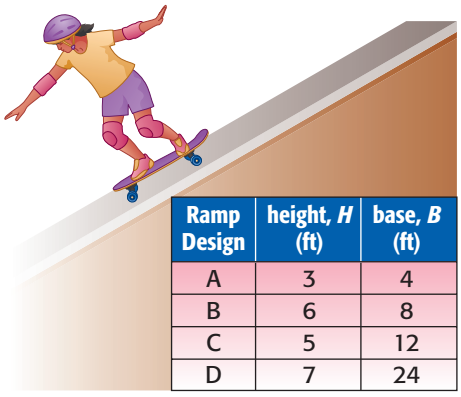
legs
hypotenuse
Pythagorean Theorem
converse

REVIEW Vocabulary

right angle an angle with a measure of 90°

MINI Lab

SPORTS When viewed from the side, the shape of some wooden skateboarding ramps is a right triangle. The dimensions of four possible ramps of this type are given in the table. Copy this table.



Ramp Design	height, H (ft)	base, B (ft)
A	3	4
B	6	8
C	5	12
D	7	24

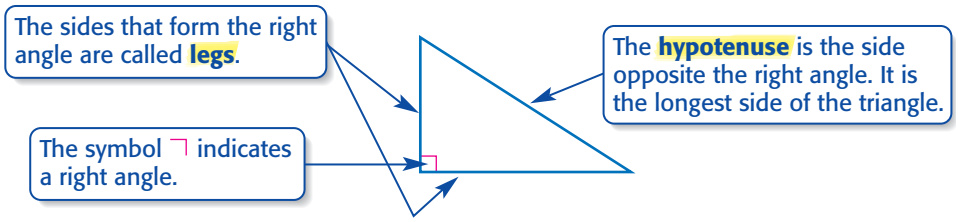
STEP 1 Draw a side-view model of each ramp on grid paper, letting the width of one grid equal 1 foot.

STEP 2 Cut each ramp out and use your grid paper to find the length of the ramp, which is the longest side of your model. Write these measures in a new column labeled *length, L (ft).*

STEP 3 Finally, add a column labeled $H^2 + B^2$. Calculate each of these values and place them in your table.

1. What is the relationship between the values in the $H^2 + B^2$ column and the values in the L column?
2. How could you use a value in the $H^2 + B^2$ column to find a corresponding value in the L column?

A right triangle is a triangle with one right angle.



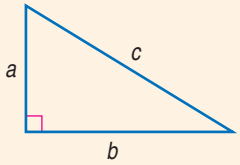
The **Pythagorean Theorem** describes the relationship between the lengths of the legs and the hypotenuse for *any* right triangle.

KEY CONCEPT Pythagorean Theorem

Words In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

Symbols $c^2 = a^2 + b^2$

Model

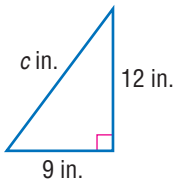


You can use the Pythagorean Theorem to find the length of a side of a right triangle when you know the other two sides.

EXAMPLES Find the Length of a Side

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

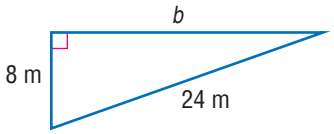
1



$c^2 = a^2 + b^2$	Pythagorean Theorem
$c^2 = 9^2 + 12^2$	Replace a with 9 and b with 12.
$c^2 = 81 + 144$	Evaluate 9^2 and 12^2.
$c^2 = 225$	Add 81 and 144.
$c = \pm\sqrt{225}$	Definition of square root
$c = 15$ or -15	Simplify.

The equation has two solutions, 15 and −15. However, the length of a side must be positive. So, the hypotenuse is 15 inches long.

2



$a^2 + b^2 = c^2$	Pythagorean Theorem
$8^2 + b^2 = 24^2$	Replace a with 8 and c with 24.
$64 + b^2 = 576$	Evaluate 8^2 and 24^2.
$64 - 64 + b^2 = 576 - 64$	Subtract 64 from each side.
$b^2 = 512$	Simplify.
$b = \pm\sqrt{512}$	Definition of square root
$b \approx 22.6$ or -22.6	Use a calculator.

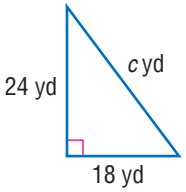
The length of side b is about 22.6 meters.

STUDY TIP

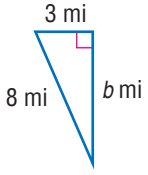
Check for Reasonableness
The hypotenuse is always the longest side in a right triangle. Since 22.6 is less than 24, the answer is reasonable.

CHECK Your Progress

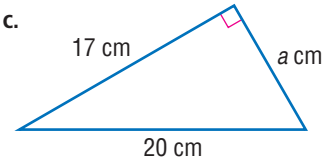
a.



b.



c.



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If you reverse the parts of the Pythagorean Theorem, you have formed its **converse**. The converse of the Pythagorean Theorem is also true.

KEY CONCEPT — Converse of Pythagorean Theorem

If the sides of a triangle have lengths a, b, and c units such that $c^2 = a^2 + b^2$, then the triangle is a right triangle.

EXAMPLE 3 Identify a Right Triangle

The measures of three sides of a triangle are 15 inches, 8 inches, and 17 inches. Determine whether the triangle is a right triangle.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$17^2 \stackrel{?}{=} 15^2 + 8^2$ $c = 17$, $a = 15$, $b = 8$

$289 \stackrel{?}{=} 225 + 64$ Evaluate 17^2, 15^2, and 8^2.

$289 = 289$ ✓ Simplify.

The triangle is a right triangle.

STUDY TIP

Draw a Picture
When solving a problem, it is often helpful to draw a picture to represent the situation.

CHECK Your Progress **Determine whether each triangle with sides of given lengths is a right triangle. Justify your answer.**

d. 36 mi, 48 mi, 60 mi

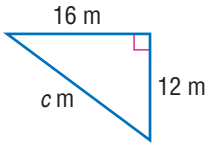
e. 4 ft, 7 ft, 5 ft

CHECK Your Understanding

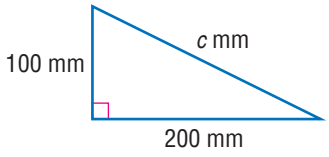
Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

Example 1 (p. 163)

1.

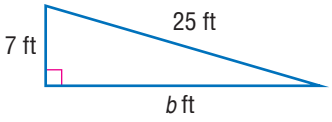


2.

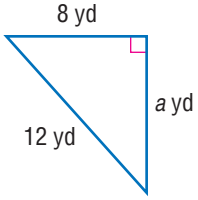


Example 2 (p. 163)

3.



4.



Example 1 (p. 163)

5. The hypotenuse of a right triangle is 12 inches, and one of its legs is 7 inches. Find the length of the other leg. Round to the nearest tenth if necessary.

Example 3 (p. 164)

Determine whether each triangle with sides of given lengths is a right triangle. Justify your answer.

6. 5 in., 10 in., 12 in.

7. 9 m, 40 m, 41 m

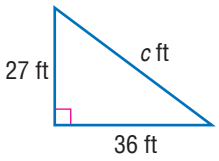
Exercises

HOMEWORK HELP

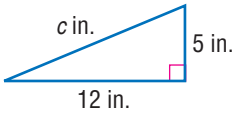
For Exercises	See Examples
8, 9	1
10–13	2
14–19	3

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

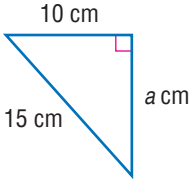
8.



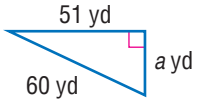
9.



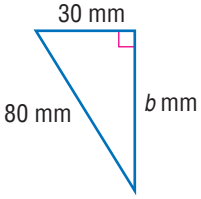
10.



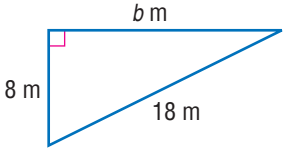
11.



12.



13.



Determine whether each triangle with sides of given lengths is a right triangle. Justify your answer.

14. 28 yd, 195 yd, 197 yd

15. 30 cm, 122 cm, 125 cm

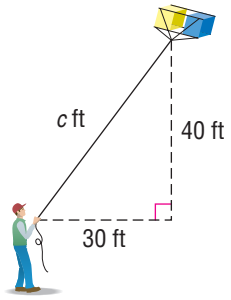
16. 24 m, 143 m, 145 m

17. 135 in., 140 in., 175 in.

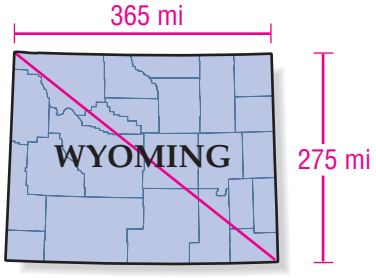
18. 56 ft, 65 ft, 16 ft

19. 44 cm, 70 cm, 55 cm

20. KITES Paulo is flying a kite as shown below. Find the length of the kite string.



21. GEOGRAPHY Calculate the length of the diagonal of the state of Wyoming.



Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

22. b, 99 mm; c, 101 mm

23. a, 48 yd; b, 55 yd

24. a, 17 ft; c, 20 ft

25. a, 23 in.; b, 18 in.

26. b, 4.5 m; c, 9.4 m

27. b, 5.1 m; c, 12.3 m

EXTRA PRACTICE

See pages 684, 710.

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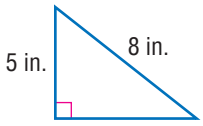
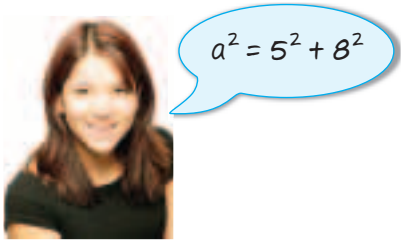
28. TRAVEL The Research Triangle in North Carolina is formed by Raleigh, Durham, and Chapel Hill. Is this triangle a right triangle? Explain.

H.O.T. Problems

29. **OPEN ENDED** State three measures that could be the side measures of a right triangle. Justify your answer.

30. **FIND THE ERROR** Catalina and Morgan are finding the length of the third side of the right triangle. Who is correct? Explain your reasoning.





Catalina

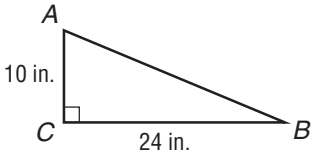
Morgan

31. **CHALLENGE** The whole numbers 3, 4, and 5 are called Pythagorean triples because they satisfy the Pythagorean Theorem. Find three other sets of Pythagorean triples.

32. **WRITING IN MATH** Explain why you can use any two sides of a right triangle to find the third side.

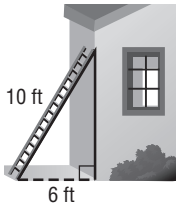
STANDARDS PRACTICE

33. What is the perimeter of the triangle ABC?



A 26 in. **C** 60 in.
B 34 in. **D** 68 in.

34. The base of a ten-foot ladder stands six feet from a house.



How many feet up the side of the house does the ladder reach?

F 4.0 ft **H** 8.0 ft
G 5.8 ft **J** 11.7 ft

Spiral Review

Replace each ● with <, >, or = to make each a true sentence. (Lesson 3-4)

35. $\sqrt{12}$ ● 3.5 36. $\sqrt{41}$ ● 6.4 37. $5.\overline{6}$ ● $\frac{17}{3}$ 38. $\sqrt{55}$ ● $7.\overline{4}$

39. **ALGEBRA** Estimate the solution of $x^2 = 77$ to the nearest integer. (Lesson 3-2)

GET READY for the Next Lesson

PREREQUISITE SKILL **Solve each equation. Check your solution.** (Lesson 1-9)

40. $57 = x + 24$ 41. $82 = 54 + y$ 42. $71 = 35 + z$ 43. $64 = a + 27$

3-6 Using The Pythagorean Theorem

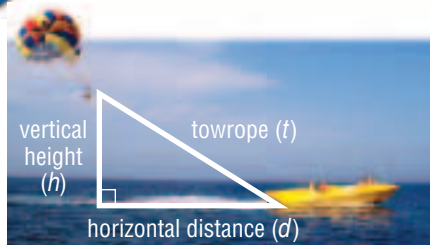
Main IDEA

Solve problems using the Pythagorean Theorem.

Standard 7MG3.3 Know and understand the Pythagorean theorem and its converse **and use it to find the length of the missing side of a right triangle and the lengths of other line segments** and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

GET READY for the Lesson

PARASAILING In parasailing, a towrope is used to attach a parasailor to a boat.

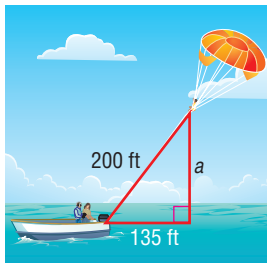


1. What type of triangle is formed by the horizontal distance, the vertical height, and the length of the towrope?
2. Write an equation that can be used to find the length of the towrope.

The Pythagorean Theorem can be used to solve a variety of problems.

Real-World EXAMPLE

1 **PARASAILING** **Find the height of the parasailor above the surface of the water.**



Notice that the vertical and horizontal distances, along with the length of the rope form a right triangle. Use the Pythagorean Theorem.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$200^2 = a^2 + 135^2$	Replace c with 200 and b with 135.
$40{,}000 = a^2 + 18{,}225$	Evaluate 200^2 and 135^2.
$40{,}000 - 18{,}225 = a^2 + 18{,}225 - 18{,}225$	Subtract 18,225 from each side.
$21{,}775 = a^2$	Simplify.
$\pm\sqrt{21{,}775} = a$	Definition of square root
148 or $-148 \approx a$	Simplify.

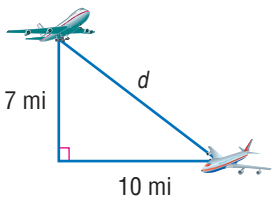
The parasailor is about 148 feet above the surface of the water.

STUDY TIP

Square Roots
In most real-world situations, only the positive square root is considered.

CHECK Your Progress

a. **AVIATION** Write an equation that can be used to find the distance between the planes. Then solve. Round to the nearest tenth.



STANDARDS EXAMPLE

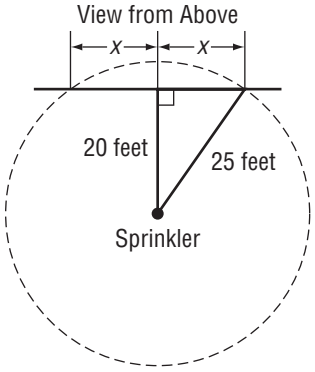
2 A circular lawn sprinkler with a range of 25 feet is placed 20 feet from the edge of a lawn. Find the length of the section of the lawn's edge that is within the range of the sprinkler.

A 15 ft

B 20 ft

C 25 ft

D 30 ft



Read the Item

From the diagram, you know that the distance of the sprinkler from the lawn's edge, the sprinkler's range, and a section of the lawn's edge all form a right triangle. The section of the lawn's edge within the range of the sprinkler is twice the section forming the right triangle.

Solve the Item

Use the Pythagorean Theorem.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$20^2 + x^2 = 25^2$ $a = 20$, $b = x$, and $c = 25$.

$400 + x^2 = 625$ Evaluate 20^2 and 25^2.

$400 - 400 + x^2 = 625 - 400$ Subtract 400 from each side.

$x^2 = 225$ Simplify.

$x = \pm\sqrt{225}$ Definition of square root

$x = 15$ or -15 Simplify.

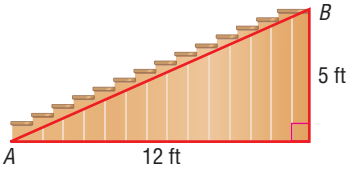
The length of the section of the lawn's edge within the sprinkler's range is $x + x$ or $15 + 15 = 30$ feet. Therefore, choice D is correct.

Test-Taking Tip

Pythagorean Triples Look for measures that are multiples of a 3-4-5 right triangle.
25 = 5 • 5
20 = 4 • 5
x = 3 • 5 or 15

CHECK Your Progress

b. If the "rise" of the stairs of a building is 5 feet and the "run" is 12 feet, how long is it from point A to point B?



F 13 ft

G 12 ft

H 11 ft

J 10 ft

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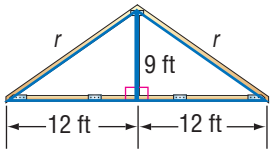
CHECK Your Understanding

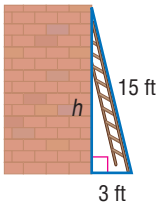
Example 1 (p. 167)

Write an equation that can be used to answer the question. Then solve. Round to the nearest tenth if necessary.

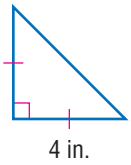
1. How long is each rafter?



2. How high does the ladder reach?

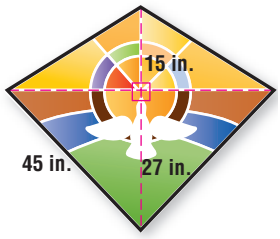


3. **GEOMETRY** An *isosceles* right triangle is a right triangle in which both legs are equal in length. If one leg of an isosceles triangle is 4 inches long, what is the length of the hypotenuse?



Example 2 (p. 168)

4. **STANDARDS PRACTICE** Abigail designed a stained glass window in the shape of a kite. What is the perimeter of the window?



A 108 in. **C** 162 in.

B 114 in. **D** 168 in.

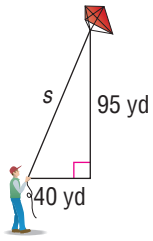
Exercises

HOMEWORK HELP

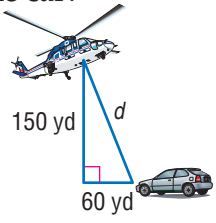
For Exercises	See Examples
5–10	1
21, 22	2

Write an equation that can be used to answer the question. Then solve. Round to the nearest tenth if necessary.

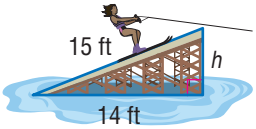
5. How long is the kite string?



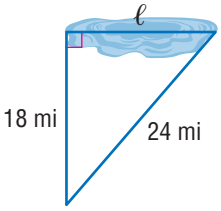
6. How far is the helicopter from the car?



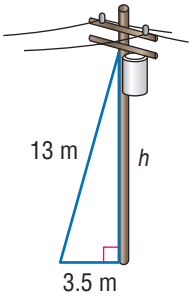
7. How high is the ski ramp?



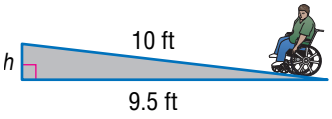
8. How long is the lake?



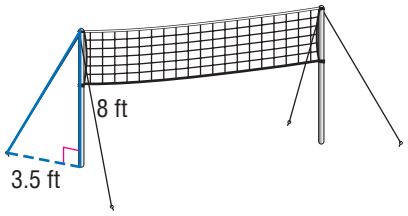
9. How high is the wire attached to the pole?



10. How high is the wheel chair ramp?



11. **VOLLEYBALL** Two ropes and two stakes are needed to support each pole holding the volleyball net. Find the length of each rope.

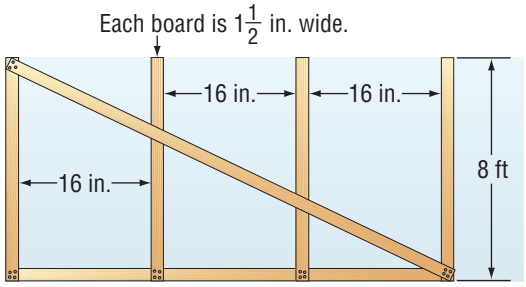


12. **GEOGRAPHY** Suppose Greenville, Rock Hill, and Columbia form a right triangle. What is the distance from Columbia to Greenville?

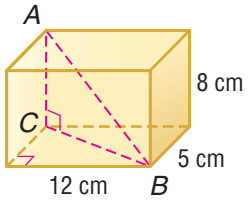
Real-World Link
Televisions are advertised by their diagonal measure. The most common sizes are 27–32 inches.

13. **ENTERTAINMENT** Connor loves to watch movies in the widescreen format on his television. He wants to buy a new television with a screen that is at least 25 inches by 13.6 inches. What diagonal size television meets Connor's requirements?

14. **CONSTRUCTION** Home builders add corner bracing to give strength to a house frame. How long will the brace need to be for the frame shown?



15. **GEOMETRY** Find the length of the diagonal $\overline{AB}$ in the rectangular prism at the right. (*Hint:* First find the length of $\overline{BC}$.)



EXTRA PRACTICE
See pages 684, 710.
MathOnline
Self-Check Quiz at ca.gr7math.com

16. **FIND THE DATA** Refer to the California Data File on pages 16–19. Choose some data and write a real-world problem in which you would use the Pythagorean Theorem.

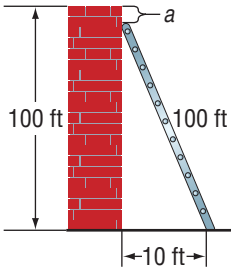
H.O.T. Problems

17. **OPEN ENDED** Write a problem that can be solved by using the Pythagorean Theorem. Then explain how to solve the problem.

18. **Which One Doesn't Belong?** Each set of numbers represents the side measures of a triangle. Identify the set that does not belong with the other three. Explain your reasoning.

5–12–13	10–24–26	5–7–9	8–15–17

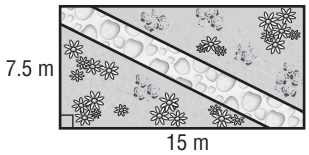
19. **CHALLENGE** Suppose a ladder 100 feet long is placed against a vertical wall 100 feet high. How far would the top of the ladder move down the wall by pulling out the bottom of the ladder 10 feet? Explain your reasoning.



20. **WRITING IN MATH** The length of the hypotenuse of an isosceles right triangle is $\sqrt{288}$ units. Explain how to find the length of a leg.

STANDARDS PRACTICE

21. Ms. Johnson designed a rectangular garden. She plans to build a walkway through the garden as shown.



Which measure is closest to the length of the walkway?

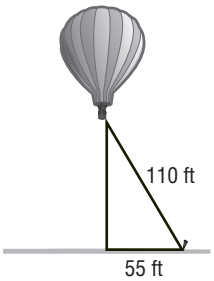
A 8 m

B 11 m

C 17 m

D 23 m

22. A hot air balloon is tethered to the ground as shown.



How high above the ground is the balloon?

F 55.0 ft

G 95.3 ft

H 123.0 ft

J 163.5 ft

Spiral Review

23. **GEOMETRY** Determine whether a triangle with sides 20 inches, 48 inches, and 52 inches long is a right triangle. Justify your anwer. (Lesson 3-5)

24. Order $\sqrt{45}$, $6.\overline{6}$, 6.75, and 6.7 from least to greatest. (Lesson 3-4)

Add or subtract. Write in simplest form. (Lesson 2-6)

25. $-3\frac{2}{3} + \left(-5\frac{3}{4}\right)$

26. $-1\frac{1}{8} - 7\frac{3}{4}$

27. $\frac{3}{5} - 4\frac{1}{2}$

28. $4\frac{7}{8} + \left(-6\frac{5}{6}\right)$

29. **ARCHAEOLOGY** Stone tools found in Ethiopia are estimated to be 2.5 million years old. That is about 700,000 years older than similar tools found in Tanzania. Write and solve an addition equation to find the age of the tools found in Tanzania. (Lesson 1-9)

GET READY for the Next Lesson

PREREQUISITE SKILL **Graph each point on the same coordinate plane.** (Page 686)

30. $T(5, 2)$

31. $A(-1, 3)$

32. $M(-5, 0)$

33. $D(-2, -4)$

Extend 3-6

Geometry Lab
Graphing Irrational Numbers

Main IDEA

Graph irrational numbers.

Standard 7MG3.2 **Understand and use coordinate graphs to** plot simple figures, **determine lengths** and areas related to them, and determine their image under translations and reflections. **Standard 7MR2.5** **Use a variety of methods such as** words, numbers, symbols, charts, graphs, tables, **diagrams, and models, to explain mathematical reasoning.**

In Lesson 3-2, you found approximate locations for irrational numbers on a number line. You can also accurately graph irrational numbers.

ACTIVITY

Graph $\sqrt{34}$ on a number line as accurately as possible.

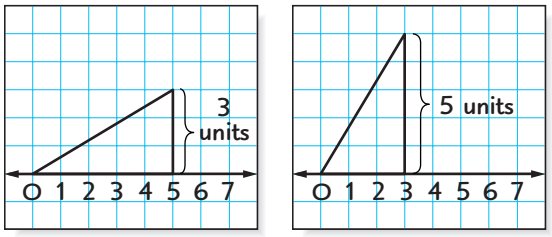
STEP 1 Find two numbers with squares that have a sum of 34.

$34 = 25 + 9$

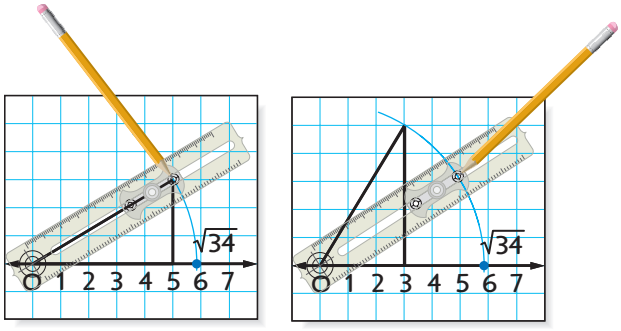
$34 = 5^2 + 3^2$

The hypotenuse of a triangle with legs that measure 5 and 3 units will measure $\sqrt{34}$ units.

STEP 2 Draw a number line on grid paper. Then draw a triangle whose legs measure 5 and 3 units.



STEP 3 Adjust your compass to the length of the hypotenuse. Place the compass at 0 and draw an arc that intersects the number line. The point of intersection corresponds to the number $\sqrt{34}$.



CHECK Your Progress **Graph each irrational number.**

a. $\sqrt{10}$ b. $\sqrt{13}$ c. $\sqrt{17}$ d. $\sqrt{8}$

Analyze the Results

1. Explain how you decide what lengths to make the legs of the right triangle when graphing an irrational number.
2. Explain how the graph of $\sqrt{2}$ can be used to graph $\sqrt{3}$.
3. **MAKE A CONJECTURE** Do you think you could graph the square root of any whole number? Explain your reasoning.

3-7 Geometry: Distance on the Coordinate Plane

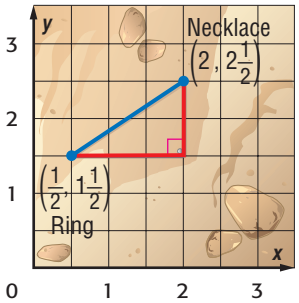
Main IDEA

Graph rational numbers on the coordinate plane. Find the distance between two points on the coordinate plane.

Standard 7MG3.2 **Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them,** and determine their image under translations and reflections.

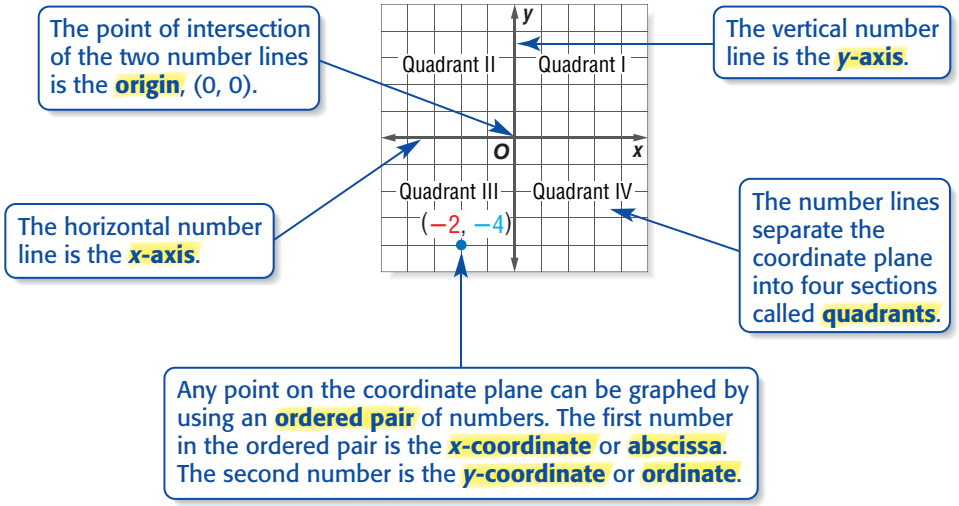
GET READY for the Lesson

ARCHAEOLOGY An archaeologist sets up a grid with string to keep careful record of the location of objects she finds at a dig. She finds a ring at $\left(\frac{1}{2}, 1\frac{1}{2}\right)$ and a necklace at $\left(2, 2\frac{1}{2}\right)$.



1. What does each colored line on the graph represent?
2. What type of triangle is formed by the lines?
3. What are the lengths of the two red lines?

Recall that you can locate a point by using a coordinate system similar to the grid used by the archaeologist. It is called a **coordinate plane**.



NEW Vocabulary

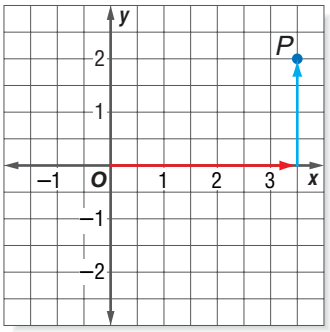
coordinate plane
origin
y-axis
x-axis
quadrants
ordered pair
x-coordinate
abscissa
y-coordinate
ordinate

EXAMPLES Name an Ordered Pair

1 **Name the ordered pair for point *P*.**

- Start at the origin.
- Move right to find the *x*-coordinate of point *P*, which is $3\frac{1}{2}$.
- Move up to find the *y*-coordinate, which is 2.

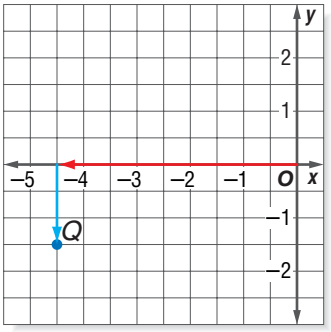
So, the ordered pair for point *P* is $\left(3\frac{1}{2}, 2\right)$.



2 **Name the ordered pair for point Q.**

- Start at the origin.
- Move left to find the x-coordinate of point Q, which is $-4\frac{1}{2}$.
- Move down to find the y-coordinate, which is $-1\frac{1}{2}$.

So, the ordered pair for point Q is $\left(-4\frac{1}{2}, -1\frac{1}{2}\right)$.



CHECK Your Progress

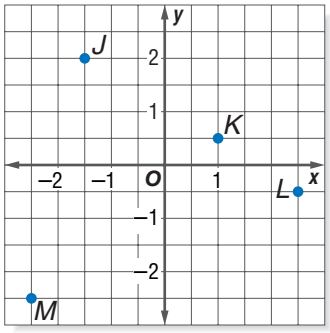
Name the ordered pair for each point.

a. J

b. K

c. L

d. M

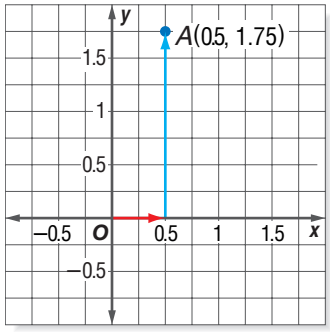


EXAMPLES Graphing Ordered Pairs

Graph and label each point.

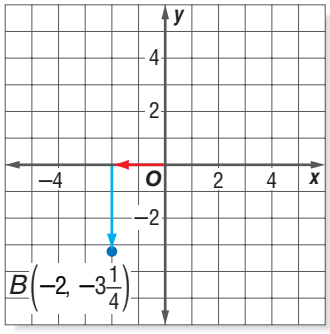
3 A**(0.5, 1.75)**

- Start at the origin and move 0.5 unit to the right. Then move up 1.75 units.
- Draw a dot and label it $A(0.5, 1.75)$.



4 $B\left(-2, -3\frac{1}{4}\right)$

- Start at the origin and move 2 units to the left. Then move down $3\frac{1}{4}$ or 3.25 units.
- Draw a dot and label it $B\left(-2, -3\frac{1}{4}\right)$.



CHECK Your Progress

Graph and label each point.

e. $R\left(2\frac{1}{4}, 3\frac{1}{2}\right)$ f. $S(-1.5, 3)$ g. $T\left(-\frac{1}{2}, -3\frac{3}{4}\right)$

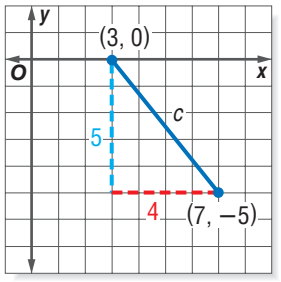
STUDY TIP

Look Back You can review **graphing integers on the coordinate plane** on page 668.

You can use the Pythagorean Theorem to find the distance between two points on the coordinate plane.

EXAMPLE Find Distance on the Coordinate Plane

5 **Graph the ordered pairs (3, 0) and (7, −5). Then find the distance c between the two points.**



$c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = 4^2 + 5^2$ Replace a with 4 and b with 5.

$c^2 = 41$ $4^2 + 5^2 = 16 + 25$ or 41

$\sqrt{c^2} = \pm\sqrt{41}$ Definition of square root

$c \approx \pm 6.4$ Use a calculator.

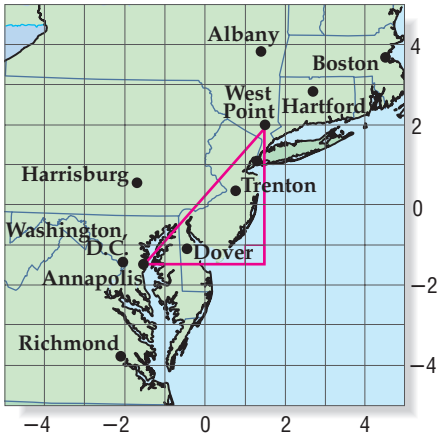
The points are about 6.4 units apart.

CHECK Your Progress **Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth.**

h. (2, 0), (5, −4) **i.** (1, 3), (−2, 4) **j.** (−3,−4), (2, −1)

Real-World EXAMPLE

6 **MAPS** **On the map, each unit represents 45 miles. West Point, New York, is located at $\left(1\frac{1}{2}, 2\right)$ and Annapolis, Maryland, is located at $\left(-1\frac{1}{2}, -1\frac{1}{2}\right)$. What is the approximate distance between West Point and Annapolis?**



Let c represent the distance between West Point and Annapolis. Then $a = 3$ and $b = 3.5$.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = 3^2 + 3.5^2$ Replace a with 3 and b with 3.5.

$c^2 = 21.25$ $3^2 + 3.5^2 = 9 + 12.25$ or 21.25

$\sqrt{c^2} = \pm\sqrt{21.25}$ Definition of square root

$c \approx \pm 4.6$ The map distance is about 4.6 units.

Since each map unit equals 45 miles, the distance between the cities is 4.6 • 45 or about 207 miles.

Real-World Link

The United States Military Academy, also known as West Point, graduates more than 900 officers each year. The same is true for the United States Naval Academy, which is located in Annapolis.

Source: www.usma.edu

CHECK Your Progress

k. **SPORTS** On a University of Southern California map, Cromwell field is located at $(2\frac{1}{2}, 3\frac{1}{2})$ and Dedeaux Field at $(1\frac{1}{2}, 4\frac{1}{2})$. Graph these points. If each map unit is 0.1 mile, about how far apart are the fields?

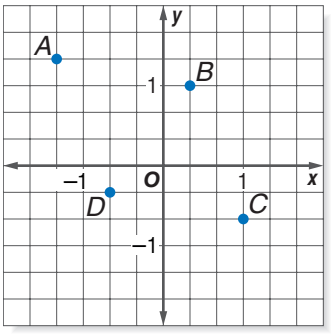
Personal Tutor

Online **Personal Tutor at** ca.gr7math.com

CHECK Your Understanding

Examples 1, 2 (pp. 173–174)

Name the ordered pair for each point.



1. A
2. B
3. C
4. D

Examples 3, 4 (p. 174)

Graph and label each point.

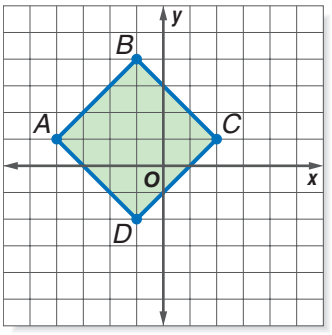
5. $J\left(\frac{1}{4}, 3\frac{1}{2}\right)$
6. $K\left(-1, -2\frac{3}{4}\right)$
7. $L(4.5, -2.25)$
8. $M(-2.5, 2.5)$

Example 5 (p. 175)

Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary.

9. $(1, 5), (3, 1)$
10. $(-1, 0), (2, 7)$
11. $(-5.5, -2), (2.5, 3)$

12. **GEOMETRY** Square $ABCD$ is graphed on the coordinate plane. What is the length of each side? What is the area? Round to the nearest tenth.



Example 6 (p. 175)

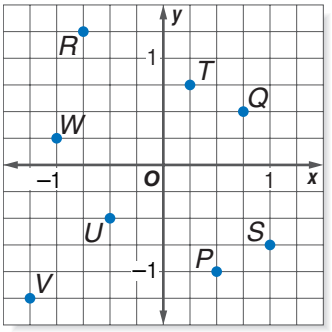
13. **PARKS** On a park map, the ranger station is located at (2.5, 3.5) and the nature center is located at (0.5, 4). Each unit in the map is equal to 0.5 mile. Graph the ordered pairs. What is the approximate distance between the ranger station and the nature center?

Exercises

HOMEWORK HELP

For Exercises	See Examples
14–21	1
22–27	2, 3
28–33	4, 5
34–35	6

Name the ordered pair for each point.



14. P
15. Q
16. R
17. S
18. T
19. U
20. V
21. W

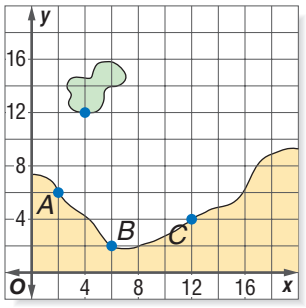
Graph and label each point.

22. $E\left(\frac{3}{4}, 2\frac{1}{4}\right)$
23. $F\left(\frac{2}{5}, 1\frac{1}{2}\right)$
24. $G\left(-3, 4\frac{2}{3}\right)$
25. $H\left(-2\frac{1}{4}, 3\frac{4}{5}\right)$
26. $J(4.3, -3.1)$
27. $K(-3.75, -0.5)$

Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary.

28. $(4, 5), (2, 2)$
29. $(6, 2), (1, 0)$
30. $(-3, 4), (1, 3)$
31. $(-5, 1), (2, 4)$
32. $(2.5, -1), (-3.5, -5)$
33. $(4, -2.3), (-1, -6.3)$

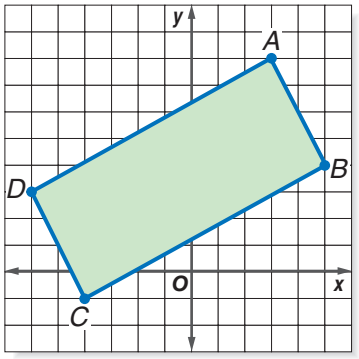
34. **NAVIGATION** A ferry sets sail from an island located at (4, 12) on the map at the right. Its destination is Ferry Landing B at (6, 2). How far will the ferry travel if each unit on the grid is 0.5 mile?



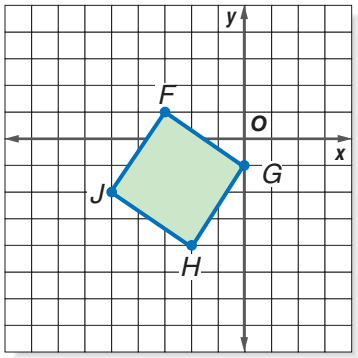
35. **GEOGRAPHY** On a map of Florida, Clearwater is located at (3, 2.5), and Jacksonville is located at (8.5, 14.5). Each unit on the map equals 16.5 miles. Graph the ordered pairs. What is the approximate distance between the cities?

Find the area of each rectangle.

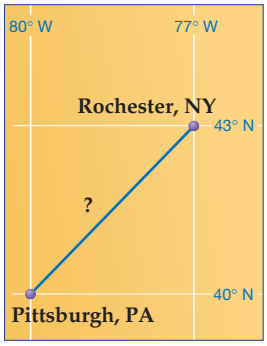
36.



37.



38. **TRAVEL** Rochester, New York, has a longitude of 77° W and a latitude of 43° N. Pittsburgh, Pennsylvania, is located at 80° W and 40° N. At this longitude/latitude, each degree is about 53 miles. Find the distance between Rochester and Pittsburgh.



39. **GEOMETRY** If one point is located at (−5, 4) and another point is located at (−8, −2), find the distance between the points.

EXTRA PRACTICE
See pages 684, 710.
Math Online
Self-Check Quiz at ca.gr7math.com

H.O.T. Problems

40. **CHALLENGE** Apply what you have learned about distance on the coordinate plane to determine the coordinates of the endpoints of a line segment that is neither horizontal nor vertical and has a length of 5 units.

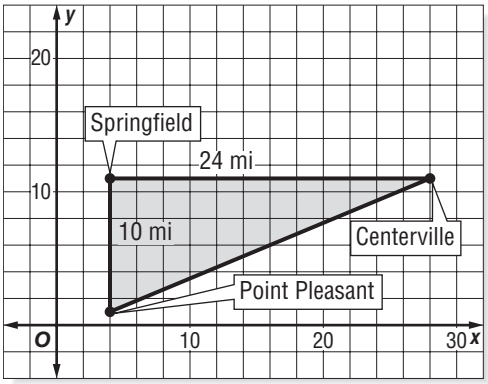
41. **SELECT A TOOL** Kendra needs to find the distance between the points $A(-2.4, 3.7)$ and $B(4.6, -1.3)$. Which of the following tools will be most useful to Kendra? Justify your selection(s). Then use the tool(s) to solve the problem.

calculator | paper and pencil | real objects

42. **WRITING IN MATH** In your own words, explain how to find the length of a non-vertical and a non-horizontal segment whose endpoints are (x_1, y_1) and (x_2, y_2).

STANDARDS PRACTICE

43. The map shows the location of the towns of Springfield, Centerville, and Point Pleasant.

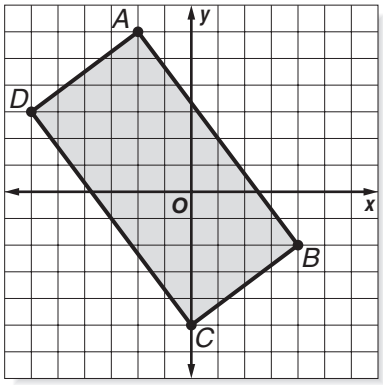


What is the shortest distance between Centerville and Point Pleasant?

A 14 mi **C** 26 mi

B 22 mi **D** 34 mi

44. Rectangle *ABCD* is graphed on the coordinate plane.



Find the area of rectangle *ABCD*.

F 30 units^2

G 50 units^2

H 60 units^2

J 100 units^2

Spiral Review

45. **HIKING** Hunter hikes 3 miles south and then turns and hikes 7 miles east. How far is he from his starting point? (Lesson 3-6)

GEOMETRY **Find the missing side of each right triangle. Round to the nearest tenth if necessary.** (Lesson 3-5)

46. a, 15 cm; b, 18 cm

47. b, 14 in.; c, 17 in.

48. a, 36 km; b, 40 km

49. **ENERGY** Electricity costs $6\frac{1}{2}¢$ per kilowatt-hour. Of that cost, $3\frac{1}{4}¢$ goes toward the cost of the fuel. What fraction of the cost goes toward fuel? (Lesson 2-4)

ALGEBRA **Write and solve an equation to find each number.** (Lesson 1-9)

50. The product of a number and 8 is 56.

51. The quotient of a number and 7 is −14.

Cross-Curricular Project

Math and Geography

Bon Voyage! It's time to complete your project. Use the information and data you have gathered about cruise packages and destination activities to prepare a video or brochure. Be sure to include a diagram and itinerary with your project.

MathOnline **Cross-Curricular Project at** ca.gr7math.com

CHAPTER 3

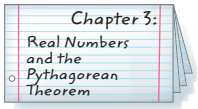
Study Guide and Review

Download Vocabulary Review from ca.gr7math.com

FOLDABLES Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

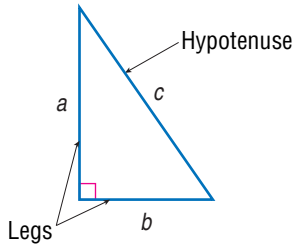
Square Roots and Irrational Numbers

(Lessons 3-1, 3-2, and 3-4)

- A square root of a number is one of its two equal factors.
- An irrational number is a number that cannot be expressed as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Pythagorean Theorem (Lessons 3-5 to 3-7)

- In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



- If the sides of a triangle have lengths a, b, and c units such that $c^2 = a^2 + b^2$, then the triangle is a right triangle.

Key Vocabulary

coordinate plane (p. 173)
hypotenuse (p. 162)
irrational number (p. 155)
legs (p. 162)
perfect square (p. 144)
Pythagorean Theorem (p. 162)
radical sign (p. 144)
real number (p. 155)
square root (p. 144)
Venn diagram (p. 152)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. The number 11 is a perfect square.
2. The symbol that is used to indicate a square root is the radical sign.
3. A real number is a number that cannot be expressed as a fraction.
4. If the measures of the sides of a triangle are 6 inches, 8 inches and 10 inches, then the triangle is a right triangle.
5. The opposite of squaring a number is finding a converse.
6. A Venn diagram uses overlapping rectangles to organize information and solve problems.
7. The hypotenuse is the shortest side of a right triangle.
8. The Pythagorean Theorem states that the sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.

CHAPTER 3

Study Guide and Review

Lesson-by-Lesson Review

3-1 Square Roots (pp. 144–147)

Find each square root.

9. $\sqrt{81}$
10. $\pm\sqrt{225}$
11. $-\sqrt{64}$
12. $\sqrt{6.25}$

13. **FARMING** Pecan trees are planted in square patterns to take advantage of land space and for ease in harvesting. For 289 trees, how many rows should be planted and how many trees should be planted in each row?

Example 1 **Find $\sqrt{36}$.**

Since $6^2 = 36$, $\sqrt{36} = 6$.

Example 2 **Find $-\sqrt{169}$.**

Since $(-13)(-13) = 169$, $-\sqrt{169} = -13$.

Example 3 **Find $\pm\sqrt{1.21}$.**

Since $(1.1)^2 = 1.21$ and $(-1.1)^2 = 1.21$, $\pm\sqrt{1.21} = \pm 1.1$.

3-2 Estimating Square Roots (pp. 148–151)

Estimate to the nearest whole number.

14. $\sqrt{32}$
15. $\sqrt{42}$
16. $\sqrt{230}$
17. $\sqrt{96}$
18. $\sqrt{150}$
19. $\sqrt{8}$
20. $\sqrt{50.1}$
21. $\sqrt{19.25}$

22. **ALGEBRA** Estimate the solution of $b^2 = 60$ to the nearest integer.

Example 4 **Estimate $\sqrt{135}$ to the nearest whole number.**

$121 < 135 < 144$ Write an inequality.

$11^2 < 135 < 12^2$ $121 = 11^2$ and $144 = 12^2$

$11 < \sqrt{135} < 12$ Take the square root of each number.

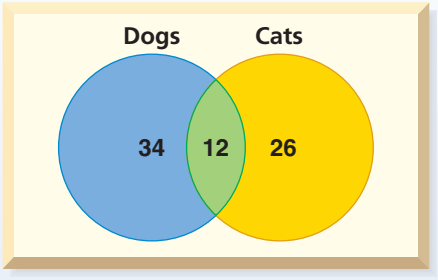
Since 135 is closer to 144 than to 121, the best whole number estimate is 12.

3-3 PSI: Use a Venn Diagram (pp. 152–153)

23. **APARTMENTS** An apartment complex offers 15 apartments with a view of the river, 8 with two bedrooms, and 6 that have both selections. How many have only a view of the river?

24. **LANGUAGE** At Madison Middle School, 95% of the students speak English fluently, 65% speak Spanish fluently, and 60% speak both English and Spanish fluently. What percent of the students speak only Spanish fluently?

Example 5 **The Venn diagram shows the number of dog and cat owners.**



So, 34 people own only dogs, 26 people own only cats, and 12 own both.

Mixed Problem Solving
For mixed problem-solving practice, see page 710.

3-4 The Real Number System (pp. 155–159)

Name all sets of numbers to which each real number belongs.

25. $-\sqrt{19}$
26. $0.\overline{3}$
27. 7.43
28. -12
29. $\sqrt{32}$
30. 101

31. **MEASUREMENT** The area of a square vegetable garden is 360 square meters. To the nearest hundredth meter, what is the perimeter of the garden?

Example 6 **Name all sets of numbers to which $-\sqrt{33}$ belongs.**

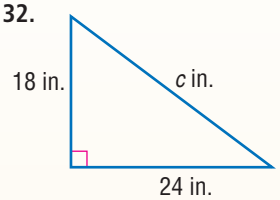
$-\sqrt{33} \approx -5.744562647$

Since the decimal does not terminate or repeat, it is an irrational number.

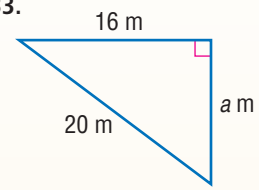
3-5 The Pythagorean Theorem (pp. 162–166)

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

32.



33.



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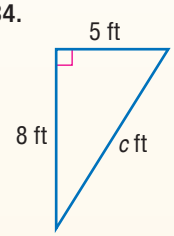
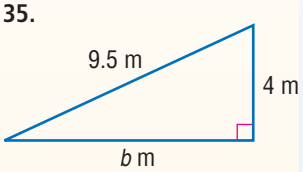


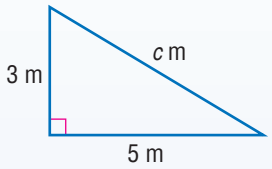
35.



36. a, 5 in.; c, 6 in.
37. a, 6 cm; b, 7 cm

38. **GEOMETRY** Lolita drew a right triangle where the hypotenuse was 17 inches and one of the legs was 8 inches. What was the length of the third side?

Example 7 **Write an equation you could use to find the length of the hypotenuse of the right triangle. Then find the missing length.**



$c^2 = a^2 + b^2$ Pythagorean Theorem
$c^2 = 3^2 + 5^2$ Replace a with 3 and b with 5.
$c^2 = 9 + 25$ Evaluate 3^2 and 5^2.
$c^2 = 34$ Simplify.
$c = \pm\sqrt{34}$ Definition of square root
$c \approx \pm 5.8$ Use a calculator.

The hypotenuse is about 5.8 meters long.

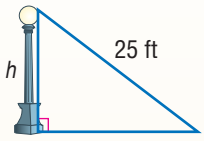
CHAPTER 3

Study Guide and Review

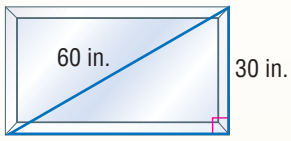
3-6 Using the Pythagorean Theorem (pp. 167–171)

Write an equation that can be used to answer the question. Then solve. Round to the nearest tenth if necessary.

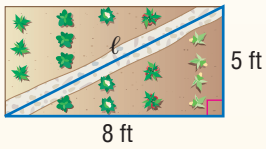
39. How tall is the light?



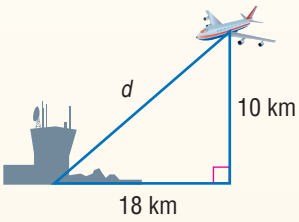
40. How wide is the window?



41. How long is the walkway?

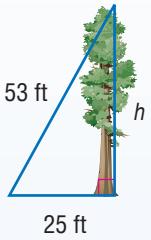


42. How far is the plane from the airport?



43. GEOMETRY A rectangle is 12 meters by 7 meters. What is the length of one of its diagonals?

Example 8 **Write an equation that can be used to find the height of the tree. Then solve.**



Use the Pythagorean Theorem to write the equation $53^2 = h^2 + 25^2$. Then solve the equation.

$$53^2 = h^2 + 25^2$$
$$2{,}809 = h^2 + 625$$
$$2{,}809 - 625 = h^2 + 625 - 625$$
$$2{,}184 = h^2$$
$$\pm\sqrt{2{,}184} = h$$
$$\pm 46.7 \approx h \quad \text{Use a calculator.}$$

The height of the tree is about 47 feet.

3-7 Geometry: Distance on the Coordinate Plane (pp. 173–178)

Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary.

44. (0, −3), (5, 5)

45. (−1, 2), (4, 8)

46. (−2, 1.5), (2, 3.6)

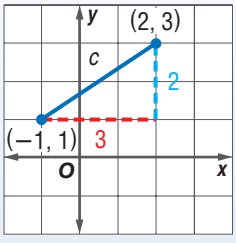
47. (−6, 2), (−4, 5)

48. (3, 4.2), (−2.1, 0)

49. (−1, 3), (2, 4)

50. GEOMETRY The coordinates of points R and S are (4, 3) and (1, 6). What is the distance between the points? Round to the nearest tenth if necessary.

Example 9 **Graph the ordered pairs (2, 3) and (−1, 1). Then find the distance between the points.**



$$c^2 = a^2 + b^2$$
$$c^2 = 3^2 + 2^2$$
$$c^2 = 9 + 4$$
$$c^2 = 13$$
$$c = \sqrt{13}$$
$$c \approx 3.6$$

The distance is about 3.6 units.

CHAPTER 3

Practice Test

Find each square root.

1. $\sqrt{225}$
2. $-\sqrt{0.25}$
3. $\pm\sqrt{\frac{36}{49}}$

4. **STANDARDS PRACTICE** Which list shows the numbers in order from least to greatest?

   A $2.\overline{2}, 2\frac{1}{5}, 2.25, \sqrt{5}$

   B $2\frac{1}{5}, 2.\overline{2}, \sqrt{5}, 2.25$

   C $\sqrt{5}, 2.25, 2\frac{1}{5}, 2.\overline{2}$

   D $2.25, \sqrt{5}, 2.\overline{2}, 2\frac{1}{5}$

Estimate to the nearest whole number.

5. $\sqrt{67}$
6. $\sqrt{118}$
7. $\sqrt{82}$

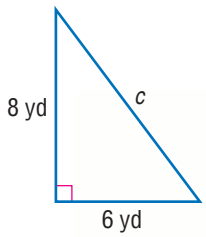
Name all sets of numbers to which each real number belongs.

8. $-\sqrt{64}$
9. $6.\overline{13}$
10. $\sqrt{14}$

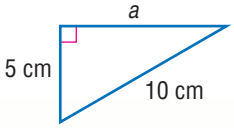
11. **FOOD** Gino's Pizzeria conducted a survey of 50 customers. The results showed that 15 people liked cheese pizza and 25 liked pepperoni. Of those customers, 4 people liked both cheese and pepperoni pizza. How many people liked neither cheese nor pepperoni pizza? Use a Venn diagram.

Write an equation you could use to find each length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

12.



13.

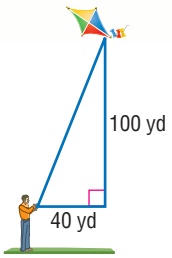


14. a, 55 in.; b, 48 in.
15. b, 12 ft; c, 20 ft

Determine whether each triangle with sides of given lengths is a right triangle. Justify your answer.

16. 12 in., 20 in., 24 in.
17. 34 cm, 30 cm, 16 cm
18. 15 ft, 25 ft, 20 ft
19. 7 yd, 14 yd, 35 yd

20. **STANDARDS PRACTICE** Justin is flying a kite.

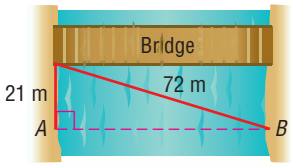


Which is closest to the length of the string?

F 70 yd
G 92 yd
H 108 yd
J 146 yd

21. **MEASUREMENT** Find the perimeter of a right triangle with legs of 10 inches and 8 inches.

22. **SURVEYING** A survey team calculated the distance across a river from point A to point B. How wide is the river at this point? Round to the nearest tenth.



Graph each pair of ordered pairs. Then find the distance between points. Round to the nearest tenth if necessary.

23. $(-2, -2), (5, 6)$
24. $\left(\frac{1}{3}, 1\right), \left(-1\frac{1}{3}, 1\frac{2}{3}\right)$
25. $(-0.5, 0.25), (0.25, -0.75)$

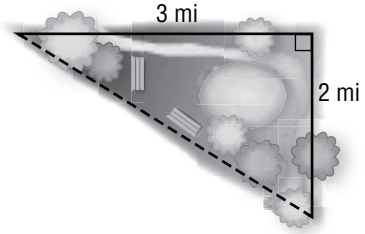
CHAPTER 3

California Standards Practice

Cumulative, Chapters 1–3

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1 Erin jogged along the track around the outer edge of a park. She ran two miles along the one edge and then 3 miles along the other edge. She then cut across the park as shown by the dotted line. How far did she jog to get back to her starting point?



A 3 miles
B 3.6 miles
C 5.2 miles
D 13 miles

2 Michelle had to choose the number closest to 5. Which irrational number should she choose?

F $\sqrt{30}$
G $\sqrt{27}$
H $\sqrt{20}$
J $\sqrt{18}$

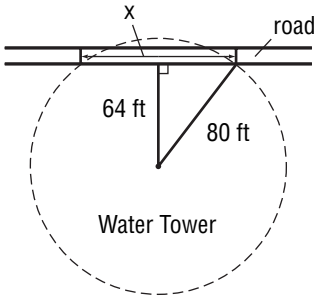
3 The Moon is about 3.84×10^5 kilometers from Earth. Which of the following represents this number in standard notation?

A 38,400,000 km
B 3,840,000 km
C 384,000 km
D 38,400 km

4 The square root of 250 is between

F 14 and 15.
G 15 and 16.
H 16 and 17.
J 17 and 18.

5 The proposed location of a new water tower intersects a section of an existing service road. Find x, the inside length of the section of road that is intersected by the water tower.



A 36 ft
B 48 ft
C 96 ft
D 112 ft

TEST-TAKING TIP

Question 5 Remember that the hypotenuse of a right triangle is always opposite the right angle.

6 Zack, Luke, and Charlie ordered a large pizza for $11.99, breadsticks for $2.99, and chicken wings for $5.99. If the three friends agree to split the cost of the food evenly, about how much will each friend pay?

F $20.79
G $7.93
H $7.32
J $6.99

7 Out of 100 students surveyed at Central Middle School, 48 are in the band, 52 play a sport, and 50 are in the drama club. Sixteen students are in both the band and the drama club, 22 students are in the drama club and play a sport, and 18 students are in the band and play a sport. Six students are in the band, play a sport, and are in the drama club. How many students are only in the drama club?

A 20 students
B 18 students
C 12 students
D 0 students

More California Standards Practice
For practice by standard, see pages CA1–CA39.

8 Molly multiplied her age by 3 and subtracted 2 from the product. She then divided the difference by 4, and added 7 to the quotient. The result was 14. Which could be the first step in finding Molly's age?

F Add 14 and 7.

G Subtract 7 from 14.

H Multiply 14 by 4.

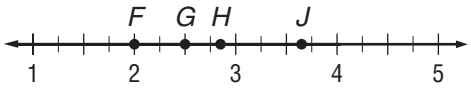
J Divide 14 by 3.

9 The diameter of a red blood cell is about 0.00074 centimeter. Which expression represents this number in scientific notation?

A 7.4×10^4 **C** 7.4×10^{-3}

B 7.4×10^3 **D** 7.4×10^{-4}

10 Which point on the number line best represents $\sqrt{8}$?



F point F **H** point H

G point G **J** point J

11 Ms. Leigh wants to organize the desks in the study hall into a square. If she has 64 desks, how many should be in each row?

A 7

B 8

C 9

D 10

12 On Monday, the high temperature in Las Vegas, Nevada, was 101°F, and the high temperature in Columbus, Ohio, was 76°F. How much warmer was it in Las Vegas than Columbus?

F 25°F **H** −25°F

G −15°F **J** 15°F

13 Which fraction is between $\frac{4}{5}$ and $\frac{5}{6}$?

A $\frac{1}{5}$ **C** $\frac{6}{7}$

B $\frac{7}{8}$ **D** $\frac{9}{11}$

14 Which of the following expressions results in a positive number?

F $2(-6)$ **H** $-2 + (-6)$

G $2 + (-6)$ **J** $2 - (-6)$

15 Student admission to the movies is $6.25. What is the total cost of tickets for you and four other students?

A $18.75 **C** $31.25

B $25.00 **D** $35.50

Pre-AP

Record your answers on a sheet of paper. Show your work.

16 Use a grid to graph and answer the following questions.

a. Graph the ordered pairs (3, 4) and (−2, 1).

b. Describe how to find the distance between the two points.

c. Find the distance between the points.

NEED EXTRA HELP?																
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Go to Lesson...	3-6	3-2	2-10	3-2	3-6	1-1	3-3	1-7	2-10	3-2	3-1	1-5	2-2	1-6	1-6	3-7
For Help with Standard...	MG3.3	NS2.4	NS1.1	NS2.4	MG3.3	NS1.2	MR2.5	AF1.1	NS1.1	NS2.4	NS2.4	NS1.2	NS1.1	NS1.7	NS1.2	MG3.2