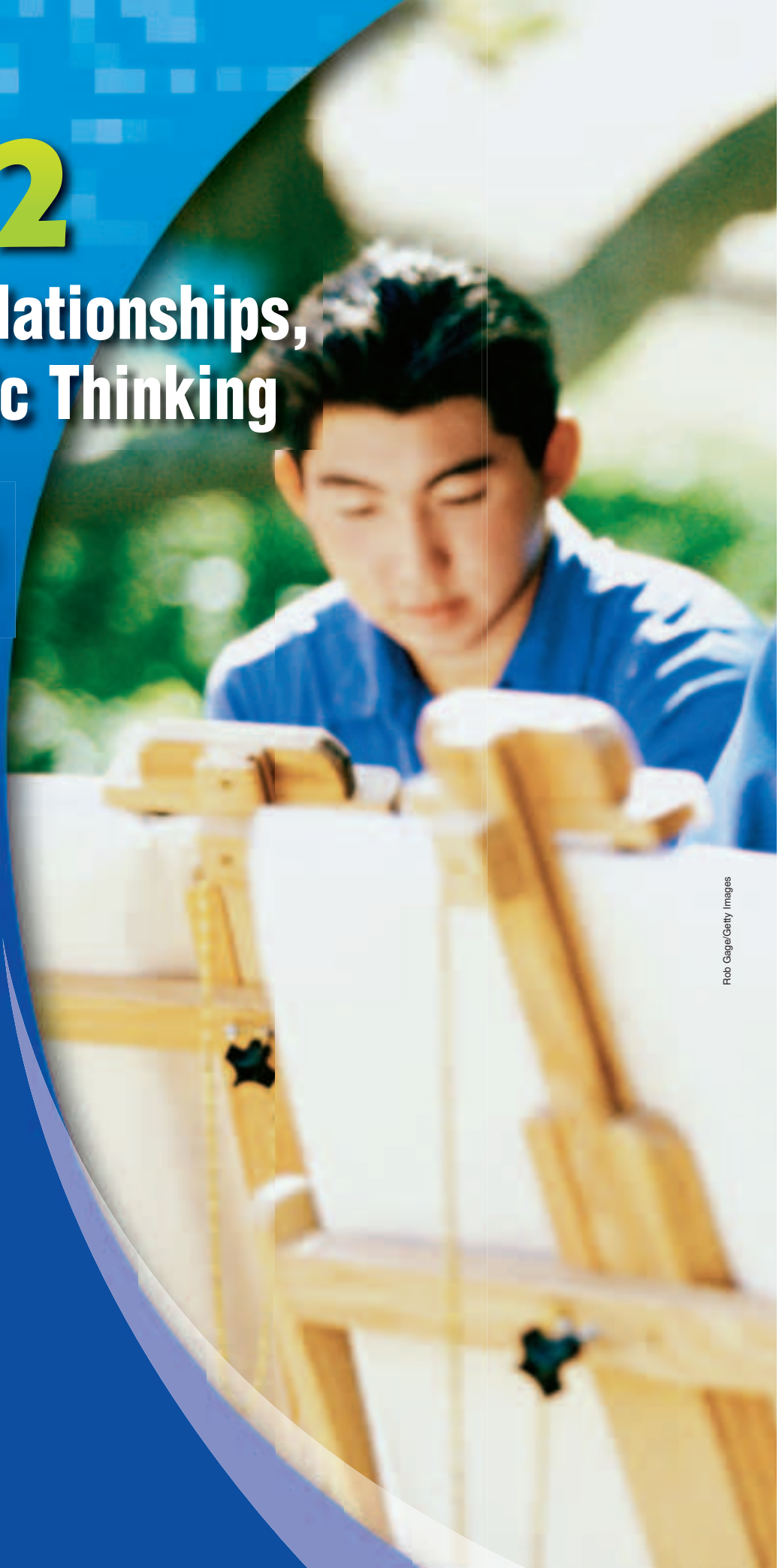
Unit 2

Patterns, Relationships, and Algebraic Thinking



Rob Gage/Getty Images

Focus

Compute with proportions and percents.

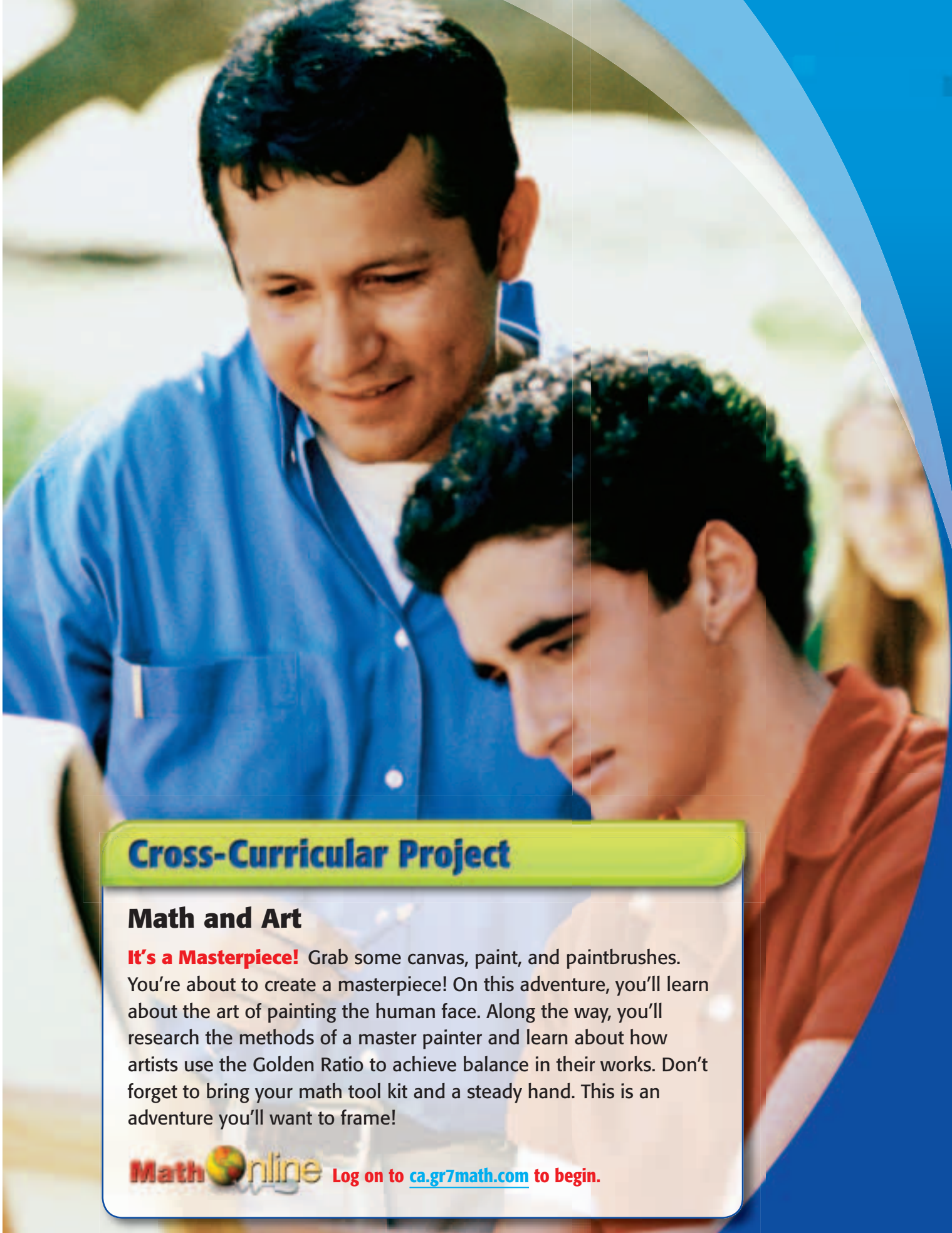
CHAPTER 4
Proportions and Similarity

BIG Idea Solve simple linear equations and inequalities over the rational numbers.

BIG Idea Choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems.

CHAPTER 5
Percent

BIG Idea Know the properties of, and compute with, rational numbers expressed in a variety of forms.



Cross-Curricular Project

Math and Art

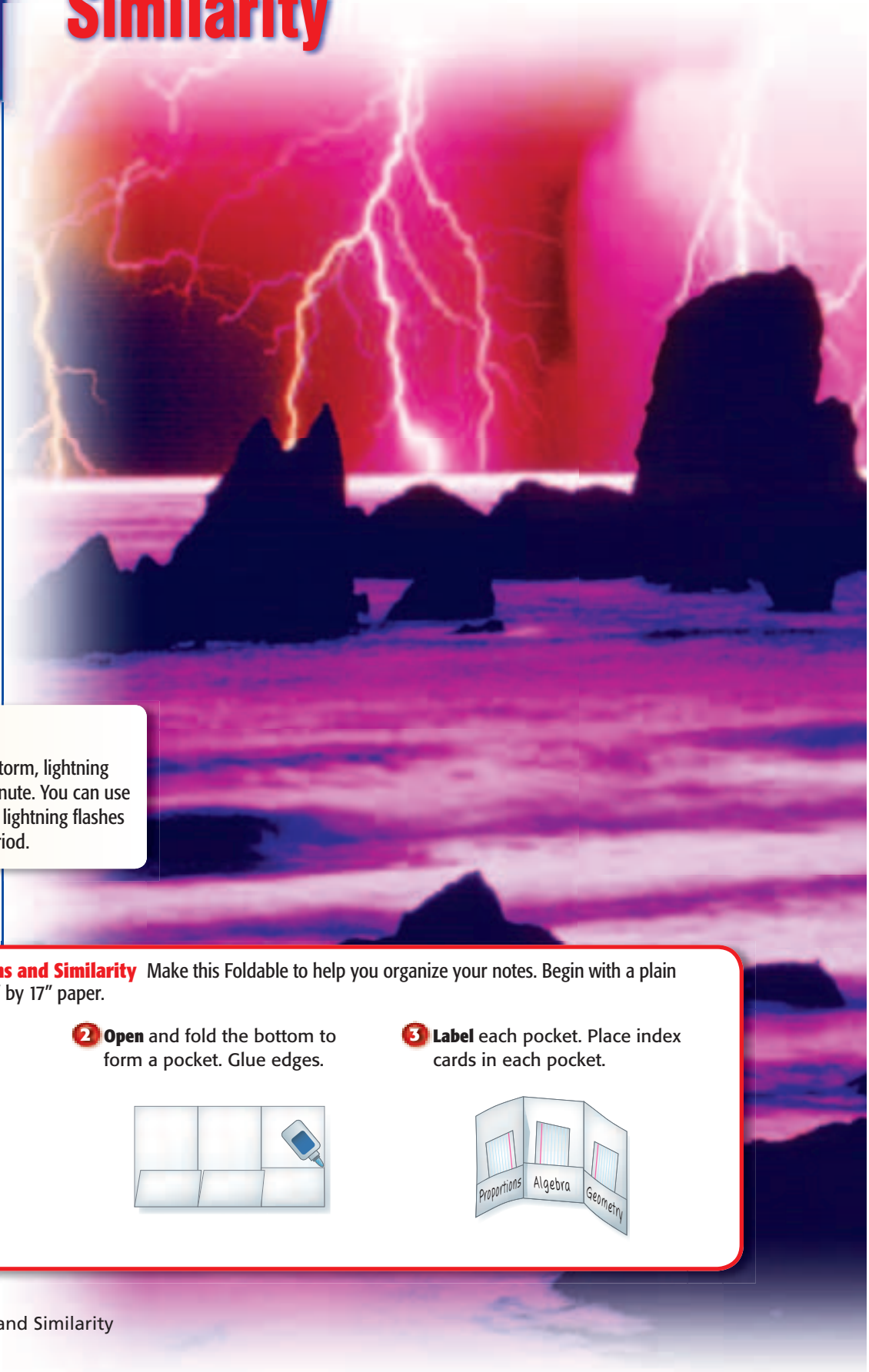
It's a Masterpiece! Grab some canvas, paint, and paintbrushes. You're about to create a masterpiece! On this adventure, you'll learn about the art of painting the human face. Along the way, you'll research the methods of a master painter and learn about how artists use the Golden Ratio to achieve balance in their works. Don't forget to bring your math tool kit and a steady hand. This is an adventure you'll want to frame!

MathOnline **Log on to ca.gr7math.com to begin.**

Rob Gage/Getty Images

CHAPTER 4

Proportions and Similarity



BIG Ideas

- **Standard 7AF4.0** Solve simple linear equations and inequalities over the rational numbers.
- **Standard 7MG1.0** Choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems.

Key Vocabulary

constant of proportionality (p. 200)
proportion (p. 198)
ratio (p. 190)
scale factor (p. 207)

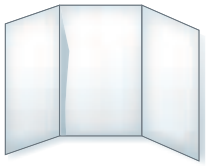
Real-World Link

Lightning During a severe thunderstorm, lightning flashed an average of 8 times per minute. You can use this rate to determine the number of lightning flashes that occurred during a 15-minute period.

FOLDABLES Study Organizer

Proportions and Similarity Make this Foldable to help you organize your notes. Begin with a plain sheet of 11″ by 17″ paper.

1. **Fold** in thirds widthwise.



2. **Open** and fold the bottom to form a pocket. Glue edges.

3. **Label** each pocket. Place index cards in each pocket.

Jim Zuckerman/CORBIS

GET READY for Chapter 4

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

Simplify each fraction. (Prior Grade)

1. $\frac{10}{24}$
2. $\frac{88}{104}$
3. $\frac{36}{81}$
4. $\frac{49}{91}$

5. **MONEY** Devon spent $18 of the $45 that he saved. Write a fraction in simplest form that represents the portion of his savings he spent. (Prior Grade)

Evaluate each expression. (Prior Grade)

6. $\frac{6-2}{5+5}$
7. $\frac{7-4}{8-4}$
8. $\frac{3-1}{1+9}$
9. $\frac{5+7}{8-6}$

Solve each equation. (Lessons 1-10)

10. $5 \cdot 6 = x \cdot 2$
11. $c \cdot 1.5 = 3 \cdot 7$
12. $12 \cdot z = 9 \cdot 4$
13. $7 \cdot 2 = 8 \cdot g$
14. $3 \cdot 11 = 4 \cdot y$
15. $b \cdot 6 = 7 \cdot 9$

16. **NUMBER SENSE** The product of a number and four is equal to the product of eight and twelve. Find the number. (Lessons 1-10)

QUICK Review

Example 1

Simplify $\frac{54}{81}$.

$\frac{54}{81} = \frac{2}{3}$ (÷ 27 above and ÷ 27 below)

Divide the numerator and denominator by their GCF, 27.

Example 2

Evaluate $\frac{11+4}{9-4}$.

$\frac{11+4}{9-4} = \frac{15}{5}$ Simplify the numerator and denominator.

$= 3$ Simplify.

Example 3

Solve $4 \cdot 6 = 8 \cdot p$.

$4 \cdot 6 = 8 \cdot p$ Write the equation.

$\frac{24}{8} = \frac{8p}{8}$ Multiply 4 by 6 and 8 by p.

$3 = p$ Divide each side by 8.

Option 2

MathOnline Take the Online Readiness Quiz at ca.gr7math.com.

4-1 Ratios and Rates

Main IDEA

Express ratios as fractions in simplest form and determine unit rates.

Standard 7AF4.2 Solve multistep problems involving rate, average speed, distance, and time or a direct variation.

Standard 7MG1.3 Use measures expressed as rates (e.g. speed, density) and measures expressed as products (e.g. person-days) **to solve problems; check the units of the solutions;** and use dimensional analysis to check the reasonableness of the answer.

NEW Vocabulary

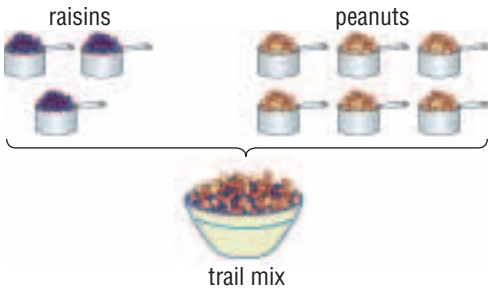
ratio
rate
unit rate

READING Math

Ratios In Example 1, the ratio 2 out of 7 means that for every 7 cats, 2 are Siamese.

GET READY for the Lesson

TRAIL MIX The diagram shows a batch of trail mix that is made using 3 scoops of raisins and 6 scoops of peanuts.



1. To make the batch of trail mix, how many scoops of raisins should you use for every 1 scoop of peanuts? Explain your reasoning.

A **ratio** is a comparison of two numbers or quantities by division. If a batch of trail mix contains 3 scoops of raisins and 6 scoops of peanuts, the ratio comparing raisins to peanuts can be written as follows.

$$3 \text{ to } 6 \qquad 3{:}6 \qquad \frac{3}{6}$$

Since a ratio can be written as a fraction, it can be simplified.

EXAMPLES Write Ratios in Simplest Form

Express each ratio in simplest form.

1 **8 Siamese cats out of 28 cats**

$\frac{8 \text{ cats}}{28 \text{ cats}} = \frac{2}{7}$ Divide the numerator and denominator by the greatest common factor, 4. Divide out common units.

The ratio of Siamese cats to cats is $\frac{2}{7}$ or 2 out of 7.

2 **10 ounces of butter to 1 pound of flour**

When writing ratios that compare quantities with the same kinds of units, convert so that they have the same unit.

$\frac{10 \text{ ounces}}{1 \text{ pound}} = \frac{10 \text{ ounces}}{16 \text{ ounces}}$ Convert 1 pound to 16 ounces.

$= \frac{5 \text{ ounces}}{8 \text{ ounces}}$ Divide the numerator and the denominator by 2. Divide out common units.

The ratio of butter to flour in simplest form is $\frac{5}{8}$ or 5:8.

CHECK Your Progress

a. 16 pepperoni pizzas out of 24 pizzas

b. 30 minutes of commercials to 2 hours of programming

A **rate** is a ratio that compares two quantities with different types of units such as $5 for 2 pounds or 130 miles in 2 hours. When a rate is simplified so it has a denominator of 1, it is called a **unit rate**. An example of a unit rate is *$6.50 per hour,* which means *$6.50 per 1 hour.*

EXAMPLE Find a Unit Rate

3 **TRAVEL** **Darrell drove 187 miles in 3 hours. What was Darrell's average rate of speed in miles per hour?**

Write the rate that expresses the comparison of miles to hours. Then find the average speed by finding the unit rate.

$$\frac{187 \text{ miles}}{3 \text{ hours}} \approx \frac{62 \text{ miles}}{1 \text{ hour}}$$

÷ 3 above the first fraction, ÷ 3 below the first fraction.

Divide the numerator and denominator by 3 to get a denominator of 1.

Darrell drove an average speed of about 62 miles per hour.

READING Math

Math Symbols The symbol ≈ is read *approximately equal to.*

CHECK Your Progress **Express each rate as a unit rate.**

c. 24 tickets for 8 rides

d. 4 inches of rain in 5 hours

Personal Tutor

Personal Tutor at ca.gr7math.com

Real-World EXAMPLE Compare Unit Rates

4 **CIVICS** **In 2000, the population of California was about 33,900,000, and the population of Kentucky was about 4,000,000. There were 53 members of the U.S. House of Representatives from California and 6 from Kentucky. In which state did a member represent more people?**

For each state, write a rate that compares the state's population to its number of representatives. Then find the unit rates.

$$\frac{33{,}900{,}000 \text{ people}}{53 \text{ representatives}} \approx \frac{640{,}000 \text{ people}}{1 \text{ representative}}$$

÷ 53 above, ÷ 53 below.

$$\frac{4{,}000{,}000 \text{ people}}{6 \text{ representatives}} \approx \frac{670{,}000 \text{ people}}{1 \text{ representative}}$$

÷ 6 above, ÷ 6 below.

A member represented more people in Kentucky than in California.

Real-World Link

In the U.S. House of Representatives, the number of representatives from each state is based on a state's population in the preceding census.

Source: www.house.gov

CHECK Your Progress

SHOPPING **Decide which is the better buy. Explain your reasoning.**

e. a 17-ounce box of cereal for $4.89 or a 21-ounce box for $5.69

f. 6 cans of green beans for $1 or 10 cans for $1.95

Peter Heimsath/Rex USA

CHECK Your Understanding

Examples 1, 2 (p. 190)

Express each ratio in simplest form.

1. 12 missed days out of 180 days
2. 12 wins to 18 losses
3. 6 inches of water for 7 feet of snow
4. 3 quarts of soda : 1 gallon of juice

Example 3 (p. 191)

Express each rate as a unit rate.

5. $50 for 4 days of work
6. 3 pounds of dog food in 5 days

Example 4 (p. 191)

7. **SHOPPING** You can buy 4 Granny Smith apples at Ben's Mart for $0.95. SaveMost sells 6 of the same quality apples for $1.49. Which store has the better buy? Explain your reasoning.

Exercises

HOMEWORK HELP

For Exercises	See Examples
8–11	1
12–15	2
16–21	3
22–23	4

Express each ratio in simplest form.

8. 14 chosen out of 70 who applied
9. 28 out of 100 doctors disagree
10. 33 stores open to 18 closed
11. 56 boys to 64 girls participated
12. 1 cup vinegar in 8 pints of water
13. 2 yards wide : 10 feet long
14. 20 centimeters out of 1 meter cut
15. 2,500 pounds for 1 ton of steel

16. **BASEBALL** In 2005, Hank Aaron was still the MLB career all-time hitter, with 3,771 hits in 3,298 games. What was Aaron's average number of hits per game?

17. **CARS** Manufacturers must publish a car's gas mileage or the average number of miles one can expect to drive per gallon of gasoline. The test of a new car resulted in 2,250 miles being driven using 125 gallons of gas. Find the car's expected gas mileage.

Real-World Link
Gas mileage can be improved by as much as 3.3% by keeping tires inflated to the proper pressure.
Source: www.fueleconomy.gov

Express each rate as a unit rate.

18. 153 points in 18 games
19. 350 miles on 15 gallons
20. 100 meters in 12 seconds
21. 1,473 people entered in 3 hours

22. **ELECTRONICS** A 20-gigabyte digital music player sells for $249. A similar 30-gigabyte player sells for $349. Which player offers the better price per gigabyte of storage? Explain.

23. **MEASUREMENT** Logan ran a 200-meter race in 25.24 seconds, and Scott ran a 400-meter race in 52.77 seconds. Who ran faster, Logan or Scott? Explain.

24. **MAGAZINES** Which costs more per issue, an 18-issue subscription for $40.50 or a 12-issue subscription for $33.60? Explain.

JupiterImages/Comstock

EXTRA PRACTICE

See pages 685, 711.

Math Online

Self-Check Quiz at ca.gr7math.com

25. **TRAVEL** Three people leave at the same time from Rawson to travel to Huntsville. Sarah averaged 45 miles per hour for the first third of the trip, 55 miles per hour for the second third, and 75 miles per hour for the last third. Darnell averaged 55 miles per hour for the first half of the trip and 70 miles per hour for the second half. Megan drove at a steady speed of 60 miles per hour the entire trip. Who arrived at Huntsville first? Explain.

H.O.T. Problems

26. **Which One Doesn't Belong?** Identify the situation that does not describe the same type of relationship as the other two. Explain your reasoning.

She reads two pages for every one page he reads.	Sam has three more chips than Wes has.	Jana has half as many CDs as Beth has.

27. **CHALLENGE** Luisa and Rachel have some trading cards. The ratio of Luisa's cards to Rachel's cards is 3:1. If Luisa gives Rachel 2 cards, the ratio will be 2:1. How many cards does Luisa have? Explain.

28. **WRITING IN MATH** Write about a real-world situation that can be represented by the ratio 2:5.

STANDARDS PRACTICE

29. Lucy typed 210 words in 5 minutes, and Yvonne typed 336 words in 8 minutes. Based on these rates, which statement is true?

A Lucy's rate was 3-words-per-minute slower than Yvonne's.

B Lucy's rate was 25.2-words-per-minute faster than Yvonne's.

C Lucy's rate was about 15.8-words-per-minute faster than Yvonne's.

D Lucy's rate was equal to Yvonne's.

30. Jackson drove 70 miles per hour for 4 hours and then 55 miles per hour for 2 hours to go to a conference. How far did Jackson drive in all?

F 390 miles

G 360 miles

H 320 miles

J 280 miles

Spiral Review

GEOMETRY **Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth.** (Lesson 3-7)

31. (1, 4), (6, −3)

32. (−1, 5), (3, −2)

33. (−5, −2), (−1, 0)

34. (−2, −3), (3, 1)

35. **MEASUREMENT** A square floor exercise mat measures 40 feet on each side. Find the length of the mat's diagonal. (Lesson 3-6)

GET READY for the Next Lesson

PREREQUISITE SKILL **Write each expression as a decimal.** (Lesson 2-1)

36. $\frac{19}{5}$

37. $\frac{3}{8}$

38. $\frac{12.4}{4}$

39. $\frac{2.5}{5}$

4-2 Proportional and Nonproportional Relationships

Main IDEA

Identify proportional and nonproportional relationships.

Preparation for Standard 7AF3.4 Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities.

GET READY for the Lesson

PIZZA Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges $8 for each medium pizza.

1. Copy and complete the table to determine the cost for different numbers of pizzas ordered.

Pizzas Ordered	1	2	3	4
Cost ($)	8			

2. For each number of pizzas, write the relationship of the cost and number of pizzas as a ratio in simplest form. What do you notice?

In the example above, notice that while the number of pizzas ordered and the cost both change or vary, the ratio of these quantities remains the same, a constant $8 per pizza.

$$\frac{\text{cost of order}}{\text{pizzas ordered}} = \frac{8}{1} = \frac{16}{2} = \frac{24}{3} = \frac{32}{4} \text{ or } \$8 \text{ per pizza}$$

This relationship is expressed by saying that the cost of an order is *proportional* to the number of pizzas ordered.

If two quantities are **proportional**, then they have a constant ratio. For relationships in which this ratio is not constant, the two quantities are said to be **nonproportional**.

NEW Vocabulary

proportional
nonproportional

EXAMPLES Identify Proportional Relationships

1 **PIZZA** **Uptown Pizzeria sells medium pizzas for $7 each but charges a $3 delivery fee per order. Is the cost of an order proportional to the number of pizzas ordered?**

Find the cost for 1, 2, 3, and 4 pizzas and make a table to display numbers and cost.

Cost ($)	10	17	24	31
Pizzas Ordered	1	2	3	4

For each number of pizzas, write the relationship of the cost and number of pizzas as a ratio in simplest form.

$\frac{\text{cost of order}}{\text{pizzas ordered}} \rightarrow \frac{10}{1}$ or 10 $\quad \frac{17}{2}$ or 8.5 $\quad \frac{24}{3}$ or 8 $\quad \frac{31}{4}$ or 7.75

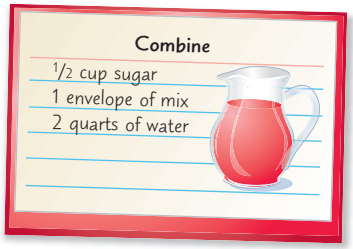
Since the ratios of the two quantities are not the same, the cost of an order is *not* proportional to the number of pizzas ordered. The relationship is nonproportional.

STUDY TIP

Common Error Even though there may be an adding pattern in both sets of values, a proportional relationship may not exist. In Example 1, as the number of pizzas increases by 1, the cost increases by 7, but the ratio of these values is not the same.

MathOnline Extra Examples at ca.gr7math.com

2 **BEVERAGES** **You can use the recipe shown to make a healthier version of a popular beverage. Is the amount of mix used proportional to the amount of sugar used?**



Find the amount of mix and sugar needed for different numbers of batches and make a table to show these mix and sugar measures.

Cups of Sugar	$\frac{1}{2}$	1	$1\frac{1}{2}$	2
Envelopes of Mix	1	2	3	4
Quarts of Water	2	4	6	8

For each number of cups of sugar, write the relationship of the cups and number of envelopes of mix as a ratio in simplest form.

$$\frac{\text{cups of sugar}}{\text{envelopes of mix}} \rightarrow \frac{\frac{1}{2}}{1} = \frac{0.5}{1} \text{ or } 0.5 \quad \frac{1}{2} \text{ or } 0.5 \quad \frac{1\frac{1}{2}}{3} = \frac{1.5}{3} \text{ or } 0.5 \quad \frac{2}{4} \text{ or } 0.5$$

Since the ratios between the two quantities are all equal to 0.5, the amount of mix used is proportional to the amount of sugar used.

READING in the Content Area

For strategies in reading this lesson, visit ca.gr7math.com.

CHECK Your Progress

a. **BEVERAGES** In Example 2, is the amount of sugar used proportional to the amount of water used?

b. **MONEY** At the beginning of the school year, Isabel had $120 in the bank. Each week, she deposits another $20. Is her account balance proportional to the number of weeks since she started school?

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CHECK Your Understanding

Examples 1, 2 (pp. 194–195)

1. **ELEPHANTS** An adult elephant drinks about 225 liters of water each day. Is the number of days that an elephant's water supply lasts proportional to the number of liters of water the elephant drinks?

2. **PACKAGES** A package shipping company charges $5.25 to deliver a package. In addition, they charge $0.45 for each pound over one pound. Is the cost to ship a package proportional to the weight of the package?

3. **SCHOOL** At a certain middle school, every homeroom teacher is assigned 28 students. There are 3 teachers who do not have a homeroom. Is the number of students at this school proportional to the number of teachers?

4. **JOBS** Andrew earns $18 per hour for mowing lawns. Is the amount of money he earns proportional to the number of hours he spends mowing?

Exercises

HOMEWORK HELP

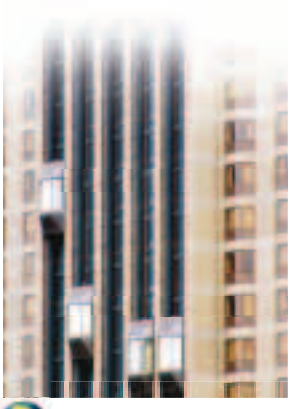
For Exercises	See Examples
5–12	1, 2

5. **RECREATION** The Vista Marina rents boats for $25 per hour. In addition to the rental fee, there is a $12 charge for fuel. Is the number of hours you can rent the boat proportional to the total cost?

6. **ELEVATORS** An elevator *ascends* or goes up at a rate of 750 feet per minute. Is the height to which the elevator ascends proportional to the number of minutes it takes to get there?

7. **PLANTS** Kudzu is a vine that grows an average of 7.5 feet every 5 days. Is the number of days of growth proportional to the length of the vine as measured on the last day?

8. **TEMPERATURE** To convert a temperature in degrees Celsius to degrees Fahrenheit, multiply the Celsius temperature by $\frac{9}{5}$ and then add 32°. Is a temperature in degrees Fahrenheit proportional to its equivalent temperature in degrees Celsius?



Real-World Link
Ascending at a speed of 1,000 feet per minute, the five outside elevators of the Westin St. Francis are the fastest glass elevators in San Francisco.
Source: sfvisitor.org

ADVERTISING **For Exercises 9 and 10, use the following information.**
On Saturday, Querida gave away 416 coupons for a free appetizer at a local restaurant. The next day, she gave away about 52 coupons an hour.

9. Is the number of coupons Querida gave away on Sunday proportional to the number of hours she worked that day?

10. Is the total number of coupons Querida gave away on Saturday and Sunday proportional to the number of hours she worked on Sunday?

SHOPPING **For Exercises 11 and 12, use the following information.**
MegaMart collects a sales tax equal to $\frac{1}{16}$ of the retail price of each purchase and sends this money to the state government.

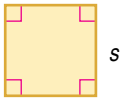
11. Is the amount of tax collected proportional to the cost of an item before tax is added?

12. Is the amount of tax collected proportional to the cost of an item after tax has been added?

MEASUREMENT **For Exercises 13 and 14, determine whether the measures described for the figure shown are proportional.**

13. the length of a side and the perimeter

14. the length of a side and the area



POSTAGE **For Exercises 15 and 16, use the table below that shows the price to mail a first-class letter for various weights.**

15. Is the cost to mail a letter proportional to its weight? Explain your reasoning.

16. Find the cost to mail a letter that weighs 5 ounces. Justify your answer.

Weight (oz)	1	2	3	4	5
Cost ($)	0.39	0.63	0.87	1.11	

EXTRA PRACTICE
See pages 685, 711.
Math Online
Self-Check Quiz at ca.gr7math.com

age fotostock/SuperStock

H.O.T. Problems

17. **OPEN ENDED** Give one example of a proportional relationship and one example of a nonproportional relationship. Justify your examples.

18. **CHALLENGE** This year Andrea celebrated her 10th birthday, and her brother Carlos celebrated his 5th birthday. Andrea noted that she was now twice as old as her brother was. Is the relationship between their ages proportional? Explain your reasoning using a table of values.

19. **WRITING IN MATH** Luke uses $200 in birthday money to purchase some $20 DVDs. He claims that the amount of money remaining after his purchase is proportional to the number of DVDs he decides to buy, because the DVDs are each sold at the same price. Is his claim valid? If his claim is false, name two quantities in this situation that are not proportional.

STANDARDS PRACTICE

20. Mr. Martinez is comparing the price of oranges from several different markets. Which market's pricing guide is based on a constant unit price?

A

Farmer's Market	
Number of Oranges	Total Cost ($)
5	3.50
10	6.00
15	8.50
20	11.00

C

Central Produce	
Number of Oranges	Total Cost ($)
5	3.00
10	6.00
15	9.00
20	12.00

B

The Fruit Place	
Number of Oranges	Total Cost ($)
5	3.50
10	6.50
15	9.50
20	12.50

D

Green Grocer	
Number of Oranges	Total Cost ($)
5	3.00
10	5.00
15	7.00
20	9.00

Spiral Review

Express each ratio in simplest form. (Lesson 4-1)

21. 40 working hours out of 168 hours

22. 2 inches of shrinkage to 1 yard of material

23. **GEOMETRY** The vertices of right triangle ABC are located at $A(-2, -5)$, $B(-2, 8)$, and $C(1, 4)$. Find the perimeter of the triangle. (Lesson 3-7)

ALGEBRA Write and solve an equation to find each number. (Lesson 1-10)

24. The product of -9 and a number is 45.

25. A number divided by 4 is -16.

GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation. Check your solution. (Lesson 1-9)

26. $5 \cdot x = 6 \cdot 10$

27. $8 \cdot 3 = 4 \cdot y$

28. $2 \cdot d = 3 \cdot 5$

29. $2.1 \cdot 7 = 3 \cdot a$

4-3 Solving Proportions

Main IDEA

Use proportions to solve problems.

Standard 7AF4.2 **Solve multistep problems involving** rate, average speed, distance, and time or **a direct variation.**

GET READY for the Lesson

NUTRITION Part of the nutrition label from a granola bar is shown at the right.

1. Write a ratio in simplest form that compares the number of Calories from fat to the total number of Calories.
2. Suppose you plan to eat two such granola bars. Write a ratio comparing the number of Calories from fat to the total number of Calories.
3. Is the number of Calories from fat proportional to the total number of Calories for one and two bars? Explain your reasoning.

Nutrition Facts
Serving Size 1 Bar (28g)
Servings Per Container 10

Amount Per Serving	
Calories 110	Calories from Fat 20
	% Daily Value*
Total Fat 2g	3%
Saturated Fat 0.5g	2%
Cholesterol 0mg	0%
Sodium 70mg	3%

In the example above, the ratios of Calories from fat to Calories for one or two granola bars are equal or **equivalent ratios** because they simplify to the same ratio, $\frac{2}{11}$. One way of expressing a proportional relationship like this is by writing a proportion.

$$\frac{20 \text{ Calories from fat}}{110 \text{ Calories}} = \frac{40 \text{ Calories from fat}}{220 \text{ Calories}}$$

NEW Vocabulary

equivalent ratios
proportion
cross products
constant of proportionality

KEY CONCEPT Proportion

Words A **proportion** is an equation stating that two ratios or rates are equivalent.

Examples

Numbers	Algebra
$\frac{6}{8} = \frac{3}{4}$	$\frac{a}{b} = \frac{c}{d}, b \neq 0, d \neq 0$

Consider the following proportion.

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a}{\cancel{b}_1} \cdot \cancel{b}^1 d = \frac{c}{\cancel{d}_1} \cdot b\cancel{d}^1$$ Multiply each side by bd and divide out common factors.

$$ad = bc$$ Simplify.

The products ad and bc are called the **cross products** of this proportion. The cross products of any proportion are equal. You can use cross products to *solve proportions* in which one of the quantities is not known.

$$\frac{6}{8} = \frac{3}{4} \quad \begin{matrix} \rightarrow 8 \cdot 3 = 24 \\ \rightarrow 6 \cdot 4 = 24 \end{matrix}$$ The cross products are equal.

STUDY TIP

Cross Products If the cross products of two ratios are equal, then the ratios form a proportion. If the cross products are *not* equal, the ratios do *not* form a proportion.

Concepts in Motion
Interactive Lab ca.gr7math.com

EXAMPLE Write and Solve a Proportion

1 TEMPERATURE **After 2 hours, the air temperature had risen 7°F. Write and solve a proportion to find the amount of time it will take at this rate for the temperature to rise an additional 13°F.**

Write a proportion. Let t represent the time in hours.

temperature → $\frac{7}{2} = \frac{13}{t}$ ← temperature
time → ← time

$\frac{7}{2} = \frac{13}{t}$ Write the proportion.

$7 \cdot t = 2 \cdot 13$ Find the cross products.

$7t = 26$ Multiply.

$\frac{7t}{7} = \frac{26}{7}$ Divide each side by 7.

$t \approx 3.7$ Simplify.

It will take about 3.7 hours to rise an additional 13°F.

CHECK Your Progress **Solve each proportion.**

a. $\frac{x}{4} = \frac{9}{10}$ b. $\frac{2}{34} = \frac{5}{y}$ c. $\frac{7}{3} = \frac{n}{2.1}$

Real-World Career . . .
How Does a Medical Technologist Use Math? A medical technologist uses proportional reasoning to analyze blood samples.

MathOnline
For more information, go to ca.gr7math.com.

You can use ratios to make predictions in situations involving proportions.

Real-World EXAMPLE

2 BLOOD **A microscope slide shows 37 red blood cells and 23 blood cells that are not red blood cells. How many red blood cells would be expected in a sample of the same blood that has 925 blood cells?**

red blood cells → $\frac{37}{23 + 37}$ or $\frac{37}{60}$
total blood cells →

Write and solve a proportion. Let r represent the number of red blood cells in the bigger sample.

red blood cells → $\frac{37}{60} = \frac{r}{925}$ ← red blood cells
total blood cells → ← total blood cells

$37 \cdot 925 = 60 \cdot r$ Find the cross products.

$34{,}225 = 60r$ Multiply.

$\frac{34{,}225}{60} = \frac{60r}{60}$ Divide each side by 60.

$570.4 \approx r$ Simplify.

You would expect to find about 570 red blood cells.

CHECK Your Progress

d. **RECYCLING** Recycling 2,000 pounds of paper saves about 17 trees. Write and solve a proportion to determine how many trees you would expect to save by recycling 5,000 pounds of paper.

Personal Tutor

MathOnline **Personal Tutor at** ca.gr7math.com

Matt Meadows

You can also use the constant ratio to write an equation expressing the relationship between two proportional quantities. The constant ratio is also called the **constant of proportionality**.

EXAMPLE 3 Write and Use an Equation

ALGEBRA **Jaycee bought 8 gallons of gasoline for $22.32. Write an equation relating the cost to the number of gallons of gasoline. How much would Jaycee pay for 11 gallons at this same rate? for 20 gallons?**

Find the constant of proportionality between cost and gallons.

$\frac{\text{cost in dollars}}{\text{gasoline in gallons}} = \frac{22.32}{8}$ or 2.79 The cost is $2.79 per gallon.

Words	The cost is $2.79 times the number of gallons.
Variable	Let c represent the cost. Let g represent the number of gallons.
Equation	$c = 2.79 \cdot g$

Use this equation to find the cost for 11 and 20 gallons sold at the same rate.

$c = 2.79g$ ←	Write the equation.	→ $c = 2.79g$
$c = 2.79(11)$ ←	Replace g with the number of gallons.	→ $c = 2.79(20)$
$c = 30.69$ ←	Multiply.	→ $c = 55.80$

The cost for 11 gallons is $30.69 and for 20 gallons is $55.80.

STUDY TIP

Checking Your Equation You can check to see if the equation you wrote is accurate by testing the two known quantities.

$c = 2.79g$

$22.32 = 2.79(8)$

$22.32 = 22.32$

CHECK Your Progress

e. **ALGEBRA** Olivia typed 2 pages in 15 minutes. Write an equation relating the number of minutes m to the number of pages p typed. If she continues typing at this rate, how many minutes will it take her to type 10 pages? to type 25 pages?

CHECK Your Understanding

Example 1 (p. 199)

Solve each proportion.

1. $\frac{1.5}{6} = \frac{10}{p}$
2. $\frac{3.2}{9} = \frac{n}{36}$
3. $\frac{41}{x} = \frac{5}{2}$

For Exercises 4 and 5, assume all situations are proportional.

Example 2 (p. 199)

4. **TEETH** For every 7 people who say they floss daily, there are 18 people who say they do not. Write and solve a proportion to determine out of 65 people how many you would expect to say they floss daily.

Example 3 (p. 200)

5. **TUTORING** Amanda earns $28.50 tutoring for 3 hours. Write an equation relating her earnings m to the number of hours h she tutors. How much would Amanda earn tutoring for 2 hours? for 4.5 hours?

MathOnline **Extra Examples at** ca.gr7math.com

Exercises

HOMEWORK HELP

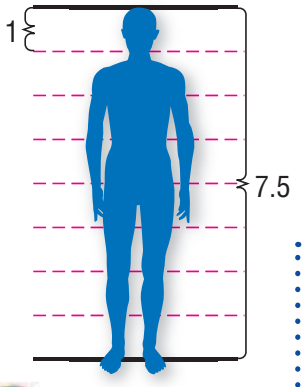
For Exercises	See Examples
6–15	1
16–19	2
20–25	3

Solve each proportion.

6. $\frac{k}{7} = \frac{32}{56}$
7. $\frac{x}{13} = \frac{18}{39}$
8. $\frac{44}{p} = \frac{11}{5}$
9. $\frac{2}{w} = \frac{0.4}{0.7}$
10. $\frac{6}{25} = \frac{d}{30}$
11. $\frac{2.5}{6} = \frac{h}{9}$
12. $\frac{3.5}{8} = \frac{a}{3.2}$
13. $\frac{48}{9} = \frac{72}{n}$

For Exercises 14–21, assume all situations are proportional.

14. **COOKING** Evarado paid $1.12 for a dozen eggs. Write and solve a proportion to determine the ingredient cost of the 3 eggs Evarado needs for a recipe.
15. **TRAVEL** A certain vehicle can travel 483 miles on 14 gallons of gasoline. Write and solve a proportion to determine how many gallons of gasoline this vehicle will need to travel 600 miles.
16. **ILLNESS** For every person who actually has the flu, there are 6 people who have flu-like symptoms resulting from a cold. If a doctor sees 40 patients, write and solve a proportion to determine how many of these you would expect to have a cold.
17. **LIFE SCIENCE** For every left-handed person, there are about 4 right-handed people. If there are 30 students in a class, write and solve a proportion to predict the number of students who are right-handed.



Real-World Link
Although people vary in size and shape, in general, people do not vary in proportion.
Source: *Art Talk*

PEOPLE **For Exercises 18 and 19, use the following information.**
The head height to overall height ratio for an adult is given in the diagram at the left. Write and solve a proportion to predict the following measures.

18. the height of an adult who has a head height of 9.6 inches
19. the head height of an adult who is 64 inches tall

20. **PHOTOGRAPHY** It takes 2 minutes to print out 3 digital photos. Write an equation relating the number of photos n to the number of minutes m. At this rate, how long will it take to print 10 photos? 14 photos?
21. **MEASUREMENT** A 20-pound object on Earth weighs $3\frac{1}{3}$ pounds on the Moon. Write an equation relating the weight m of an object on the Moon to the weight a of the object on Earth. How much does an object weigh on the Moon if it weighs 96 pounds on Earth? 128 pounds on Earth?

MEASUREMENT **For Exercises 22–25, use the table to write an equation relating the two measures. Then find the missing quantity. Round to the nearest hundredth.**

Customary System To Metric System
1 in. ≈ 2.54 cm
1 mi ≈ 1.61 km
1 gal ≈ 3.78 L
1 lb ≈ 0.454 kg

22. 12 in. = ■ cm
23. 20 mi = ■ km
24. 2 L = ■ gal
25. 45 kg = ■ lb

EXTRA PRACTICE
See pages 685, 711.
Math Online
Self-Check Quiz at ca.gr7math.com

26. **FIND THE DATA** Refer to the California Data File on pages 16–19. Choose some data and write a real-world problem that could be solved by writing and solving a proportion.

27. **MEASUREMENT** A 5-pound bag of grass seed covers 2,000 square feet. An opened bag has 3 pounds of seed remaining in it. Will this be enough to seed a 14-yard by 8-yard piece of land? Explain your reasoning.

H.O.T. Problems

28. **OPEN ENDED** List two other amounts of cinnamon and sugar, one larger and one smaller, that are proportional to $1\frac{1}{2}$ tablespoons of cinnamon for every 3 tablespoons of sugar. Justify your answers.

CHALLENGE Solve each equation.

29. $\frac{2}{3} = \frac{18}{x + 5}$

30. $\frac{x - 4}{10} = \frac{7}{5}$

31. $\frac{4.5}{17 - x} = \frac{3}{8}$

32. **WRITING IN MATH** Explain why it might be easier to write an equation to represent a proportional relationship rather than using a proportion.

STANDARDS PRACTICE

33. Michael paid $24 for 3 previously-viewed DVDs at Play-It-Again Movies. Which equation can he use to find the cost c of purchasing 12 previously-viewed DVDs from this same store?

A $c = 12 \cdot 24$ **C** $c = 12 \cdot 8$

B $c = 24 \cdot 4$ **D** $c = 72 \cdot 36$

34. An amusement park line is moving about 4 feet every 15 minutes. At this rate, approximately how long will it take for a person at the back of the 50-foot line to reach the front of the line?

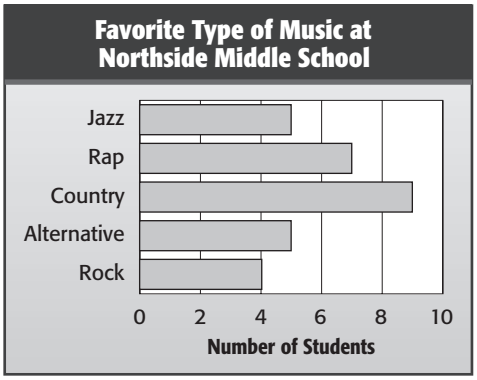
F 1 hour

G 3 hours

H 5 hours

J 13 hours

35. The graph shows the results of a survey of 30 Northside students.



Which proportion can be used to find n, the number preferring country music out of 440 Northside students?

A $\frac{30}{9} = \frac{n}{440}$ **C** $\frac{n}{9} = \frac{30}{400}$

B $\frac{440}{n} = \frac{9}{30}$ **D** $\frac{9}{30} = \frac{n}{440}$

Spiral Review

36. **MONEY** Cassie deposits $40 in a savings account. The money earns $1.40 per month in simple interest, and she makes no further deposits. Is her account balance proportional to the number of months since her initial deposit? (Lesson 4-2)

37. **SHOPPING** Which is the better buy: 1 pound 4 ounces of cheese for $4.99 or 2 pounds 6 ounces for $9.75? Explain your reasoning. (Lesson 4-1)

GET READY for the Next Lesson

38. **PREREQUISITE SKILL** Jacquelyn pays $8 for fair admission but then must pay $0.75 for each ride. If she rides five rides, what is the total cost at the fair? (Lesson 1-1)

Extend 4-3

Geometry Lab
The Golden Rectangle

Main IDEA

Find the value of the golden ratio.

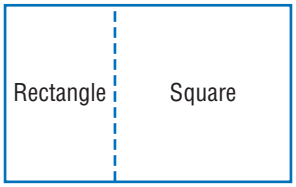
Standard 7MR1.2 **Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.**
Standard 7NS1.3 **Convert fractions to decimals** and percents **and use these representations in** estimations, **computations, and applications.**

ACTIVITY

STEP 1 Cut out a rectangle that measures 34 units long by 21 units wide. Using your calculator, find the ratio of the length to the width. Express it as a decimal to the nearest hundredth. Record your data in a table like the one below.

length	34	21			
width	21	13			
ratio					
decimal					

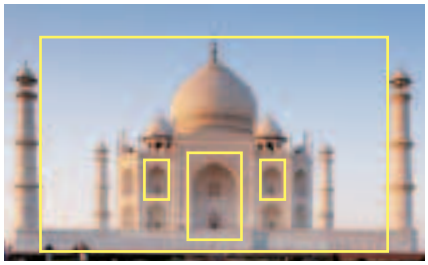
STEP 2 Cut this rectangle into two parts, in which one part is the largest possible square and the other part is a rectangle. Record the rectangle's length and width. Write the ratio of length to width. Express it as a decimal to the nearest hundredth and record in the table.



STEP 3 Repeat the procedure described in Step 2 until the remaining rectangle measures 3 units by 5 units.

Analyze the Results

1. Describe the pattern in the ratios you recorded.
2. **MAKE A CONJECTURE** If the rectangles you cut out are described as *golden rectangles,* what is the value of the *golden ratio?*
3. Write a definition of golden rectangle. Use the word *ratio* in your definition. Then describe the shape of a golden rectangle.
4. Determine whether all golden rectangles are similar. Explain your reasoning.
5. **RESEARCH** There are many examples of the golden rectangle in architecture. One is shown at the right. Use the Internet or another resource to find three places where the golden rectangle is used in architecture.



Doug Corrance/Taxi/Getty Images

4-4 Problem-Solving Investigation

MAIN IDEA: Solve problems by drawing a diagram.

Standard 7MR2.5 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, **diagrams,** and models, **to explain mathematical reasoning.** **Standard 7AF4.2 Solve multistep problems involving rate,** average speed, distance, and time or a direct variation.

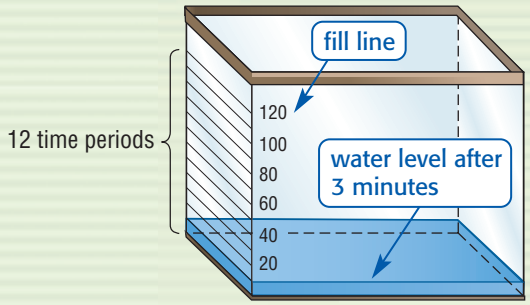
P.S.I. TEAM +

e-Mail: DRAW A DIAGRAM

YOUR MISSION: Draw a diagram to solve the problem.

THE PROBLEM: How long will it take to fill a 120-gallon aquarium?

GABRIELLA: It's been 3 minutes and this 120-gallon tank is only at the 10-gallon mark. I wonder how much longer it will take. Let's draw a diagram to help us picture what's happening.

EXPLORE	The tank holds 120 gallons of water. After 3 minutes, the tank has 10 gallons of water in it. How many more minutes will it take to fill the tank?
PLAN	Draw a diagram showing the water level after every 3 minutes.
SOLVE	The tank will be filled after twelve 3-minute time periods. This is a total of 12 × 3 or 36 minutes. 
CHECK	The tank is filling at a rate of 10 gallons every 3 minutes, which is about 3 gallons per minute. So, a 120-gallon tank will take about 120 ÷ 3 or 40 minutes to fill. An answer of 36 minutes is reasonable.

Analyze The Strategy

1. Describe another method the students could have used to find the number of 3-minute time periods it would take to fill the tank.
2. WRITING IN MATH Write a problem that is more easily solved by drawing a diagram. Then draw a diagram and solve the problem.

J. Strange/KS Studio

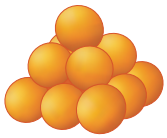
Mixed Problem Solving

For Exercises 3–5, use the *draw a diagram* strategy to solve the problem.

3. **AQUARIUM** Refer to the problem at the beginning of the lesson. Jack fills another 120-gallon tank at the same time Gabriella is filling the first 120-gallon tank. After 3 minutes, his tank has 12 gallons in it. How much longer will it take Gabriella to fill her tank than Jack?

4. **LOGGING** It takes 20 minutes to cut a log into 5 equal-size pieces. How long will it take to cut a similar log into 3 equal-size pieces?

5. **GEOMETRY** A stock clerk is piling oranges in the shape of a square-based pyramid, as shown. If the pyramid is to have five layers, how many oranges will he need?



Use any strategy to solve Exercises 6–11. Some strategies are shown below.

PROBLEM-SOLVING STRATEGIES
- Use the four-step plan.
- Look for a pattern.
- Use a Venn diagram.
- Draw a diagram.

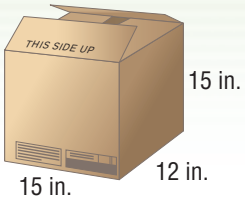
6. **MONEY** Mi-Ling has only nickels in her pocket. Julian has only quarters in his, and Aisha has only dimes in hers. Hannah approached all three for a donation for the school fund-raiser. What is the least each person could donate so that each one gives the same amount?

TECHNOLOGY **For Exercises 7 and 8, use the diagram and the information below.**
Seven closed shapes are used to make the digits 0 to 9 on a digital clock. (The number 1 is made using the line segments on the right side of the figure.)

7. In forming these digits, which line segment is used most often?

8. Which line segment is used the least?

9. **TILES** Three-inch square tiles that are 2 inches high are being packaged into boxes like the one below. If the tiles must be laid flat, how many will fit in one box?



10. **DESSERTS** At a birthday party, 12 people chose cake for dessert and 8 people chose ice cream. Five people chose both cake and ice cream. How many people had dessert?

11. **SCHOOL** Of the 30 students in a science class, 19 like to do chemistry labs, 15 prefer physical science labs, and 7 like to do both. How many students like chemistry labs but not physical science labs?

Select the Operation

For Exercises 12–14, select the appropriate operation(s) to solve the problem. Justify your selection(s) and solve the problem.

12. **MEASUREMENT** An amusement park features giant statues of comic strip characters. If you multiply one character's height by 4 and add 1 foot, you will find the height of its statue. If the statue is 65 feet tall, how tall is the character?

13. **SPORTS** The width of a tennis court is ten feet more than one-third its length. If the court is 78 feet long, what is its perimeter?

14. **FLIGHTS** A DC-11 jumbo jet carries 345 passengers with 38 in first-class and the rest in coach. For a day flight, a first-class ticket from Los Angeles to Chicago costs $650, and a coach ticket costs $230. What will be the ticket sales if the flight is full?

4-5 Similar Polygons

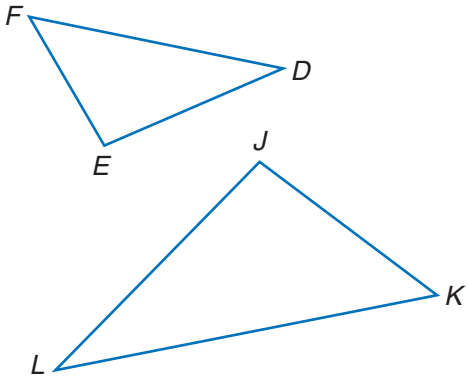
Main IDEA

Identify similar polygons and find missing measures of similar polygons.

Reinforcement of Standard 6NS1.3 Use proportions to solve problems. Use cross multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.

MINI Lab

Follow the steps below to discover how the triangles at the right are related.



STEP 1 Copy both triangles onto tracing paper.

STEP 2 Measure and record the sides of each triangle.

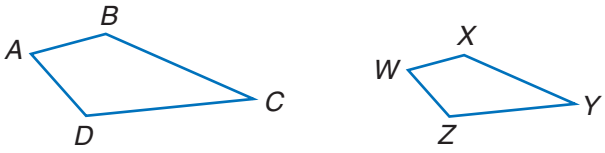
STEP 3 Cut out both triangles.

1. Compare the angles of the triangles by matching them up. Identify the angle pairs that have equal measure.
2. Express the ratios $\frac{DF}{LK}$, $\frac{EF}{JK}$, and $\frac{DE}{LJ}$ as decimals to the nearest tenth.
3. What do you notice about the ratios of these sides of matching triangles?

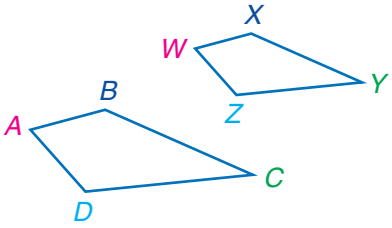
NEW Vocabulary

polygon
similar
corresponding parts
congruent
scale factor

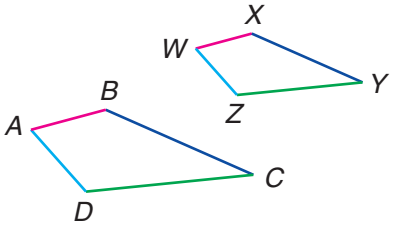
A **polygon** consists of a sequence of consecutive line segments in a plane, placed end to end to form a simple closed figure. Polygons that have the same shape are called **similar** polygons. In the figure below, polygon $ABCD$ is similar to polygon $WXYZ$. This is written as polygon $ABCD \sim$ polygon $WXYZ$.



The parts of similar figures that "match" are called **corresponding parts**.



Corresponding Angles
$\angle A \leftrightarrow \angle W$, $\angle B \leftrightarrow \angle X$, $\angle C \leftrightarrow \angle Y$, $\angle D \leftrightarrow \angle Z$



Corresponding Sides
$\overline{AB} \leftrightarrow \overline{WX}$, $\overline{BC} \leftrightarrow \overline{XY}$, $\overline{CD} \leftrightarrow \overline{YZ}$, $\overline{DA} \leftrightarrow \overline{ZW}$

READING Math

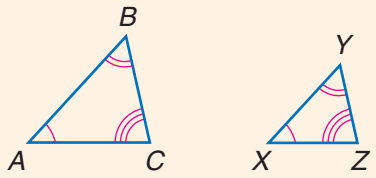
Congruence The symbol ≅ is read *is congruent to.* Arcs are used to show congruent angles.

The similar triangles in the Mini Lab suggest the following.

KEY CONCEPT Similar Polygons

Words If two polygons are similar, then
- their corresponding angles are **congruent**, or have the same measure, and
- the measures of their corresponding sides are proportional.

Model

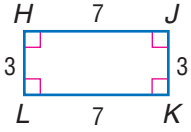


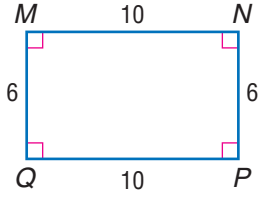
$\triangle ABC \sim \triangle XYZ$

Symbols $\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$, and $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$

EXAMPLE 1 Identify Similar Polygons

Determine whether rectangle *HJKL* is similar to rectangle *MNPQ*. Explain.





First, check to see if corresponding angles are congruent.

Since the two polygons are rectangles, all of their angles are right angles. Therefore, all corresponding angles are congruent.

Next, check to see if corresponding sides are proportional.

$$\frac{HJ}{MN} = \frac{7}{10} \quad \frac{JK}{NP} = \frac{3}{6} \text{ or } \frac{1}{2} \quad \frac{KL}{PQ} = \frac{7}{10} \quad \frac{LH}{QM} = \frac{3}{6} \text{ or } \frac{1}{2}$$

Since $\frac{7}{10}$ and $\frac{1}{2}$ are not equivalent ratios, rectangle *HJKL* is *not* similar to rectangle *MNPQ*.

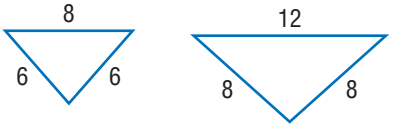
STUDY TIP

Common Error
Do not assume that two polygons are similar just because their corresponding angles are congruent. Their corresponding sides must also be proportional.

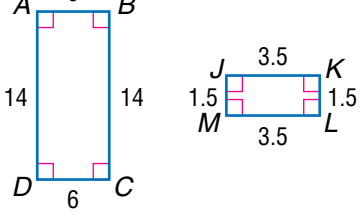
CHECK Your Progress

Determine whether these polygons are similar. Explain.

a.



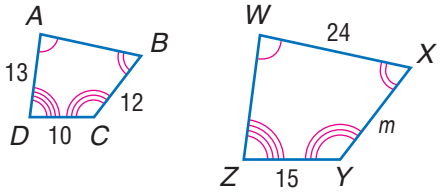
b.



The ratio of the lengths of two corresponding sides of two similar polygons is called the **scale factor**. You can use the scale factor of similar figures or a proportion to find missing measures.

EXAMPLE Find Missing Measures

2 **GEOMETRY** Given that polygon $WXYZ \sim$ polygon $ABCD$, find the missing measure.



READING Math

Segment Measure The *measure* of $\overline{XY}$ is written as XY. It represents a number.

METHOD 1 **Write a proportion.**

The missing measure m is the length of $\overline{XY}$. Write a proportion that relates corresponding sides of the two polygons.

polygon $WXYZ \rightarrow \frac{XY}{BC} = \frac{YZ}{CD} \leftarrow$ polygon $WXYZ$
polygon $ABCD \rightarrow$ $\leftarrow$ polygon $ABCD$

$\frac{m}{12} = \frac{15}{10}$ $XY = m$, $BC = 12$, $YZ = 15$, and $CD = 10$.

$m \cdot 10 = 12 \cdot 15$ Find the cross products.

$10m = 180$ Multiply.

$m = 18$ Divide each side by 10.

METHOD 2 **Use the scale factor to write an equation.**

Find the scale factor from polygon $WXYZ$ to polygon $ABCD$ by finding the ratio of corresponding sides with known lengths.

scale factor: $\frac{YZ}{CD} = \frac{15}{10}$ or $\frac{3}{2}$ The scale factor is the constant of proportionality.

Words	A length on polygon *WXYZ*	is $\frac{3}{2}$ times as long as	a corresponding length on polygon *ABCD*.
Variable	Let m represent the measure of $\overline{XY}$.		
Equation	$m = \frac{3}{2} \cdot 12$		

$m = \frac{3}{2}(12)$ Write the equation.

$m = 18$ Multiply.

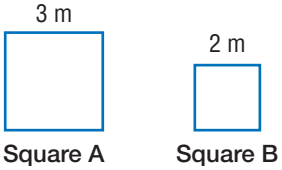
CHOOSE Your Method **Find each missing measure above.**

c. WZ d. AB

STUDY TIP

Scale Factor In Example 2, the scale factor from polygon *ABCD* to polygon *WXYZ* is $\frac{2}{3}$, which means that a length on polygon *ABCD* is $\frac{2}{3}$ as long as a length on polygon *WXYZ*.

Square A $\sim$ square B with a scale factor of 3:2. Notice the relationship between the scale factor and the ratio of their perimeters.



Square	Perimeter
A	12 m
B	8 m

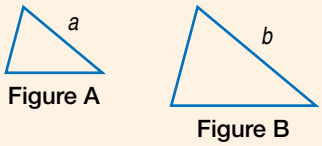
perimeter of square A $\rightarrow$ $\frac{12}{8} = \frac{3}{2}$ or 3:2
perimeter of square B $\rightarrow$

This and other related examples suggest the following.

KEY CONCEPTS — Ratios of Similar Figures

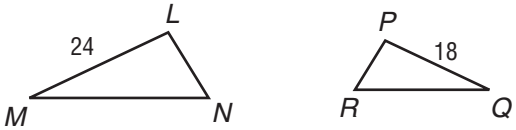
Words If two figures are similar with a scale factor of $\frac{a}{b}$, then the perimeters of the figures have a ratio of $\frac{a}{b}$.

Model



STANDARDS EXAMPLE

3 Triangle LMN is similar to triangle PQR. If the perimeter of $\triangle LMN$ is 64 units, what is the perimeter of $\triangle PQR$?



A 108 units

B 96 units

C 48 units

D 36 units

Read the Item You know that the two triangles are similar, and you know the measures of two corresponding sides and the perimeter of $\triangle LMN$. You need to find the perimeter of $\triangle PQR$.

Solve the Item Triangle $LMN \sim$ triangle PQR with a scale factor of $\frac{24}{18}$ or $\frac{4}{3}$. The ratio of the perimeters of $\triangle LMN$ to $\triangle PQR$ is also $\frac{4}{3}$. Write and solve a proportion. Let x represent the perimeter of $\triangle PQR$.

perimeter of $\triangle LMN$ → $\frac{64}{x} = \frac{4}{3}$ ← perimeter of $\triangle PQR$ — Scale factor relating $\triangle LMN$ to $\triangle PQR$

$64 \cdot 3 = 4 \cdot x$ — Find the cross products.

$192 = 4x$ — Multiply.

$\frac{192}{4} = \frac{4x}{4}$ — Divide each side by 4.

$48 = x$ — Simplify.

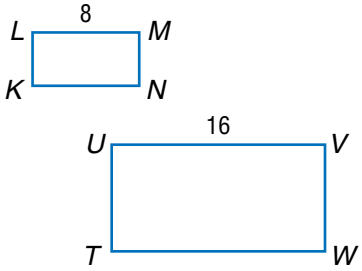
The answer is C.

Test-Taking Tip

Similarity Statements In naming similar triangles, the order of the vertices indicates the corresponding parts. Read the similarity statement carefully to be sure that you compare corresponding parts.

CHECK Your Progress

e. Rectangle $KLMN$ is similar to rectangle $TUVW$. If the perimeter of rectangle $KLMN$ is 32 units, what is the perimeter of rectangle $TUVW$?



F 128 units

G 96 units

H 64 units

J 40 units

Personal Tutor

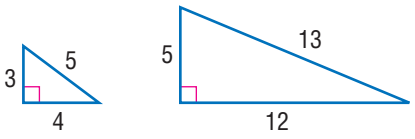
Online **Personal Tutor at** ca.gr7math.com

CHECK Your Understanding

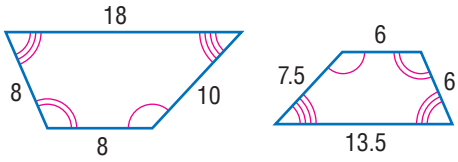
Example 1 (p. 207)

Determine whether each pair of polygons is similar. Explain.

1.

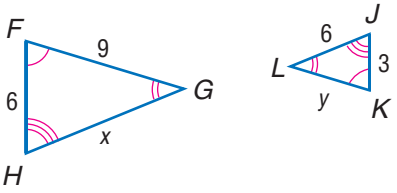


2.



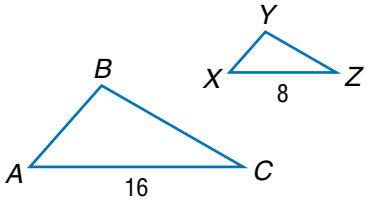
Example 2 (p. 208)

3. In the figure at the right, $\triangle FGH \sim \triangle KLJ$. Write and solve a proportion to find each missing side measure.



Example 3 (p. 209)

4. **STANDARDS PRACTICE** $\triangle ABC$ is similar to $\triangle XYZ$. If the perimeter of $\triangle ABC$ is 40 units, what is the perimeter of $\triangle XYZ$?



A 10 units **C** 40 units

B 20 units **D** 80 units

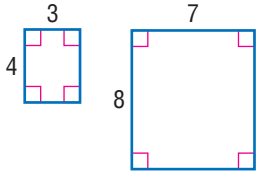
Exercises

HOMEWORK HELP

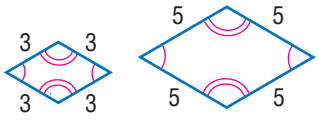
For Exercises	See Examples
5–8	1
9–12	2
18, 19	3

Determine whether each pair of polygons is similar. Explain.

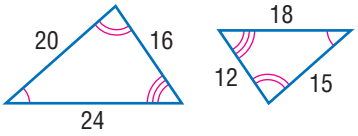
5.



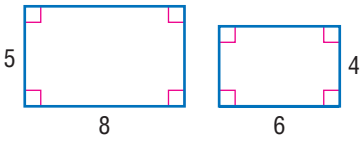
6.



7.

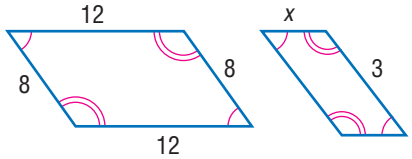


8.

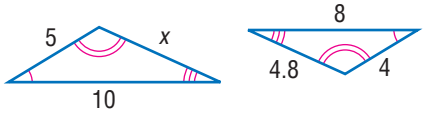


Each pair of polygons is similar. Write and solve a proportion to find each missing side measure.

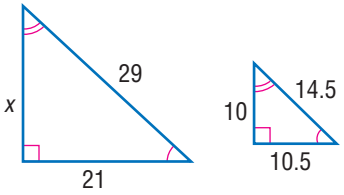
9.



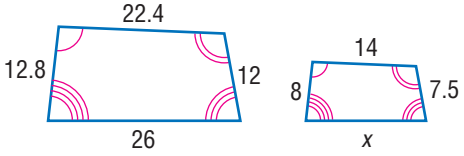
10.



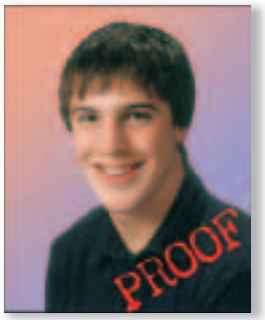
11.



12.



13. YEARBOOK The scale factor from the original proof at the right to the reduced picture for a yearbook will be 8:5. Find the dimensions of the pictures as they will appear in the yearbook.



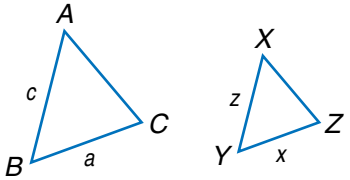
5 in.
4 in.

14. MOVIES When projected onto a movie screen, the image from a film is 9 meters wide and 6.75 meters high. If the image from this same film is projected so that it appears 8 meters wide, what is the height of the projected image?

EXTRA PRACTICE
See pages 686, 711.
Math Online
Self-Check Quiz at ca.gr7math.com

H.O.T. Problems

15. CHALLENGE *True* or *false*? If $\triangle ABC \sim \triangle XYZ$, then $\frac{a}{c} = \frac{x}{z}$. Justify your answer.



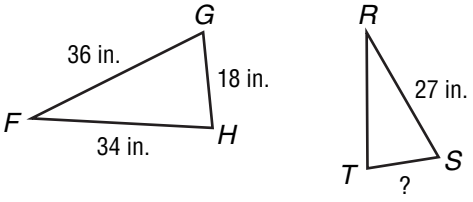
WRITING IN MATH **Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.**

16. Any two rectangles are similar.

17. Any two squares are similar.

STANDARDS PRACTICE

18. Triangle FGH is similar to triangle RST.

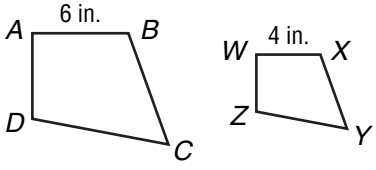


What is the length of $\overline{TS}$?

A $13\frac{1}{2}$ inches **C** 24 inches

B $22\frac{2}{3}$ inches **D** $25\frac{1}{2}$ inches

19. Quadrilateral $ABCD$ is similar to quadrilateral $WXYZ$.



If the area of quadrilateral $ABCD$ is 54 square units, what is the area of quadrilateral $WXYZ$?

F 13.5 inches^2 **H** 27 inches^2

G 24 inches^2 **J** 36 inches^2

Spiral Review

20. ROCK CLIMBING Grace is working her way up a climbing wall. Every 5 minutes she is able to climb 6 feet, but then loses her footing, slips back 1 foot, and decides to rest for 1 minute. If the rock wall is 30 feet tall, how long will it take her to reach the top? Use the *draw a diagram* strategy. (Lesson 4-4)

21. BAKING A recipe calls for 4 cups of flour for 64 cookies. How much flour is needed for 96 cookies? (Lesson 4-3)

GET READY for the Next Lesson

PREREQUISITE SKILL **Graph and connect each pair of ordered pairs.** (Lesson 3-6)

22. $(-2.5, 1.5), (1.5, -3.5)$ **23.** $\left(-2, -1\frac{1}{2}\right), \left(4, 3\frac{1}{2}\right)$ **24.** $\left(-2\frac{1}{3}, 1\right), \left(2, 3\frac{2}{3}\right)$

John Evans

CHAPTER 4

Mid-Chapter Quiz

Lessons 4-1 through 4-5

Express each ratio in simplest form. (Lesson 4-1)

1. 32 out of 100 dentists
2. 12 tickets chosen out of 60 tickets
3. 300 points in 20 games

Express each rate as a unit rate. (Lesson 4-1)

4. 750 yards in 25 minutes
5. $420 for 15 tickets
6. 40 laps in 6 races

7. **STANDARDS PRACTICE** In her last race, Bergen swam 1,500 meters in 30 minutes. On average, how many meters did she swim per minute? (Lesson 4-1)

   A 25

   B 30

   C 40

   D 50

8. **ICE CREAM** In one 8-hour day, Bella's Ice Cream Shop sold 72 cones of vanilla ice cream. In one hour, they sold 9 cones of vanilla ice cream. Is the total number of cones sold in one hour proportional to the number of cones sold during the day? (Lesson 4-2)

9. **DISHES** Jack washed 60 plates in 30 minutes. It took him 3 minutes to wash 6 plates. Is the number of plates washed in 3 minutes proportional to the total number of plates he washed in 30 minutes? (Lesson 4-2)

Solve each proportion. (Lesson 4-3)

10. $\frac{33}{r} = \frac{11}{2}$
11. $\frac{x}{36} = \frac{15}{24}$
12. $\frac{5}{9} = \frac{4.5}{a}$

13. **STANDARDS PRACTICE** There are 2 cubs for every 3 adults in a certain lion pride. If the pride has 8 cubs, how many adults are there? (Lesson 4-3)

    F 12 **G** 16 **H** 24 **J** 48

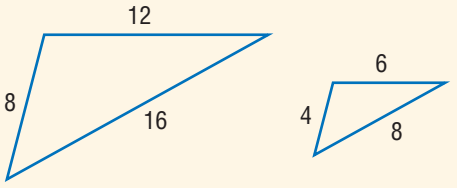
14. **MEASUREMENT** Light travels approximately 1,860,000 miles in 10 seconds. How long will it take light to travel 93,000,000 miles from the Sun to Earth? (Lesson 4-3)

15. **TELEVISION** A typical 30-minute TV program has about 8 minutes of commercials. At that rate, how many commercial minutes are shown during a 2-hour TV movie? (Lesson 4-3)

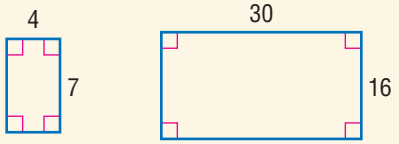
16. **MOVIES** A section of a theater is arranged so that each row has the same number of seats. You are seated in the 5th row from the front and the 3rd row from the back. If your seat is 6th from the left and 2nd from the right, how many seats are in this section of the theater? Use the *draw a diagram* strategy. (Lesson 4-4)

Determine whether each pair of polygons is similar. Explain. (Lesson 4-5)

17.



18.



19. **MEASUREMENT** Dollhouse furniture is similar in shape to full-sized furniture. A dollhouse chair is 6 inches high and 2.5 inches wide. If a full-sized chair is 36 inches tall, how wide is the chair? (Lesson 4-5)

4-6 Measurement: Converting Length, Weight/Mass, Capacity, and Time

Main IDEA

Convert customary and metric units of length, weight or mass, capacity, and time.

Standard 7MG1.1 **Compare weights, capacities, geometric measures, times,** and temperatures **within and between measurement systems (e.g. miles per hour and feet per second, cubic inches to cubic centimeters).**

NEW Vocabulary

unit ratio

GET READY for the Lesson

Jesse Owens set a record of 9.4 seconds for the 100-yard dash at the Big Ten track meet in Ann Arbor, Michigan, on May 25, 1935. The next year at the 1936 Olympic Games in Berlin, he astounded the world by matching the world record of 10.3 seconds in the 100-meter race. How did the lengths of the races compare?

1. A *yard* is a unit of length in the customary system. Name another unit of length in the customary system.
2. A *meter* is a unit of length in the metric system. Name another unit of length in the metric system.
3. Explain why the college race was measured in yards and the Olympic race was measured in meters.

The relationships among the most commonly used customary and metric units of length, weight or mass, capacity, and time are shown in the table below.

KEY CONCEPT Measurement Conversions

Customary Units	Metric Units
Length	
1 foot (ft) = 12 inches (in.) 1 yard (yd) = 3 feet 1 mile (mi) = 5,280 feet	1 meter (m) = 1,000 millimeters (mm) 1 meter = 100 centimeters (cm) 1 kilometer (km) = 1,000 meters
Weight	**Mass**
1 pound (lb) = 16 ounces (oz) 1 ton (T) = 2,000 pounds	1 gram (g) = 1,000 milligrams (mg) 1 kilogram (kg) = 1,000 grams
Capacity	
1 cup (c) = 8 fluid ounces (fl oz) 1 pint (pt) = 2 cups 1 quart (qt) = 2 pints 1 gallon (gal) = 4 quarts	1 liter (L) = 1,000 milliliters (mL) 1 kiloliter (kL) = 1,000 liters
Time	
1 minute (min) = 60 seconds (s) 1 hour (h) = 60 minutes 1 day (d) = 24 hours	1 week (wk) = 7 days 1 year (yr) = 365 days

Bettmann/CORBIS

Each of the relationships in the table can be written as a unit ratio. Like a unit rate, a **unit ratio** is one in which the denominator is 1 unit.

$$\frac{3 \text{ ft}}{1 \text{ yd}} \qquad \frac{2{,}000 \text{ lb}}{1 \text{ T}} \qquad \frac{1{,}000 \text{ m}}{1 \text{ km}} \qquad \frac{24 \text{ h}}{1 \text{ d}}$$

Notice that the numerator and denominator of each fraction above are equivalent, so the value of each ratio is 1. You can multiply by a unit ratio of this type to *convert* or change from larger units to smaller units.

EXAMPLE Convert Larger Units to Smaller Units

1 **Convert 12 yards to feet.**

Since 1 yard = 3 feet, the unit ratio is $\frac{3 \text{ ft}}{1 \text{ yd}}$.

$12 \text{ yd} = 12 \text{ yd} \cdot \frac{3 \text{ ft}}{1 \text{ yd}}$ Multiply by $\frac{3 \text{ ft}}{1 \text{ yd}}$.

$= 12 \cancel{\text{yd}} \cdot \frac{3 \text{ ft}}{1 \cancel{\text{yd}}}$ Divide out common units, leaving the desired unit, feet.

$= 12 \cdot 3 \text{ ft or } 36 \text{ ft}$ Multiply.

So, 12 yards = 36 feet.

STUDY TIP

You should always write the units to ensure that the correct units are being cancelled.

CHECK Your Progress

Complete each conversion.

a. 27 yd = ■ ft

b. $3\frac{1}{2}$ qt = ■ pt

c. 5 km = ■ m

d. 7.5 L = ■ mL

To convert from smaller units to larger units, multiply by the *reciprocal* of the appropriate unit ratio.

AP Photo/Stefan Paltera

Real-World Link
The Rose Bowl, "The Granddaddy of Them All," has been a sellout attraction every year since 1947.
Source: tournamentofroses.com

EXAMPLE Convert Smaller Units to Larger Units

2 **BANNERS** **Carleta needs 450 centimeters of material to make a banner for a parade. How many meters of material does she need?**

$450 \text{ cm} = 450 \text{ cm} \cdot \frac{1 \text{ m}}{100 \text{ cm}}$ Since 1 meter = 100 centimeters, multiply by $\frac{1 \text{ m}}{100 \text{ cm}}$.

$= 450 \cancel{\text{cm}} \cdot \frac{1 \text{ m}}{100 \cancel{\text{cm}}}$ Divide out common units, leaving the desired unit, meter.

$= \frac{450}{100} \text{ m or } 4.5 \text{ m}$ Multiply.

So, Carleta needs 4.5 meters of material.

CHECK Your Progress

Complete each conversion.

e. 56 oz = ■ lb

f. 48 in. = ■ ft

g. 150 mL = ■ L

h. 4,000g = ■ kg

MathOnline **Extra Examples at** ca.gr7math.com

REVIEW Vocabulary

dimensional analysis The process of including units of measurement when you compute. (p. 98)

You can also use dimensional analysis to convert *between* measurement systems. The table shows conversion factors for units of length, capacity, and mass or weight.

KEY CONCEPT Metric/Customary Measurement Conversions

Length	Capacity and Mass or Weight
1 in. ≈ 2.54 cm	1 fl oz ≈ 29.574 mL
1 ft ≈ 0.305 m	1 pt ≈ 0.473 L
1 yd ≈ 0.914 m	1 qt ≈ 0.946 L
1 mi ≈ 1.609 km	1 gal ≈ 3.785 L
1 cm ≈ 0.394 in.	1 oz ≈ 28.35 g
1 m ≈ 1.094 yd	1 lb ≈ 0.454 kg
1 km ≈ 0.621 mi	

STUDY TIP

Dimensional Analysis
Choose conversion factors that allow you to divide out common units.

EXAMPLE Convert Between Systems

3 **Convert 9 centimeters to inches.**

METHOD 1

Use 1 in. ≈ 2.54 cm.

$9 \text{ cm} \approx 9 \text{ cm} \cdot \frac{1 \text{ in.}}{2.54 \text{ cm}}$ Since 1 in. ≈ 2.54 cm, multiply by $\frac{1 \text{ in.}}{2.54 \text{ cm}}$.

$9 \text{ cm} \approx 9 \text{ } \cancel{\text{cm}} \cdot \frac{1 \text{ in.}}{2.54 \text{ } \cancel{\text{cm}}}$ Divide out common units, leaving the desired unit, inch.

$\approx \frac{9 \text{ in.}}{2.54}$ or 3.54 in. Multiply.

METHOD 2

Use 1 cm ≈ 0.394 in.

$9 \text{ cm} \approx 9 \text{ cm} \cdot \frac{0.394 \text{ in.}}{1 \text{ cm}}$ Multiply by $\frac{0.394 \text{ in.}}{1 \text{ cm}}$.

$9 \text{ cm} \approx 9 \text{ } \cancel{\text{cm}} \cdot \frac{0.394 \text{ in.}}{1 \text{ } \cancel{\text{cm}}}$ Divide out common units, leaving the desired unit, inch.

≈ 9 • 0.394 in. or 3.54 in. Multiply.

So, 9 centimeters is approximately 3.54 inches.

CHOOSE Your Method

Complete each conversion. Round to the nearest hundredth.

i. 6 oz = ■ g j. 5 km = ■ mi k. 6 yd = ■ m l. 2 L = ■ qt

Online Personal Tutor at ca.gr7math.com

EXAMPLE 4 Convert Units Using Multiple Steps

ANIMALS **A sloth's top speed is 1.9 kilometers per hour. How fast is this in feet per seconds?**

To convert kilometers to feet, use conversion factors relating kilometers to miles and miles to feet.

To convert hours to seconds, use conversion factors relating hours to minutes and minutes to seconds.

$$\frac{1.9 \text{ km}}{1 \text{ h}} \cdot \frac{1 \text{ mi}}{1.609 \text{ km}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}}$$

$$= \frac{1.9 \cancel{\text{ km}}}{1 \cancel{\text{ h}}} \cdot \frac{1 \cancel{\text{ mi}}}{1.609 \cancel{\text{ km}}} \cdot \frac{5280 \text{ ft}}{1 \cancel{\text{ mi}}} \cdot \frac{1 \cancel{\text{ h}}}{60 \cancel{\text{ min}}} \cdot \frac{1 \cancel{\text{ min}}}{60 \text{ s}}$$ Divide out common units.

$$= \frac{10{,}032 \text{ ft}}{5{,}792.4 \text{ s}}$$ Multiply.

$$= \frac{1.73 \text{ ft}}{1 \text{ s}}$$ Divide.

The sloth's top speed is 1.73 feet per second.

CHECK Your Progress

m. A vehicle can travel 11 kilometers per 1 liter of gasoline. How many miles per gallon is this?

CHECK Your Understanding

Examples 1, 2 (p. 214)

Complete.

1. 5 lb = ■ oz
2. $8\frac{2}{3}$ yd = ■ ft
3. 630 min = ■ h
4. 686 cm = ■ m
5. **FISH** The average weight of a bass in a certain pond is 40 ounces. About how many pounds does a bass weigh?

Examples 3, 4 (p. 215–216)

Complete each conversion. Round to the nearest hundredth if necessary.

6. 6 in. ≈ ■ cm
7. 1.6 cm ≈ ■ in.
8. 4 qt ≈ ■ L
9. 50 mL ≈ ■ fl oz
10. 50 mph ≈ ■ ft/s
11. 50 gal/h ≈ ■ L/min
12. 350 cm/s ≈ ■ in./min
13. 15 km/min ≈ ■ mi/h
14. How many inches are in 54 centimeters?
15. Convert 17 miles to kilometers.
16. **COOKING** For a holiday dinner, Joanna peeled 2 pounds of potatoes in 15 minutes. How many ounces did she peel per minute?
17. **MILEAGE** A certain vehicle travels an average of 18 miles per gallon of gasoline. How many kilometers can it travel per one liter of gasoline?

Exercises

HOMEWORK HELP

For Exercises	See Examples
18–28	1, 2
29–38	3
39–46	4

Complete.

18. 22 ft = ■ yd
19. 104 oz = ■ lb
20. 4 lb = ■ oz
21. 6 gal = ■ qt
22. $2\frac{1}{2}$ pt = ■ c
23. $5\frac{1}{2}$ c = ■ fl oz
24. 75 min = ■ h
25. $3\frac{3}{4}$ mi = ■ ft
26. 9,000 lb = ■ T
27. How many pounds are in 76 ounces?
28. Convert 11,400 milligrams to grams.

Complete each conversion. Round to the nearest hundredth if necessary.

29. 5 in. ≈ ■ cm
30. 5 gal ≈ ■ L
31. 15 cm ≈ ■ in.
32. 17 m ≈ ■ yd
33. 2 L ≈ ■ qt
34. 10 mL ≈ ■ fl oz
35. 2,000 lb ≈ ■ kg
36. 63.5 kg ≈ ■ lb
37. Convert 1.4 quarts to milliliters.
38. How many pounds are there in 19 kilograms?

Complete each conversion. Round to the nearest hundredth if necessary.

39. 20 oz/min ≈ ■ qt/day
40. 70 mi/h ≈ ■ ft/s
41. 16 fl oz/h ≈ ■ mL/min
42. 150 fl oz/day ≈ ■ L/h
43. 52 mi/h ≈ ■ km/min
44. 15 gal/h ≈ ■ L/min
45. In meters per second, how fast is 1,550 feet per minute?
46. A storage bin is being filled at a rate of 2,350 pounds per hour. What is the rate in kilograms per minute?

Determine which is greater.

47. 3 gal, 10 L
48. 14 oz, 0.4 kg
49. 4 mi, 6.2 km

ROLLER COASTERS **For Exercises 50–51, use the table that lists the fastest and tallest roller coasters on three different continents.**

Fastest Roller Coasters		
Continent	**Name**	**Speed**
Asia	Dodonpa	172 kph
Europe	Stealth	128 kph
North America	Kingda Ka	128 mph
Tallest Roller Coasters		
Continent	**Name**	**Height**
Asia	Steel Dragon 2000	97 m
Europe	Silver Star	73 m
North America	Kingda Ka	456 ft

Source: rcdb.com

50. Order the roller coasters from greatest to least speeds.
51. Order the roller coasters from tallest to shortest.
52. **WATER** Which is greater: 64 fluid ounces of water or 2 liters of water? Explain your reasoning.
53. **FOOD** Which is greater: a 1.5-pound box of raisins or a 650-gram box of raisins? Explain your reasoning.

EXTRA PRACTICE

See pages 686, 711.

Math Online

Self-Check Quiz at ca.gr7math.com

H.O.T. Problems

54. **FIND THE ERROR** Pedro and Alex are converting 2 liters. Who is correct? Explain your reasoning.

Pedro	Alex
2.144 qt	0.946 pt

55. **CHALLENGE** To make it around the track, a roller coaster must achieve a speed of at least 76 miles per hour. At top speed, the coaster traveled 136 meters in 4.3 seconds. Is the coaster traveling fast enough to make it completely around the track? Explain.

56. **WRITING IN MATH** Refer to the information at the beginning of the lesson. Explain how you can compare the 100-yard dash and the 100-meter dash. Compare Owens' records in the two events.

STANDARDS PRACTICE

57. How many millimeters are in 5 centimeters?

A 0.05

B 0.5

C 50

D 500

58. 120 kilometers per hour is the same rate as which of the following?

F 2 kilometers per second

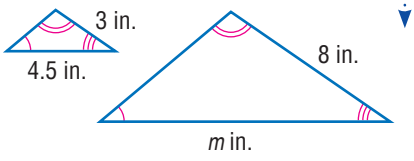
G 2 kilometers per minute

H 12 kilometers per minute

J 720 kilometers per second

Spiral Review

59. The triangles at the right are similar. Write and solve a proportion to find the missing measure. (Lesson 4-5)



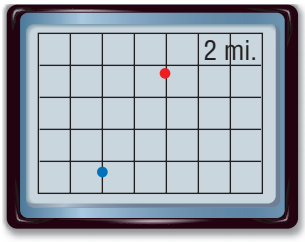
Solve each proportion. (Lesson 4-3)

60. $\frac{5}{4} = \frac{y}{12}$

61. $\frac{120}{b} = \frac{24}{60}$

62. $\frac{0.6}{5} = \frac{1.5}{n}$

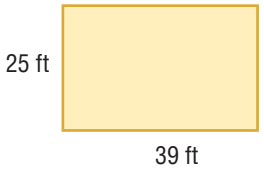
63. **TECHNOLOGY** A hiker uses her GPS (Global Positioning System) receiver to find how much farther she needs to go to get to her stopping point for the day. She is at the red dot on her GPS receiver screen, and the blue dot shows her destination. How much farther does she need to travel? (Lesson 3-7)



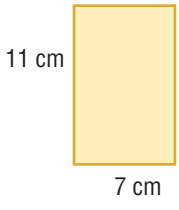
GET READY for the Next Lesson

PREREQUISITE SKILL **Find the area of each rectangle.** (p. 674)

64.



65.



Extend 4-6

Spreadsheet Lab
Converting Measures

Main IDEA

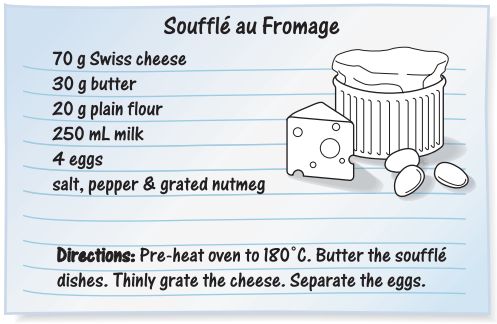
Use a spreadsheet to solve problems involving conversions of measurements within and between systems.

Standard 7MG1.1 **Compare weights, capacities, geometric measures, times,** and temperatures **within and between measurement systems (e.g. miles per hour and feet per second, cubic inches to cubic centimeters).** **Standard 7MR3.2** Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

You can use a spreadsheet to convert measurements.

ACTIVITY

COOKING **Your cooking class exchanges recipes with a cooking class in France. The class in France sends the following recipe for a *Soufflé au Fromage*, or cheese soufflé. Find the amount of Swiss cheese, butter, and flour in ounces.**



To solve the problem, set up a spreadsheet.

	A	B	C	D	E
1		Amount	Metric Unit	Amount	Customary Unit
2	Conversion Relationship	28.35	grams	1	ounce
3					
4	Ingredient	Amount	Metric Unit	Amount	Customary Unit
5	Swiss cheese	70	grams	=D2/B2*B5	ounces
6	butter	30	grams		ounces
7	plain flour	20	grams		ounces
8					

Sheet 1 Sheet 2 Sheet 3

Analyze the Results

1. Explain the formula in D5.
2. What formulas should be entered in cells D6 and D7?
3. What would you enter into cells B3, C3, D3, and E3 to convert the amount of milk in the recipe from milliliters to fluid ounces?
4. What would you enter into cells A8, B8, C8, D8, and E8 to convert the amount of milk in the recipe to fluid ounces?
5. **OPEN ENDED** Find another recipe in which ingredients are given in grams or milliliters. Use a spreadsheet to convert these measures into ounces or fluid ounces.

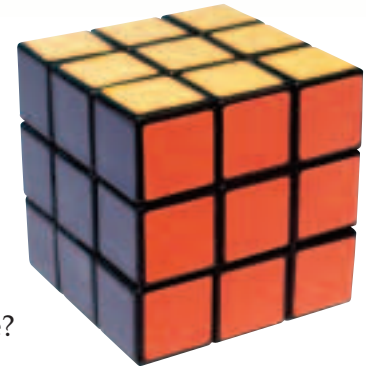
4-7 Measurement: Converting Square Units and Cubic Units

Main IDEA

Convert square and cubic units of length, weight or mass, capacity, and time in both customary and metric systems.

Standard 7MG1.1 Compare weights, capacities, geometric measures, times, and temperatures **within and between measurement systems (e.g. miles per hour and feet per second, cubic inches to cubic centimeters).** **Standard 7MG2.4 Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or [1 ft^2] = 144 in^2], 1 cubic inch is approximately 16.38 cubic centimeters or [1 in^3] = [16.38 cm^3]).**

GET READY for the Lesson



GAMES A puzzle cube can help you understand how to convert measures of area and volume.

1. Look at one face of a puzzle cube. How many cubes are there along each edge? How many squares are there on one face? How many small cubes are there in all?
2. What is the relationship between the number of cubes along each edge and the number of squares on one face? between the number of cubes along each edge and the total number of small cubes?
3. How is the number of square feet in one square yard related to the number of feet in one yard?

Some units of area in the customary system are square inch (in^2), square foot (ft^2), square yard (yd^2), and square mile (mi^2). Some units of area in the metric system are square centimeter (cm^2) and square meter (m^2). Just as you used unit ratios to convert units of length, you can use unit ratios when you convert units of area.

READING Math

Units of Area and Volume Remember that ft^2 is the same as ft × ft and cm^3 is the same as cm × cm × cm.

EXAMPLES Convert Units of Area

Complete each conversion.

1 **2 ft^2 = ■ in^2**

$$2 \text{ ft}^2 = 2 \times \cancel{\text{ft}} \times \cancel{\text{ft}} \times \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} \times \frac{12 \text{ in.}}{1 \cancel{\text{ft}}}$$ Multiply by $\frac{12 \text{ in.}}{1 \text{ ft}}$.

$$= 288 \text{ in}^2$$

2 **4,800 cm^2 = ■ m^2**

$$4{,}800 \text{ cm}^2 = 4{,}800 \times \cancel{\text{cm}} \times \cancel{\text{cm}} \times \frac{1 \text{ m}}{100 \cancel{\text{cm}}} \times \frac{1 \text{ m}}{100 \cancel{\text{cm}}}$$ Multiply by $\frac{1 \text{ m}}{100 \text{ cm}}$.

$$= \frac{4{,}800 \text{ m}^2}{10{,}000}$$ Simplify.

$$= 0.48 \text{ m}^2$$

CHECK Your Progress

Complete each conversion.

a. 1.5 ft^2 = ■ in^2

b. 45 ft^2 = ■ yd^2

c. 24 cm^2 = ■ m^2

d. 3.2 km^2 = ■ m^2

Todd Yarrington

MathOnline Extra Examples at ca.gr7math.com

Some units of volume in the customary system are cubic inch (in^3), cubic foot (ft^3), cubic yard (yd^3), and cubic mile (mi^3). Some units of volume in the metric system are cubic centimeter (cm^3) and cubic meter (m^3).

EXAMPLE Convert Units of Volume

3 **BUILDING** **How many cubic yards of concrete will a builder need for a rectangular driveway that has a volume of 132 cubic feet?**

$$132 \text{ ft}^3 = 132 \times \frac{\text{ft}}{} \times \frac{\text{ft}}{} \times \frac{\text{ft}}{} \times \frac{1 \text{ yd}}{3 \text{ ft}} \times \frac{1 \text{ yd}}{3 \text{ ft}} \times \frac{1 \text{ yd}}{3 \text{ ft}}$$ Multiply by $\frac{1 \text{ yd}}{3 \text{ ft}}$.

$$= \frac{132 \text{ yd}^3}{27}$$ Multiply.

$$\approx 4.89 \text{ yd}^3$$ Simplify.

The builder needs 4.89 cubic yards of concrete.

CHECK Your Progress

e. How many cubic meters of concrete are needed for a sidewalk that has a volume of 280,000 cubic centimeters?

f. A homeowner needs 150 cubic feet of mulch. Mulch is sold by the cubic yard. How many cubic yards does he need to buy?

Online **Personal Tutor at** ca.gr7math.com

STUDY TIP

Look Back You can review conversion factors in Lesson 4-6.

You can also use conversion factors to convert area and volume *between* the customary and metric systems.

EXAMPLE Convert Between Systems

4 **Convert 12 square centimeters to square inches.**

$$12 \text{ cm}^2 = 12 \times \frac{\text{cm}}{} \times \frac{\text{cm}}{} \times \frac{1 \text{ in.}}{2.54 \text{ cm}} \times \frac{1 \text{ in.}}{2.54 \text{ cm}}$$ Multiply by $\frac{1 \text{ in.}}{2.54 \text{ cm}}$.

$$= \frac{12 \text{ in}^2}{6.45}$$ Multiply.

$$\approx 1.86 \text{ in}^2$$ Simplify.

So, 12 square centimeters is approximately 1.86 square inches.

5 **Convert 7 cubic inches to cubic centimeters.**

$$7 \text{ in}^3 = 7 \times \frac{\text{in.}}{} \times \frac{\text{in.}}{} \times \frac{\text{in.}}{} \times \frac{2.54 \text{ cm}}{1 \text{ in.}} \times \frac{2.54 \text{ cm}}{1 \text{ in.}} \times \frac{2.54 \text{ cm}}{1 \text{ in.}}$$

$$= \frac{114.71 \text{ cm}^3}{1}$$ Multiply.

$$\approx 114.71 \text{ cm}^3$$ Simplify.

So, 7 cubic inches is approximately 114.71 cubic centimeters.

CHECK Your Progress

Complete each conversion. Round to the nearest hundredth.

g. $25 \text{ mi}^2 \approx$ ■ km^2.

h. $23 \text{ in}^3 \approx$ ■ cm^3

i. $750 \text{ ft}^2 =$ ■ m^2.

j. $212 \text{ km}^3 =$ ■ mi^3

CHECK Your Understanding

Examples 1, 2 (p. 220)

Complete each conversion.

1. $3\text{ ft}^2 = \blacksquare\text{ in}^2$
2. $2\text{ yd}^2 = \blacksquare\text{ ft}^2$
3. $15\text{ ft}^2 = \blacksquare\text{ yd}^2$
4. $10.8\text{ cm}^2 = \blacksquare\text{ mm}^2$
5. $148\text{ mm}^2 = \blacksquare\text{ cm}^2$
6. $0.264\text{ km}^2 = \blacksquare\text{ m}^2$
7. **REMODELING** Suppose you have a room that is 270 square feet in area. How many square yards of carpet would cover this room?

Examples 3–5 (p. 221)

Complete each conversion. Round to the nearest hundredth.

8. $1.5\text{ ft}^3 = \blacksquare\text{ in}^3$
9. $4.3\text{ yd}^3 = \blacksquare\text{ ft}^3$
10. $0.006\text{ m}^3 = \blacksquare\text{ mm}^3$
11. $2{,}400\text{ cm}^3 = \blacksquare\text{ m}^3$
12. $10\text{ ft}^2 \approx \blacksquare\text{ m}^2$
13. $144\text{ in}^2 \approx \blacksquare\text{ cm}^2$
14. $25\text{ m}^3 \approx \blacksquare\text{ yd}^3$
15. $250\text{ ft}^3 \approx \blacksquare\text{ m}^3$

Exercises

HOMEWORK HELP

For Exercises	See Examples
16–24	1, 2
25–32	3
33–42	4

Complete each conversion. Round to the nearest hundredth if necessary.

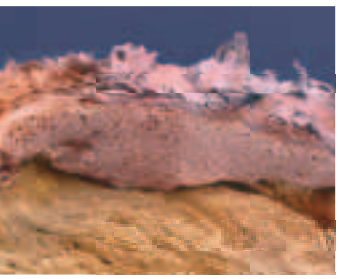
16. $1.6\text{ yd}^2 = \blacksquare\text{ ft}^2$
17. $10.4\text{ ft}^2 = \blacksquare\text{ in}^2$
18. $150\text{ ft}^2 = \blacksquare\text{ yd}^2$
19. $504\text{ in}^2 = \blacksquare\text{ ft}^2$
20. $1.6\text{ m}^2 = \blacksquare\text{ cm}^2$
21. $4{,}654\text{ cm}^2 = \blacksquare\text{ m}^2$
22. $0.058\text{ km}^2 = \blacksquare\text{ m}^2$
23. $37{,}200\text{ m}^2 = \blacksquare\text{ km}^2$
24. **BIOLOGY** The total surface area of the average adult's skin is about 21.5 square feet. Convert this measurement to square inches.

Complete each conversion. Round to the nearest hundredth if necessary.

25. $2\text{ ft}^3 = \blacksquare\text{ in}^3$
26. $0.4\text{ ft}^3 = \blacksquare\text{ in}^3$
27. $300\text{ yd}^3 = \blacksquare\text{ ft}^3$
28. $0.00397\text{ km}^3 = \blacksquare\text{ m}^3$
29. $16{,}000\text{ cm}^3 = \blacksquare\text{ m}^3$
30. $22\text{ m}^3 = \blacksquare\text{ cm}^3$
31. **BALLOONS** A standard hot air balloon holds about 2,000 cubic meters of hot air. How many cubic centimeters is this?
32. **LANDSCAPING** A landscape architect is designing the outside of a new restaurant. She needs 5 cubic yards of stone to cover a certain area. Will 100 cubic feet of stones be enough? If not, how many cubic feet are needed?

Complete each conversion. Round to the nearest hundredth.

33. $10\text{ ft}^3 \approx \blacksquare\text{ m}^3$
34. $25\text{ m}^2 \approx \blacksquare\text{ yd}^2$
35. $240\text{ in}^2 \approx \blacksquare\text{ cm}^2$
36. $2\text{ mi}^3 \approx \blacksquare\text{ km}^3$
37. $120\text{ cm}^2 \approx \blacksquare\text{ in}^2$
38. $4\text{ yd}^3 \approx \blacksquare\text{ m}^3$
39. $45\text{ in}^3 \approx \blacksquare\text{ cm}^3$
40. $108\text{ ft}^2 \approx \blacksquare\text{ m}^2$
41. $37\text{m}^3 \approx \blacksquare\text{ ft}^3$
42. **PAINT** One gallon of paint can cover 400 square feet of wall. How many square meters will one gallon of paint cover?



Real-World Link
This is a close up of a skin cell. The average person loses about 9 pounds of skin cells a year.
Source: kidshealth.org

Steve Gschmeissner/Photo Researchers, Inc.

EXTRA PRACTICE
See pages 687, 711.
Math Online
Self-Check Quiz at ca.gr7math.com

43. **MICROWAVES** The inside of a microwave oven has a volume of 1.2 cubic feet and measures 18 inches wide and 10 inches long. Using the formula $V = \ell wh$, find the depth of the microwave to the nearest tenth of an inch.

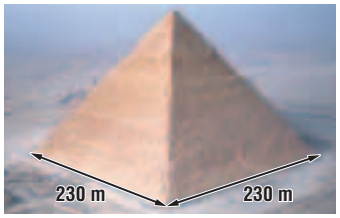
44. **MEASUREMENT** The density of gold is 19.29 grams per cubic centimeter. To the nearest hundredth, find the mass in grams of a gold bar that is 0.75 inch by 1 inch by 0.75 inch. Use the relationship 1 cubic inch ≈ 16.38 cubic centimeters.

H.O.T. Problems

45. **Which One Doesn't Belong?** Identify which equivalent measure does not belong with the other three. Explain.

5.2 yd^3	6.8 m^3	15.6 ft^3	242,611.2 in^3

46. **CHALLENGE** A hectare is a metric unit of area approximately equal to 10,000 square meters or 2.47 acres. The base of the Great Pyramid of Khufu is a 230-meter square. About how many acres does the base cover?



47. **WRITING IN MATH** Describe a real-world situation in which converting units of area or volume is necessary.

STANDARDS PRACTICE

48. The area of a roof that needs new shingles is 40 square yards. How many square feet of shingles are needed?

A 4.44 ft^2 C 360 ft^2
B 120 ft^2 D 1,600 ft^2

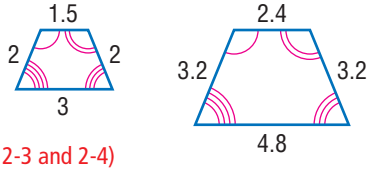
49. Approximately how many cubic feet are there in six cubic meters? Use 1 m^3 ≈ 35.31 ft^3.

F 5.89 H 41.31
G 29.31 J 211.86

Spiral Review

50. **COMPUTERS** A notebook computer has a mass of 2.25 kilograms. Approximately how many pounds does the notebook weigh? (Use 1 lb ≈ 0.4536 kg.) (Lesson 4-6)

51. Determine whether the polygons at the right are similar. Explain your reasoning. (Lesson 4-5)



Find each product or quotient. Write in simplest form. (Lessons 2-3 and 2-4)

52. $\frac{5}{12} \div \frac{3}{20}$

53. $-\frac{7}{48} \cdot \frac{9}{14}$

54. $2\frac{3}{4} \cdot 1\frac{2}{3}$

55. $-3\frac{1}{5} \div \left(-\frac{2}{3}\right)$

GET READY for the Next Lesson

PREREQUISITE SKILL **Solve.** (Lesson 4-3)

56. $\frac{3 \text{ cm}}{5 \text{ ft}} = \frac{x \text{ cm}}{9 \text{ ft}}$

57. $\frac{4 \text{ in.}}{5 \text{ mi}} = \frac{5 \text{ in.}}{x \text{ mi}}$

William Floyd Holdman/Index Stock Imagery

4-8 Scale Drawings and Models

Main IDEA

Solve problems involving scale drawings.

Standard 7MG1.2 Construct and read drawings and models made to scale.

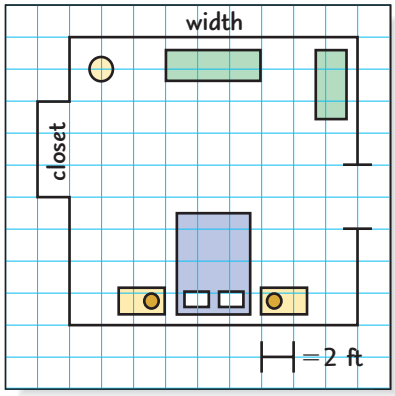
NEW Vocabulary

scale drawing
scale model
scale

GET READY for the Lesson

FLOOR PLANS The blueprint for a bedroom is given below.

1. How many units wide is the room?
2. The actual width of the room is 18 feet. Write a ratio comparing the drawing width to the actual width.
3. Simplify the ratio you found and compare it to the scale shown at the bottom of the drawing.



A **scale drawing** or a **scale model** is used to represent an object that is too large or too small to be drawn or built at actual size. The **scale** is the ratio of a length on a drawing or model to its actual length.

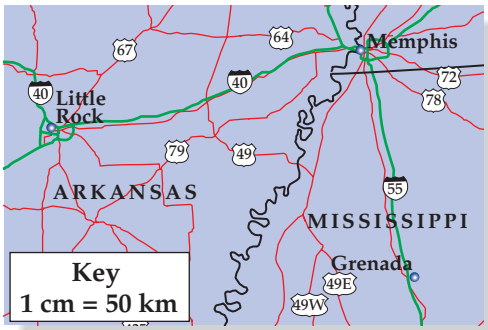
1 inch = 4 feet	1 inch represents an actual distance of 4 feet.
1:30	1 unit represents an actual distance of 30 units.

Distances on a scale drawing are proportional to distances in real life.

Real-World EXAMPLE 1 Use a Scale Drawing

GEOGRAPHY **Use the map to find the actual distance between Grenada, Mississippi, and Little Rock, Arkansas.**

Use a centimeter ruler to measure the map distance. The map distance is about 5.2 centimeters.



STUDY TIP

Scales Scales and scale factors are always written so that the drawing length comes first in the ratio.

METHOD 1 **Write and solve a proportion.**

Let x represent the actual distance to Little Rock.

Scale — **Grenada, MS to Little Rock, AR**

map → $\frac{1 \text{ cm}}{50 \text{ km}} = \frac{5.2 \text{ cm}}{x \text{ km}}$ ← map
actual → ← actual

$1 \cdot x = 50 \cdot 5.2$ Find the cross products.

$x = 260$ Simplify.

METHOD 2 **Write and solve an equation.**

Write the scale as $\frac{50 \text{ km}}{1 \text{ cm}}$, which means 50 kilometers per centimeter.

Words	The actual distance	is	50 kilometers per centimeter	of	map distance.
Variables	Let a represent the actual distance in kilometers. Let m represent the map distance in kilometers.				
Equation	a	$=$	50	$\cdot$	m

$a = 50m$ Write the equation.

$a = 50(5.2)$ or 260 Replace m with 5.2 and multiply.

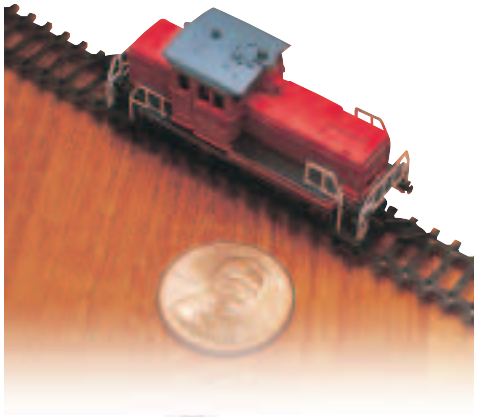
The actual distance between the two cities is about 260 kilometers.

CHOOSE Your Method

GEOGRAPHY **Use an inch ruler and the map shown to find the actual distance between each pair of cities. Measure to the nearest quarter of an inch.**

a. Spartanburg and Gastonia

b. Charlotte and Spartanburg



Real-World Link

Some of the smallest model trains are built on the Z scale. Using this scale, models are $\frac{1}{220}$ the size of real trains.

Source: www.nmra.org

Real-World EXAMPLE Find the Scale

2 **MODEL TRAINS** **A passenger car of a model train is 6 inches long. If the actual car is 80 feet long, what is the scale of the model?**

Let x represent the actual length of the train in feet corresponding to 1 inch in the model. Use a proportion.

Length of Train		Scale
model →	$\frac{6 \text{ in.}}{80 \text{ ft}} = \frac{1 \text{ in.}}{x \text{ ft}}$	← model
actual →		← actual
	$6 \cdot x = 80 \cdot 1$	Find the cross products.
	$\frac{6x}{6} = \frac{80}{6}$	Multiply. Then divide each side by 6.
	$x = 13\frac{1}{3}$	Simplify.

So, the scale is 1 inch $= 13\frac{1}{3}$ feet.

CHECK Your Progress

c. **ARCHITECTURE** The model Mr. Vicario made of the building he designed is 25.6 centimeters tall. If the actual building is to be 64 meters tall, what is the scale of his model?

Doug Martin

The scale factor for scale drawings and models is the scale written as a unitless ratio in simplest form.

EXAMPLE Find the Scale Factor

3 **Find the scale factor for the model train in Example 2.**

$$\frac{1 \text{ in.}}{13\frac{1}{3} \text{ ft}} = \frac{1 \text{ in.}}{160 \text{ in.}}$$ Convert $13\frac{1}{3}$ feet to inches by multiplying by 12.

The scale factor is $\frac{1}{160}$ or 1:160. This means that the model train is $\frac{1}{160}$ the size of the actual train.

STUDY TIP

Scale Factors
A scale factor between 0 and 1 means that the model is smaller than the actual object. A scale factor greater than 1 means that the model is larger than the actual object.

CHECK Your Progress

Find the scale factor for each scale.

d. 1 inch = 15 feet

e. 10 cm = 2.5 m

To construct a scale drawing of an object, find an appropriate scale.

Real-World EXAMPLE Construct a Scale Model

4 **SOCIAL STUDIES** **Each column of the Lincoln Memorial is 44 feet tall. Michaela wants the columns of her model to be no more than 12 inches tall. Choose an appropriate scale and use it to determine how tall she should make the model of Lincoln's 19-foot statue.**

Try a scale of 1 inch = 4 feet.

$$\frac{1 \text{ in.}}{4 \text{ ft}} = \frac{x \text{ in.}}{44 \text{ ft}}$$ ← model / ← actual

$1 \cdot 44 = 4 \cdot x$ Find the cross products.

$44 = 4x$ Multiply.

$11 = x$ Divide each side by 4.

Using this scale, the columns would be 11 inches tall.

Use this scale to find the height of the statue.

$$\frac{1 \text{ in.}}{4 \text{ ft}} = \frac{y \text{ in.}}{19 \text{ ft}}$$

$1 \cdot 19 = 4 \cdot y$

$19 = 4y$

$4\frac{3}{4} = y$

The statue should be $4\frac{3}{4}$ inches tall.

CHECK Your Progress

f. **LIFE SCIENCE** Kaliah is making a model of the human ear and wants the stirrup bone to be between 1 and 2 centimeters long. An actual stirrup bone is about 3 millimeters long. Choose an appropriate scale and use it to determine how tall his model of an actual 54-millimeter tall ear should be.

Personal Tutor

Online **Personal Tutor at** ca.gr7math.com

CHECK Your Understanding

Example 1 (p. 224)

GEOGRAPHY **Use the map and an inch ruler to find the actual distance between each pair of cities.**

1. Evansville and Louisville
2. Louisville and Elizabethtown

MONUMENTS **For Exercises 3 and 4, use the following information.**

Examples 2 and 3 (pp. 225–226)

At 555 feet tall, the Washington Monument is the highest all-masonry tower.

3. If a scale model of the monument is 9.25 inches high, what is the scale?
4. What is the scale factor for the model?

Example 4 (p. 226)

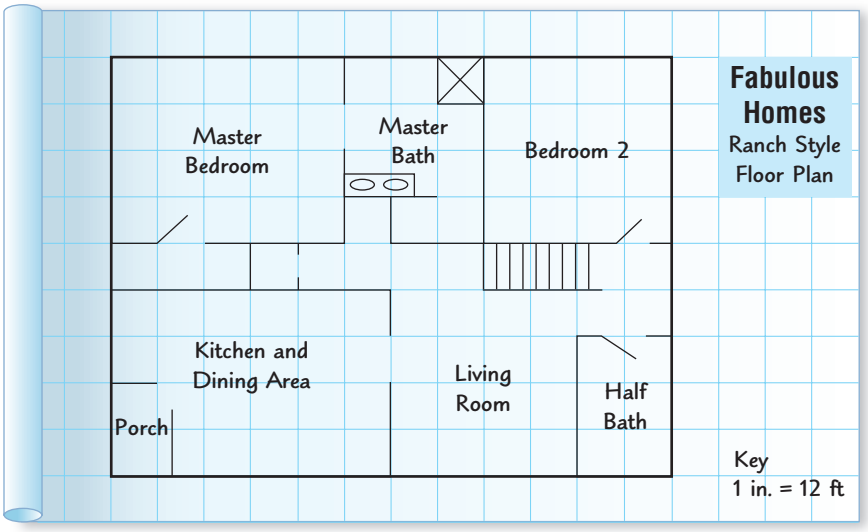
5. **DECORATING** Before redecorating, Nichelle makes a scale drawing of her bedroom on an 8.5- by 11-inch piece of paper. If the room is 10 feet wide by 12 feet long, choose an appropriate scale for her drawing and find the dimensions of the room on the drawing.

Exercises

HOMEWORK HELP

For Exercises	See Examples
6–11	1
12–13	2
14–15	3
16–17	4

FLOOR PLANS **For Exercises 6–11, use the portion of an architectural drawing shown and an inch ruler.**

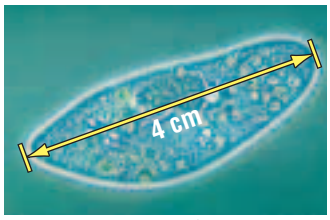


Find the actual length and width of each room. Measure to the nearest eighth of an inch.

6. half bath
7. master bath
8. porch
9. bedroom 2
10. master bedroom
11. living room

12. **MOVIES** One of the models of a dinosaur used in the filming of a movie was only 15 inches tall. In the movie, the dinosaur appeared to have an actual height of 20 feet. What was the scale of the model?

13. **LIFE SCIENCE** The paramecium shown at the right is a single-celled organism that is 0.006 millimeter long. Find the scale of the drawing.



Paramecium

14. **FLOOR PLANS** What is the scale factor of the floor plan used in Exercises 6–11? Explain its meaning.

Real-World Link
Earth has an approximate circumference of 40,000 kilometers, while the Moon has an approximate circumference of 11,000 kilometers.
Source: infoplease.com

15. **MOVIES** What is the scale factor of the model used in Exercise 12?

16. **SPIDERS** A tarantula's body length is 5 centimeters. Choose an appropriate scale for a model of the spider that is to be just over 6 meters long. Then use it to determine how long the tarantula's 9-centimeter legs should be.

17. **AIRPLANES** Dorie is building a model of a DC10 aircraft. The actual aircraft is 182 feet long and has a wingspan of 155 feet. If Dorie wants her model to be no more than 2 feet long, choose an appropriate scale for her model. Then use it to find the length and wingspan of her model.

SPACE SCIENCE **For Exercises 18 and 19, use the information at the left.**

18. Suppose you are making a scale model of Earth and the Moon. You decide to use a basketball to represent Earth. A basketball's circumference is about 30 inches. What is the scale of your model?

19. Which of the following should you use to represent the Moon in your model so it is proportional to the model of Earth in Exercise 18? (The number in parentheses is the object's circumference.) Explain.

a. a soccer ball (28 in.)
b. a tennis ball (8.25 in.)
c. a golf ball (5.25 in.)
d. a marble (4 in.)

EXTRA PRACTICE
See pages 687, 711.
Math Online
Self-Check Quiz at ca.gr7math.com

20. **TRAVEL** On a map of Illinois, the distance between Champaign and Carbondale is $6\frac{3}{4}$ inches. If the scale of the map is $\frac{1}{2}$ inch = 15 miles, about how long would it take the Kowalski family to drive from Champaign to Carbondale if they drove 60 miles per hour?

H.O.T. Problems

21. **OPEN ENDED** Choose a large or small rectangular item such as a calculator, table, or room. Find its dimensions and choose an appropriate scale for a scale drawing of the item. Then construct a scale drawing and write a problem that could be solved using your drawing.

22. **FIND THE ERROR** On a map, 1 inch represents 4 feet. Jacob and Luna are finding the scale factor of the map. Who is correct? Explain.

Jacob

Luna

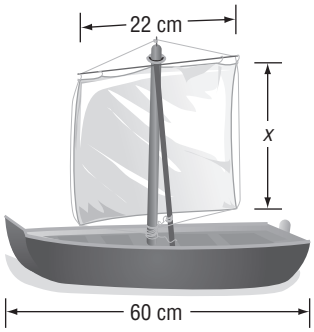
(t)NASA, (tr)M.I. Walker/Photo Researchers, (bl)RubberBall/SuperStock, (br)PNC/Getty Images

23. **CHALLENGE** Describe how you could find the scale on a map that did not have a scale printed on it.

24. **WRITING IN MATH** One model is built on a 1:75 scale. Another model of the same object is built on a 1:100 scale. Which model is larger? Explain.

STANDARDS PRACTICE

25. Jevonte is building a model of a ship with an actual length of 15 meters.



What other information is needed to find x, the height of the model's mast?

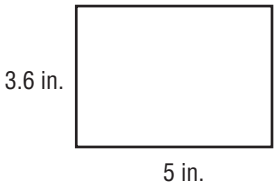
A the overall width of the ship

B the scale factor used

C the overall height of the mast

D the speed of the ship in the water

26. The actual width w of a garden is 18 feet. Use the scale drawing of the garden to find the actual length ℓ.



F 17.2 ft

G 18 ft

H 20 ft

J 25 ft

Spiral Review

Complete each conversion. Round to the nearest hundredth if necessary. (Lesson 4-7)

27. $4\text{ft}^3 = \_\_?\_\_ \text{ yd}^3$

28. $160 \text{ cm}^2 = \_\_?\_\_ \text{ m}^2$

29. $6 \text{ m}^2 = \_\_?\_\_ \text{ ft}^2$

30. **MEASUREMENT** The speed limit on a Canadian highway is 100 kilometers per hour. Approximately how fast can you drive on this highway in miles per hour? (Lesson 4-6)

31. **MEASUREMENT** Makiah has ten liters of water. She wants to pour the water into gallon jugs. To the nearest hundredth, how many gallons of water does she have? (Lesson 4-6)

Estimate each square root to the nearest whole number. (Lesson 3-2)

32. $\sqrt{11}$

33. $\sqrt{48}$

34. $-\sqrt{118}$

GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression.** (Lesson 1-2)

35. $\dfrac{45 - 33}{10 - 8}$

36. $\dfrac{85 - 67}{2001 - 1995}$

37. $\dfrac{29 - 44}{55 - 50}$

38. $\dfrac{18 - 19}{25 - 30}$

4-9 Rate of Change

Main IDEA

Find rates of change.

Preparation for Standard 7AF3.4 Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities.

GET READY for the Lesson

E-MAIL The table shows the number of entries in Alicia's e-mail contact list at the end of 2004 and 2006.

Alicia's E-mail Contact List		
Year	2004	2006
Entries	10	38

1. What is the change in the number of entries from 2004 to 2006?
2. Over what number of years did this change take place?
3. Write a rate that compares the change in the number of entries to the change in the number of years. Express your answer as a unit rate and explain its meaning.

NEW Vocabulary

rate of change

A **rate of change** is a rate that describes how one quantity changes in relation to another.

EXAMPLE 1 Find a Positive Rate of Change

E-MAIL Alicia had 62 entries in her e-mail contact list at the end of 2007. Use the information above to find the rate of change in the number of entries in her e-mail contact list between 2004 and 2007.

Year	2004	2007
Entries	10	62

+? The change or difference in the number of years is 2007–2004.

+? The change or difference in the number of entries is 62–10.

Write a rate that compares the change in each quantity.

$$\frac{\text{change in entries}}{\text{change in years}} = \frac{(62 - 10)\text{ entries}}{(2007 - 2004)\text{ years}}$$ Her contact list changed from 10 to 62 entries from 2004 to 2007.

$$= \frac{52\text{ entries}}{3\text{ years}}$$ Subtract to find the change in the number of entries and years.

$$\approx \frac{17\text{ entries}}{1\text{ year}}$$ Express this rate as a unit rate.

Since this rate is positive, Alicia's e-mail contact list *increased* or grew at an average rate of about 17 entries per year between 2004 and 2007.

STUDY TIP

Mental Math You can also find a rate of change, or unit rate, by dividing the numerator by the denominator.

CHECK Your Progress

a. **HEIGHTS** The table shows Ramon's height at ages 8 and 11. Find the rate of change in his height between these ages.

Age (yr)	8	11
Height (in.)	51	58

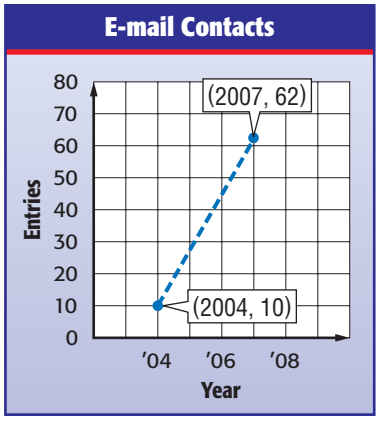
Gary Atkinson/Photonica/Getty Images

STUDY TIP

Broken Line Graph
In these 2 line graphs, the lines are broken because there are no data points between the points on the graph.

A graph of the data in Example 1 is shown at the right. The data points are connected by segments.

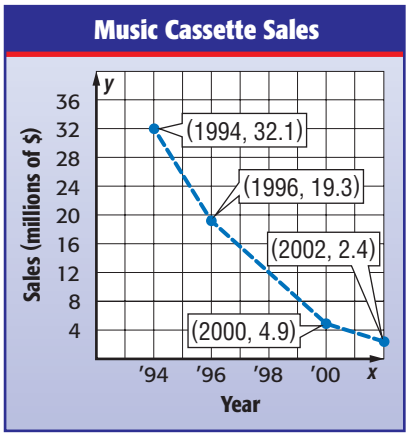
A positive rate of change is shown by a segment slanting upward from left to right.



Rates of change can also be negative.

EXAMPLE 2 Find a Negative Rate of Change

MUSIC **The graph shows cassette sales from 1994 to 2002. Find the rate of change in sales between 2000 and 2002, and describe how this rate is shown on the graph.**



Source: Recording Industry Association of America

Make a table of the data being considered using the coordinates of the points listed on the graph.

Year	Sales (millions of $)
2000	4.9
2002	2.4

Use the data to write a rate comparing the change in sales to the change in years.

$$\frac{\text{change in sales}}{\text{change in years}} = \frac{2.4 - 4.9}{2002 - 2000}$$ Sales changed from \$4.9 million to \$2.4 million from 2000 to 2002.

$$= \frac{-2.5}{2}$$ Subtract to find the change in sales amounts and years.

$$= \frac{-1.25}{1}$$ Express as a unit rate.

The rate of change was −1.25 million dollars in sales per year. The rate is negative because the cassette sales *decreased* between 2000 and 2002. This is shown on the graph by a line segment slanting downward from left to right.

STUDY TIP

Rates of Change
On a graph, the rate of change is the ratio of the change in *y*-values to the change in the *x*-values between two data points.

CHECK Your Progress

b. In the graph above, find the rate of change between 1994 and 1996.

c. Describe how this rate of change is shown on the graph.

On a graph, rates of change can be compared by measuring how fast segments rise or fall when the graph is read from left to right.

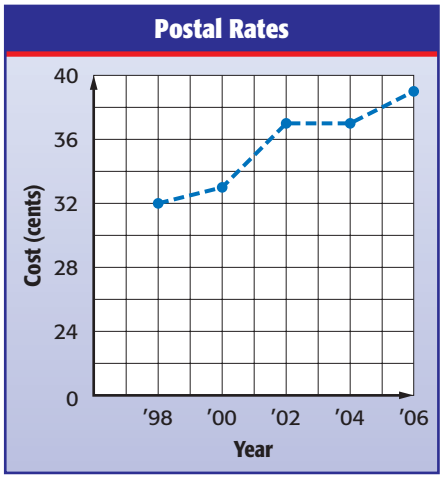
Real-World Link

In 1847, it cost 5 cents per $\frac{1}{2}$ ounce to deliver mail to locations under 300 miles away and 10 cents per $\frac{1}{2}$ ounce to deliver it to locations over 300 miles away.

Source: www.stamps.org

EXAMPLE 3 Compare Rates of Change

MAIL **The graph shows the cost in cents of mailing a 1-ounce first-class letter. Compare the rate of change between 1998 and 2000 to the rate of change between 2000 and 2002. During which period was the rate of change greatest?**



The segment from 2000 to 2002 appears steeper than the segment from 1998 to 2000. So, the rate of change between 2000 and 2002 was greater than the rate of change between 1998 and 2000.

Check Find and compare the rates of change.

From 1998 to 2000

$$\frac{\text{change in cost}}{\text{change in years}} = \frac{33 - 32}{2000 - 1998}$$

$$= \frac{1}{2} \text{ or } 0.5¢ \text{ per year}$$

From 2000 to 2002

$$\frac{\text{change in cost}}{\text{change in years}} = \frac{37 - 33}{2002 - 2000}$$

$$= \frac{4}{2} \text{ or } 2¢ \text{ per year}$$

Since $2 > 0.5$, the rate of change between 2000 and 2002 was greater than the rate of change between 1998 and 2000. ✓

CHECK Your Progress

d. NATURAL RESOURCES Use the table to make a graph of the data. During which 2-year period was the rate of change in oil production the greatest? Explain your reasoning.

Texas Oil Production				
Year	1996	1998	2000	2002
Barrels (millions)	478.1	440.6	348.9	329.8

Personal Tutor

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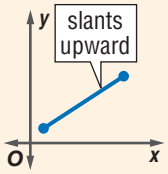
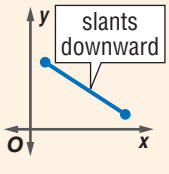
STUDY TIP

Zero Rate of Change
If a segment connecting two data points is horizontal, such as the change in the postage rate between 2002 and 2004 in Example 3, there was no change in the quantity over time.

The table below summarizes the relationship between rates of change and their graphs.

CONCEPT Summary — Rates of Change

Rate of Change	positive	negative
Real-Life Meaning	increase	decrease
Graph		

Doug Martin

CHECK Your Understanding

TEMPERATURE **For Exercises 1–3, use the information in the table at the right that shows the outside air temperature at different times during one day.**

Time	Temperature (°F)
6 A.M.	33
8 A.M.	45
12 P.M.	57
3 P.M.	57
4 P.M.	59
8 P.M.	34

Example 1 (p. 230)

1. Find the rate of temperature change in degrees per hour from 6 A.M. to 8 A.M.

Example 2 (p. 231)

2. Find the rate of temperature change between 4 P.M. and 8 P.M.

Example 3 (p. 232)

3. Make a graph of the data. During which time period was the rate of increase the greatest? Explain.

Exercises

HOMEWORK HELP

For Exercises	See Examples
4, 5, 13, 14	1
7, 8, 10, 11	2
6, 9, 12, 15	3

ADVERTISING **For Exercises 4–6, use the information in the table at the right that shows Tanisha's progress in folding flyers for the school play. She started folding at 12:55 P.M.**

Time	Flyers Folded
12:55	0
1:00	21
1:20	102
1:25	102
1:30	125

4. Find the rate of change in flyers folded per minute between 1:00 and 1:20.
5. Find her rate of change between 1:25 and 1:30.
6. Make a graph of the data. During which time period was her folding rate the greatest? Explain.

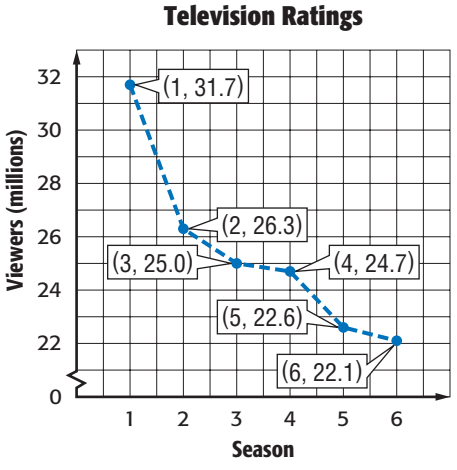
INVESTMENTS **For Exercises 7–9, use the following information.**
The value of a company's stock over a 5-day period is shown in the table.

Day	1	2	3	4	5
Value ($)	57.48	53.92	50.25	49.74	44.13

7. Determine the rate of change in value between Day 1 and Day 3.
8. What was the rate of change in value between Day 2 and Day 5?
9. Make a graph of the data. During which 2-day period was the rate of change in the stock value greatest?

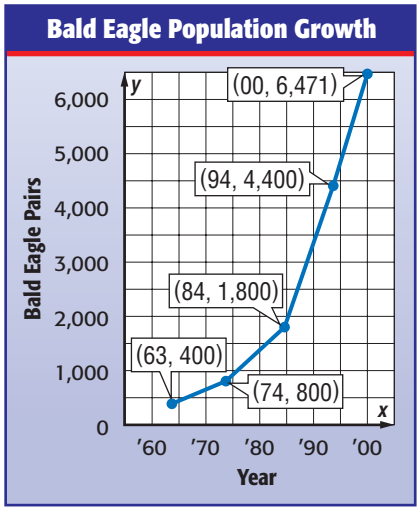
TELEVISION **For Exercises 10–12, use the information below and at the right.**
The graph shows the number of viewers who watched new episodes of a show.

10. Find the rate of change in viewership between season 1 and season 3.
11. Find the rate of change in viewership between season 2 and season 6.
12. Between which two seasons was the rate of change in viewership greatest?



BIRDS **For Exercises 13–15, use the information below and at the right.**
The graph shows the approximate number of American Bald Eagle pairs from 1963 to 2000.

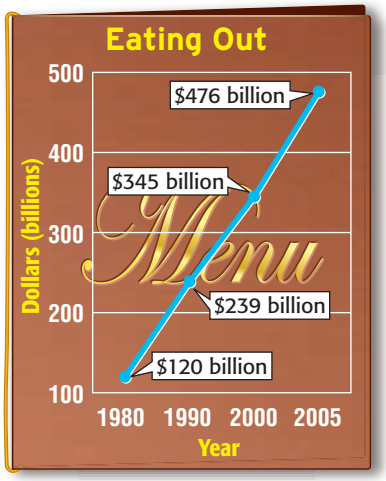
13. Find the rate of change in the number of eagle pairs from 1974 to 1994.
14. Find the rate of change in the number of eagle pairs from 1984 to 2000.
15. During which time period did the eagle population grow at the fastest rate? Explain your reasoning.



Source: birding.about.com

FAST FOOD **For Exercises 16 and 17, use the information below.**
The graph shows the estimated total of U.S. food and drink sales in billions of dollars from 1980 to 2005.

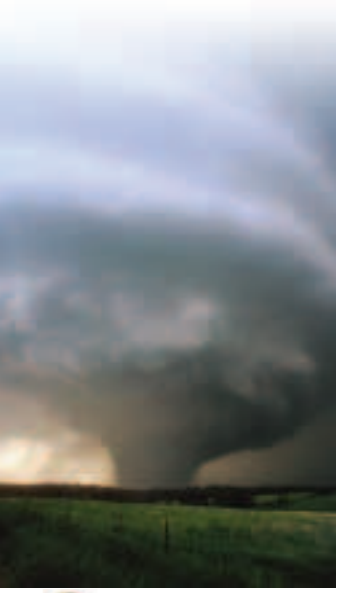
16. During which time period was the rate of change in food and drink sales greatest? Explain your reasoning.
17. Find the rate of change during that period.



Source: National Restaurant Association

SALES **For Exercises 18 and 19, use the following information.**
The National Confectioners Association reported $1,418 billion in candy sales during the 1998 winter holidays. For 2003, this figure was $1,440 billion.

18. Find the rate of change in sales from 1998 to 2003.
19. If this rate of change were to continue, what would the total candy sales during the winter holidays be in 2010? Explain your reasoning.
20. **MOLD** Nine days ago, the area covered by mold on a piece of bread was 3 square inches. Today the mold covers 9 square inches. Find the rate of change in the mold's area.



Real-World Link
On May 20, 1957, an F5 tornado touched down in Jackson, Missouri. It was the deadliest recorded tornado in Missouri's history.
Source: *missouri.edu*

TORNADOES **For Exercises 21 and 22, refer to the table at the right.**

21. Graph the data. During which interval was the rate of change in the number of tornadoes the greatest? the least?
22. Is it reasonable to state that between 2000 and 2005, the number of tornadoes in a given year changed very little? Explain.

Missouri Tornadoes

Year	Number of Tornadoes
2000	28
2001	39
2002	29
2003	84
2004	69
2005	32

Source: NOAA's National Weather Service

EXTRA PRACTICE
See pages 687, 711.
Math Online
Self-Check Quiz at ca.gr7math.com

Jim Reed/Photo Researchers

H.O.T. Problems

23. **OPEN ENDED** Create a set of gasoline price data that has a rate of change of $0.08 per gallon over a period of 4 days.

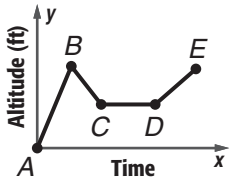
24. **NUMBER SENSE** Does the height of a candle as it burns over time show a *positive* or *negative* rate of change? Explain your reasoning.

25. **CHALLENGE** Liquid is poured at a constant rate into a beaker that is shaped like the one at the right. Draw a graph of the level of liquid in the beaker as a function of time.

26. **WRITING IN MATH** Explain the difference between the *rate of change* between a set of data values and the *change* between data values.

STANDARDS PRACTICE

27. The graph shows the altitude of a falcon over time.



Between which two points on the graph was the bird's rate of change in altitude negative?

A *A* and *B*

B *B* and *C*

C *C* and *D*

D *D* and *E*

28. Sarah earns $52 for 4 hours of work. At this rate, how many hours would she need to work to earn $975?

F 13 h **H** 75 h

G 18.75 h **J** 243.75 h

29. Ralph rode his bike an average speed of 16 miles per hour for two hours on Saturday and then an average speed of 13 miles per hour for three hours. How many miles did Ralph ride in all?

A 29 miles

B 34 miles

C 71 miles

D 74 miles

Spiral Review

30. **ARCHITECTURE** A certain building is 925 feet tall. On a scale drawing of the building, it is 18.5 inches tall. What is the scale for the drawing? (Lesson 4-8)

31. **MEASUREMENT** The area of the kitchen floor is 180 square feet. The tiles measure 36 square inches. How many tiles will it take to cover the entire floor? (Lesson 4-7)

GET READY for the Next Lesson

32. **PREREQUISITE SKILL** Michael tutors elementary school students in math 1.5 hours each week. Is the total number of hours that he spends tutoring proportional to the number of weeks he tutors during the year? Explain your reasoning. (Lesson 4-1)

4-10 Constant Rate of Change

Main IDEA

Identify proportional and nonproportional linear relationships by finding a constant rate of change.

Preparation for Standard 7AF3.4 Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities.

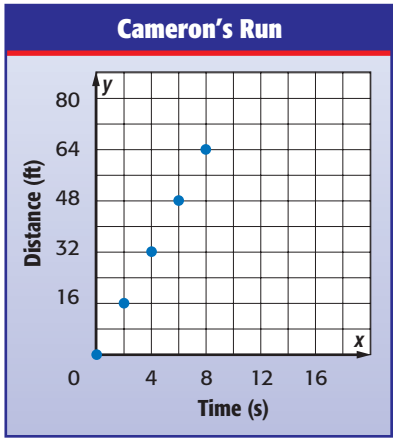
NEW Vocabulary

linear relationship
constant rate of change

GET READY for the Lesson

EXERCISE Cameron knows that after he has warmed up, he can maintain a constant running speed of 8 feet per second. This is shown in the table and in the graph.

Time (s)	0	2	4	6	8
Distance (ft)	0	16	32	48	64



1. Pick several pairs of points and find the rate of change between them. What is true of these rates?

Relationships that have straight-line graphs, like the one in the example above, are called **linear relationships**. Notice that as the time in seconds increases by 2, the distance in feet increases by 16.

		+2	+2	+2	+2
Time (s)	0	2	4	6	8
Distance (ft)	0	16	32	48	64
		+16	+16	+16	+16

Rate of Change

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{16}{2} \text{ or } 8 \text{ ft/s}$$

The rate of change between any two points in a linear relationship is the same or *constant*. A linear relationship has a **constant rate of change**.

EXAMPLE 1 Identify Linear Relationships

MONEY **The balance in an account after several transactions is shown. Is the relationship between the balance and number of transactions linear? If so, find the constant rate of change. If not, explain your reasoning.**

Number of Transactions	Balance ($)
3	170
6	140
9	110
12	80

Examine the change in the number of transactions and in the balance of the account.

	Number of Transactions	Balance ($)	
	3	170	
+3	6	140	−30
+3	9	110	−30
+3	12	80	−30

As the number of transactions increases by 3, the balance in the account decreases by $30.

STUDY TIP

Check To check the answer to Example 1, graph the data in the table, with the number of transactions on the *x*-axis and the balance in the account on the *y*-axis. If the data points fall on a line, then the relationship is linear.

Since the rate of change is constant, this is a linear relationship. The constant rate of change is $\frac{-30}{3}$ or $-\$10$ per transaction. This means that, on average, each transaction involved a \$10 *withdrawal*.

CHECK Your Progress

Determine whether the relationship between the two quantities described in each table is linear. If so, find the constant rate of change. If not, explain your reasoning.

a.

Cooling Water	
Time (min)	Temperature (°F)
5	95
10	90
15	86
20	82

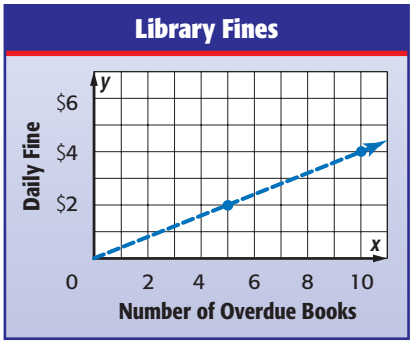
b.

Wrapping Paper	
Number of Rolls	Total Cost (\$)
2	8.50
4	17.00
6	25.50
8	34.00

Real-World Link . . . . .
With 85 branches, the New York Public Library is the world's largest public library. It has collections totaling 11.6 million items.
Source: nupl.org

EXAMPLE 2 Find a Constant Rate of Change

LIBRARIES **Find the constant rate of change for the daily fine for each overdue book in the graph shown. Interpret its meaning.**



Choose any two points on the line and find the rate of change between them.

(5, 2) ⟶ 5 books, \$2 daily fine

(10, 4) ⟶ 10 books, \$4 daily fine

$$\frac{\text{change in fine}}{\text{change in books}} = \frac{\$(4-2)}{(10-5)\text{ books}}$$

$$= \frac{\$2}{5\text{ books}}$$

$$= \frac{\$0.40}{1\text{ book}}$$

The daily fine changed from \$2 to \$4 and from 5 to 10 overdue books

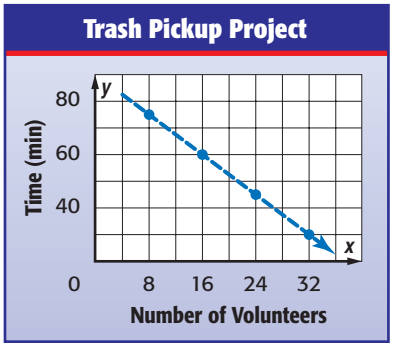
Subtract to find the change in the daily fine and number of books.

Express this rate as a unit rate.

The daily fine is \$0.40 per overdue book.

CHECK Your Progress

c. **SERVICE PROJECT** Find the constant rate of change for the time it takes to complete a highway trash pickup project for each number of volunteers in the graph shown. Interpret its meaning.



Personal Tutor

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Zoran Milich/Masterfile

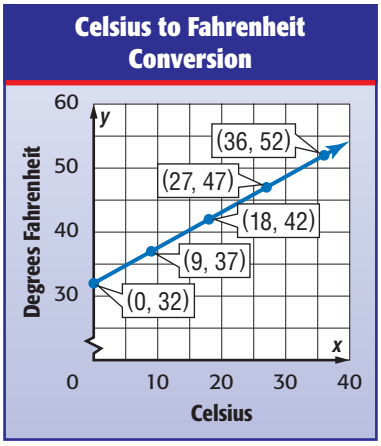
Some, but not all, linear relationships are also proportional.

EXAMPLE Identify Proportional Relationships

STUDY TIP

Look Back
To review **identifying proportional relationships,** see Lesson 4-2.

3 **TEMPERATURE** **Use the graph to determine if there is a proportional linear relationship between a temperature on the Fahrenheit scale and a temperature on the Celsius scale. Explain your reasoning.**



Since the graph of the data forms a line, the relationship between the two scales is linear. This can also be seen in the table of values created using the points on the graph.

	+9	+9	+9	+9	
Degrees Celsius	0	9	18	27	36
Degrees Fahrenheit	32	37	42	47	52
	+5	+5	+5	+5	

Constant Rate of Change

$$\frac{\text{change in °F}}{\text{change in °C}} = \frac{5}{9}$$

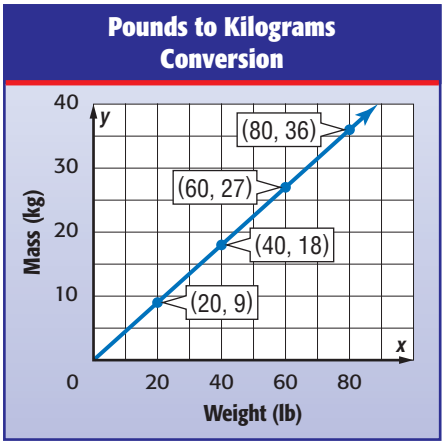
To determine if the two scales are proportional, express the relationship between the degrees for several columns as a ratio.

$$\frac{\text{degrees Fahrenheit}}{\text{degrees Celsius}} \longrightarrow \frac{37}{9} \approx 4.11 \quad \frac{42}{18} \approx 2.33 \quad \frac{47}{27} \approx 1.74 \quad \frac{52}{36} \approx 1.44$$

Since the ratios are not all the same, a temperature in degrees Celsius is *not* proportional to the same temperature in degrees Fahrenheit.

CHECK Your Progress

d. **MEASUREMENT** Use the graph to determine if there is a proportional linear relationship between the weight of an object measured in pounds and the mass of the same object measured in kilograms. Explain your reasoning.



CONCEPT Summary Proportional Linear Relationships

Words	Two quantities a and b have a proportional linear relationship if they have a constant ratio and a constant rate of change.
Symbols	$\frac{b}{a}$ is constant and $\frac{\text{change in } b}{\text{change in } a}$ is constant.

MathSpline **Extra Examples at** ca.gr7math.com

CHECK Your Understanding

Example 1 (p. 236)

Determine whether the relationship between the two quantities described in each table is linear. If so, find the constant rate of change. If not, explain your reasoning.

1.

Volume of Cube	
Side Length (cm)	Volume (cm^3)
2	8
3	27
4	64
5	125

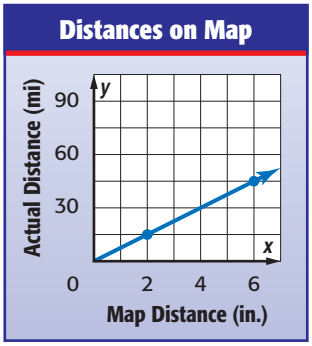
2.

Paint Needed for Chairs	
Number of Chairs	Cans of Paint
5	6
10	12
15	18
20	24

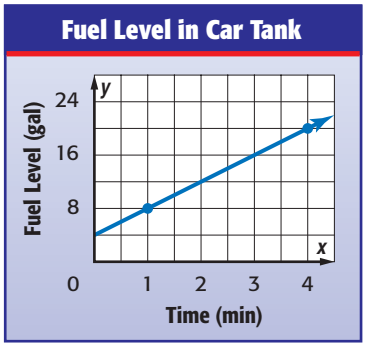
Example 2 (p. 237)

Find the constant rate of change for each graph and interpret its meaning.

3.



4.



Example 3 (p. 238)

Determine whether a proportional linear relationship exists between the two quantities shown in each of the indicated graphs. Explain your reasoning.

5. Exercise 3

6. Exercise 4

Exercises

HOMEWORK HELP

For Exercises	See Examples
7–10	1
11–16	2
17–22	3

Determine whether the relationship between the two quantities described in each table is linear. If so, find the constant rate of change. If not, explain your reasoning.

7.

Cost of Electricity to Run Personal Computer	
Time (h)	Cost (¢)
5	15
8	24
12	36
24	72

8.

Total Number of Customers Helped at Jewelry Store	
Time (h)	Total Helped
1	12
2	24
3	36
4	60

9.

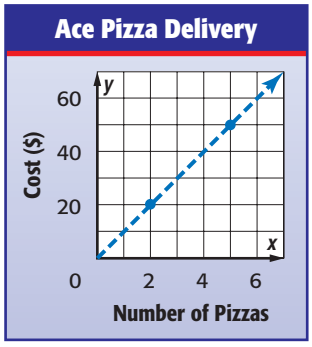
Distance Traveled by Falling Object				
Time (s)	1	2	3	4
Distance (m)	4.9	19.6	44.1	78.4

10.

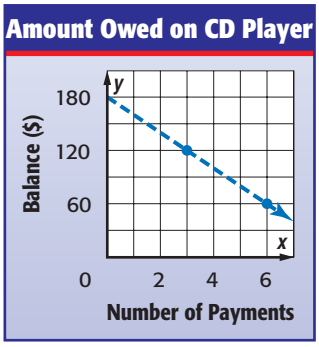
Italian Dressing Recipe				
Oil (c)	2	4	6	8
Vinegar (c)	$\frac{3}{4}$	$1\frac{1}{2}$	$2\frac{1}{4}$	3

Find the constant rate of change for each graph and interpret its meaning.

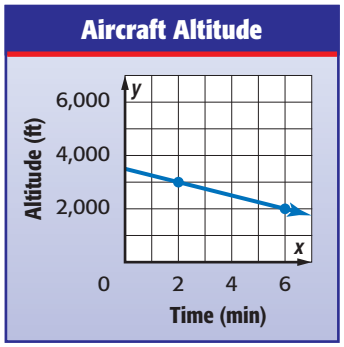
11.



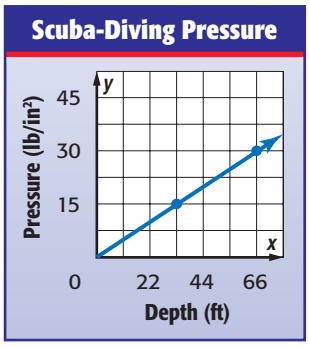
12.



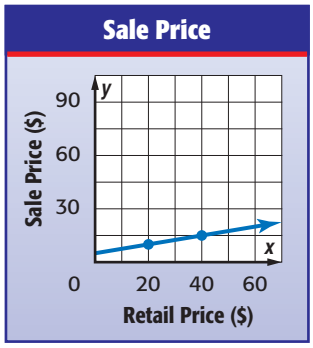
13.



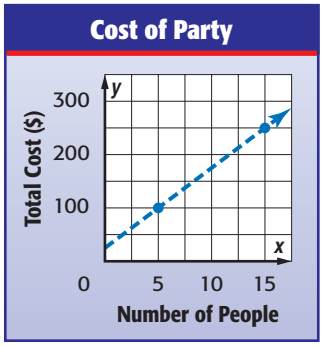
14.



15.



16.

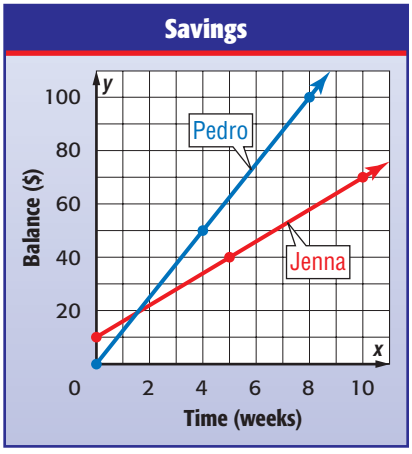


Determine whether a proportional relationship exists between the two quantities shown in each of the indicated graphs. Explain your reasoning.

17. Exercise 11
18. Exercise 12
19. Exercise 13
20. Exercise 14
21. Exercise 15
22. Exercise 16

SAVINGS For Exercises 23 and 24, use the following information.
Both Pedro and Jenna are saving money. Their savings account balances over several weeks are shown.

23. Who is saving more money each week? Explain your reasoning.
24. Whose savings are proportional to the number of weeks they have been saving? Explain.



EXTRA PRACTICE
See pages 688, 711.
Math Online
Self-Check Quiz at ca.gr7math.com

H.O.T. Problems

25. **OPEN ENDED** Graph two quantities that have a proportional linear relationship. Justify your answer.

26. **CHALLENGE** Examine the graphs in Exercises 3, 4, and 11–16, as well as your corresponding answers in Exercises 5 and 17–22. What point do all of the graphs that represent proportional linear relationships have in common?

27. **WRITING IN MATH** Write a real-world problem in which you would need to find a constant rate of change. Then solve your problem. Is the relationship described in your problem proportional? Explain.

STANDARDS PRACTICE

28. Tickets to the school play are $2.50 each. Which table contains values that fit this situation, if c represents the total cost for t tickets?

A

Cost of Play Tickets				
t	1	2	3	4
c	$2.50	$3.25	$4.00	$4.75

B

Cost of Play Tickets				
t	1	2	3	4
c	$3.50	$6.00	$8.50	$11.00

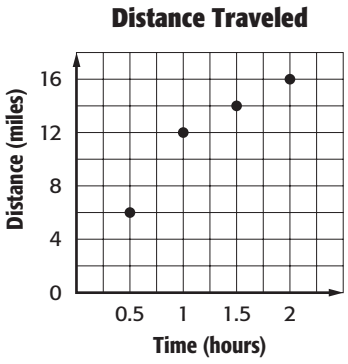
C

Cost of Play Tickets				
t	1	2	3	4
c	$3.50	$4.00	$4.50	$5.00

D

Cost of Play Tickets				
t	1	2	3	4
c	$2.50	$5.00	$7.50	$10.00

29. The graph shows the distance Bianca traveled over her 2-hour bike ride.



Which of the following statements is true?

F She traveled at a constant speed of 12 miles per hour for the entire ride.

G She traveled at a constant speed of 8 miles per hour for the last hour.

H She traveled at a constant speed of 4 miles per hour for the last hour.

J She traveled at a constant speed of 8 miles per hour for the entire ride.

Spiral Review

30. **MEASUREMENT** Three years ago, an oak tree was 4 feet 5 inches tall. Today it is 6 feet 3 inches tall. How fast did the tree grow in inches per year? (Lesson 4-9)

31. **GEOGRAPHY** On a map, two cities are 3.25 inches apart. If the scale for the map is 0.5 inch = 40 miles, how many miles apart are the cities? (Lesson 4-8)

ALGEBRA Solve each equation. Check your solution(s). (Lesson 3-1)

32. $p^2 = 0.49$

33. $t^2 = \frac{1}{144}$

34. $6{,}400 = r^2$

CHAPTER 4

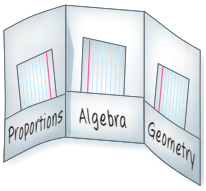
Study Guide and Review

Download Vocabulary Review from ca.gr7math.com

FOLDABLES Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Proportions (Lessons 4-2 and 4-3)

- If two related quantities are proportional, then they have a constant ratio.
- A proportion is an equation stating that two ratios or rates are equivalent.
- The cross products of a proportion are equal.

Similar Polygons (Lesson 4-5)

- If two figures are similar with a scale factor of $\frac{a}{b}$, then the perimeters of the figures have a ratio of $\frac{a}{b}$.

Measurement (Lessons 4-6 and 4-8)

- You can multiply by a unit ratio to *convert* or change from larger units to smaller units.
- To convert from smaller units to larger units, multiply by the *reciprocal* of the appropriate unit ratio.
- A scale is determined by the ratio of a given length on a drawing or model to its corresponding actual length.

Rates (Lessons 4-1, 4-9, and 4-10)

- A rate is a comparison of two quantities with different types of units.
- To find the rate of change, divide the difference in the *y*-coordinates by the difference in the *x*-coordinates.
- Two quantities *a* and *b* have a proportional linear relationship if they have a constant ratio and a constant rate of change.

Key Vocabulary

congruent (p. 207)
constant of proportionality (p. 200)
constant rate of change (p. 236)
corresponding parts (p. 206)
cross products (p. 198)
equivalent ratios (p. 198)
linear relationship (p. 236)
nonproportional (p. 194)
polygon (p. 206)
proportion (p. 198)
proportional (p. 194)
rate (p. 191)
rate of change (p. 230)
ratio (p. 190)
scale (p. 224)
scale drawing (p. 224)
scale factor (p. 207)
scale model (p. 224)
similar (p. 206)
unit rate (p. 191)
unit ratio (p. 214)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. Polygons that have the same <u>size</u> are called similar polygons.
2. A <u>unit ratio</u> is one in which the denominator is 1 unit.
3. A ratio of two measurements having <u>similar</u> units is called a rate.
4. In a relationship in which the ratio is not constant, the two quantities are said to be <u>nonproportional</u>.
5. A <u>scale</u> is the ratio of a length on a drawing or model to its corresponding actual length.
6. Comparing two numbers by <u>multiplication</u> is called a proportion.

MathOnline Vocabulary Review at ca.gr7math.com

Lesson-by-Lesson Review

4-1 Ratios and Rates (pp. 190–193)

Express each ratio in simplest form.

7. 7 chaperones for 56 students
8. 12 peaches:8 pears
9. 5 inches out of 5 feet
10. **SHOPPING** An 8-pound bag of cat food sells for $13.89. A 20-pound bag of the same brand sells for $24.79. Which is the better buy? Explain your reasoning.

Example 1 **Express the ratio *10 milliliters to 8 liters* in simplest form.**

$$\frac{10 \text{ milliliters}}{8 \text{ liters}}$$

$$= \frac{1 \cancel{\text{ milliliters}}}{8{,}000 \cancel{\text{ milliliters}}}$$ 8 liters = 8 × 1,000 or 8,000 milliliters.

$$= \frac{1}{800}$$ Divide numerator and denominator by 10.

The ratio in simplest form is $\frac{1}{800}$ or 1:800.

4-2 Proportional and Nonproportional Relationships (pp. 194–197)

11. **INTERNET** A high-speed Internet company charges $30 a month for Internet services. There is also a $30 installation fee. Is the number of months you can have high-speed Internet proportional to the total cost?
12. **WORK** On Friday, Jade washed a total of 9 vehicles in 4 hours. The next day she washed a total of 15 vehicles in 6 hours. Is the total number of vehicles she washed over the two days proportional to the time it took her to wash them?

Example 2 **Leo earns $28 for every haircut he does. Is the amount of money he earns proportional to the number of haircuts he gives?**

Haircuts	1	2	3	4
Earnings ($)	28	56	84	112

$$\frac{\text{earnings}}{\text{haircuts}} \rightarrow \frac{28}{1} \text{ or } 28 \quad \frac{56}{2} \text{ or } 28 \quad \frac{84}{3} \text{ or } 28 \quad \frac{112}{4} \text{ or } 28$$

Since these ratios are all equal to 28, the amount of money he earns is proportional to the number of people's hair he cuts.

4-3 Solving Proportions (pp. 198–202)

Solve each proportion.

13. $\frac{3}{r} = \frac{6}{8}$
14. $\frac{30}{0.5} = \frac{y}{0.25}$
15. $\frac{7}{4} = \frac{n}{2}$
16. $\frac{k}{5} = \frac{72}{8}$
17. **ANIMALS** A turtle can move 5 inches in 4 minutes. How far will it travel in 10 minutes?

Example 3 **Solve $\frac{9}{x} = \frac{4}{18}$.**

$\frac{9}{x} = \frac{4}{18}$ Write the equation.

$9 \cdot 18 = x \cdot 4$ Find the cross products.

$162 = 4x$ Multiply.

$\frac{162}{4} = \frac{4x}{4}$ Divide each side by 4.

$40.5 = x$ Simplify.

CHAPTER 4

Study Guide and Review

4-4 PSI: Draw a Diagram (pp. 204–205)

Solve. Use the *draw a diagram* strategy.

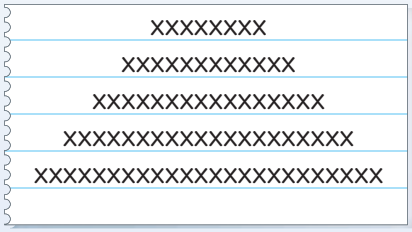
18. **CONCERTS** Nina, Tyrese, Leslie, and Ethan are going to a rock concert. In how many different orders can they enter the concert?

19. **PHYSICAL SCIENCE** A tennis ball is dropped from 12 feet above the ground. It hits the ground and bounces up half as high as it fell. This is true for each successive bounce. What height does the ball reach on the fourth bounce?

20. **MEASUREMENT** Jasmine unrolled 48 feet of carpet. This is $\frac{3}{4}$ of the total amount of carpet needed for the library. What is the total amount of carpet needed for the library?

Example 4 **A photographer is taking the eighth grade class picture. She places 8 students in the first row. Each additional row has 4 more students in it. If there are a total 80 students, how many rows will there be?**

Draw a diagram with 8 students in row one and then add 4 more students to each additional row.

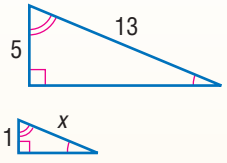


There are a total of 5 rows.

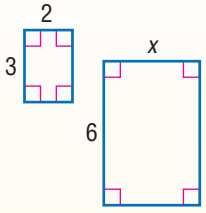
4-5 Similar Polygons (pp. 206–211)

Each pair of polygons is similar. Write a proportion to find each missing measure. Then solve.

21.



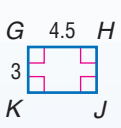
22.

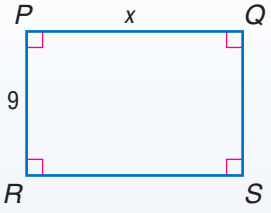


23. **MEASUREMENT** If square D has a perimeter of 49 feet and square F has a perimeter of 64 feet, what is the scale factor of the two squares?

24. **PARTY PLANNING** For your birthday party, you make a map to your house on a 3-inch wide by 5-inch long index card. How long will your map be if you use a copier to enlarge it so it is 8 inches wide?

Example 5 **Rectangle *GHJK* is similar to rectangle *PQRS*. Find the value of *x*.**





The scale factor from *GHJK* to *PQRS* is $\frac{GK}{PR}$, which is $\frac{3}{9}$ or $\frac{1}{3}$.

$\frac{GH}{PQ} = \frac{1}{3}$ Write a proportion.

$\frac{4.5}{x} = \frac{1}{3}$ $GH = 4.5$ and $PQ = x$

$4.5 \cdot 3 = 1 \cdot x$ Find the cross products.

$13.5 = x$ Multiply.

Mixed Problem Solving
For mixed problem-solving practice, see page 711.

4-6 Measurement: Converting Length, Weight/Mass, Capacity, and Time (pp. 213–218)

Complete each conversion. Round to the nearest hundredth if necessary.

25. 5 in. ≈ ■ cm **26.** 25 km ≈ ■ mi

27. Which is greater: a 10-pound weight or a 5-kilogram weight?

28. Which is greater: a 5,000-meter race or a 4-mile race?

Example 6 **Which has a greater capacity: a bottle containing 32 fluid ounces of spring water or a bottle containing 1 liter of water?**

Use the unit ratios $\frac{29.574 \text{ mL}}{1 \text{ fl oz}}$ and $\frac{1 \text{ L}}{1{,}000 \text{ mL}}$.

$$32 \text{ fl oz} \approx 32 \text{ fl oz} \cdot \frac{29.574 \text{ mL}}{1 \text{ fl oz}} \cdot \frac{1 \text{ L}}{1{,}000 \text{ mL}}$$

$$\approx 32 \cdot \frac{29.574 \text{ L}}{1{,}000} \text{ or } 0.95 \text{ L}$$

The 1-liter bottle contains more water.

4-7 Measurement: Converting Square Units and Cubic Units (pp. 220–223)

Complete each conversion. Round to the nearest hundredth if necessary.

29. 120 yd^3 = ■ ft^3 **30.** 16 m^2 ≈ ■ ft^2

31. PARKING The area of a parking lot is 375,000 square feet. How many square meters is the parking lot?

32. STORAGE The total capacity of a certain storage unit is about 23 cubic meters. How many cubic feet is the storage unit?

Example 7 **Convert 15 square centimeters to square inches.**

Use the unit ratio $\frac{1 \text{ in.}}{2.54 \text{ cm}}$.

$$15 \text{ cm}^2 \approx 15 \text{ cm}^2 \cdot \frac{1 \text{ in.}}{2.54 \text{ cm}} \cdot \frac{1 \text{ in.}}{2.54 \text{ cm}}$$

$$\approx \frac{15 \text{ in}^2}{2.54 \cdot 2.54} \text{ or } 2.33 \text{ in}^2$$

4-8 Scale Drawings and Models (pp. 224–229)

The scale on a map is 2 inches = 5 miles. Find the actual distance for each map distance.

33. 12 inches **34.** 4 inches

35. HOBBIES Mia's dollhouse is a replica of her family's townhouse. The outside dimensions of the dollhouse are 25 inches by 35 inches. If the actual outside dimensions of the townhouse are 25 feet by 35 feet, what is the scale of the dollhouse?

Example 8 **The scale on a model is 3 centimeters = 45 meters. Find the actual length for a model distance of 5 centimeters.**

$\frac{3 \text{ cm}}{45 \text{ m}} = \frac{5 \text{ cm}}{x \text{ m}}$ ← model length ← actual length

$3 \cdot x = 45 \cdot 5$ Find the cross products.

$3x = 225$ Multiply.

$x = 75$ Divide each side by 3.

The actual length is 75 meters.

CHAPTER 4

Study Guide and Review

4-9 Rate of Change (pp. 230–235)

MONEY **For Exercises 34 and 35, use the following information.**

The table below shows Victor's weekly allowance between the ages of 6 and 15.

Age (yr)	6	8	10	12	15
$ per week	1.00	2.00	2.00	3.00	5.00

34. Find the rate of change in his allowance between ages 12 and 15.

35. Was the rate of change between ages 8 and 10 positive, negative, or zero?

Example 9 **At 5 A.M., it was 54°F. At 11 A.M., it was 78°F. Find the rate of temperature change in degrees per hour.**

$$\frac{\text{change in temperature}}{\text{change in hours}} = \frac{(78 - 54)°}{(11 - 5) \text{ hours}}$$

$$= \frac{24°}{6 \text{ hours}} \text{ or } \frac{4°}{1 \text{ hour}}$$

Between 5 A.M. and 11 A.M., the temperature increased on average 4 degrees per hour.

4-10 Constant Rate of Change (pp. 236–241)

36. **RAINFALL** The amount of rainfall after several hours is shown. Is the relationship between the amount of rainfall and number of hours linear? If so, find the constant rate of change. If not, explain your reasoning.

Number of Hours	Rainfall (inches)
1	2
2	4
3	7
4	9

37. **PHONE CALL** The cost of a long-distance phone call after several minutes is shown. Is the relationship between the cost and number of minutes linear? If so, find the constant rate of change. If not, explain your reasoning.

Number of Minutes	Cost (¢)
3	7
6	14
9	21
12	28

Example 10 **The distance traveled in a car trip is shown. Is the relationship between the distance traveled and number of hours spent in the car linear? If so, find the constant rate of change. If not, explain your reasoning.**

	Number of Hours	Distance (miles)	
+2	2	120	×2
+2	4	240	×2
+2	6	360	×2
	8	480	

As the number of hours increases by two, the distance doubles. Since the rate of change is constant, this is a linear relationship. So, the constant rate of change is $\frac{120}{2}$ or 60 miles per hour. This means that for every hour they are in the car they travel 60 miles.

CHAPTER 4

Practice Test

1. Express the ratio 15 inches to 1 foot in simplest form.

2. Express 112 feet in 2.8 seconds as a unit rate.

3. **STANDARDS PRACTICE** At Flynn's Apple Orchard, 16 acres of land produced 368 bushels of apples. Which rate represents the number of bushels per acre?

   A 16:1 **C** 23:2
   B 23:1 **D** 46:1

4. **MEASUREMENT** Nick rides his bike 20 miles every two days. Is the distance Nick rides proportional to the number of days?

Solve each proportion.

5. $\frac{3}{a} = \frac{9}{12}$

6. $\frac{5}{3} = \frac{20}{y}$

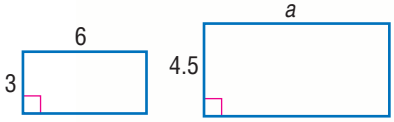
7. **NUTRITION** An 8-ounce serving of milk provides 30% of the daily value of calcium. How much milk provides 50% of the daily value of calcium?

8. **FOOD** Of the 30 students in a life skills class, 19 like to cook main dishes, 15 prefer baking desserts, and 7 like to do both. How many students like to cook main dishes, but not bake desserts? Use the *draw a diagram* strategy.

Each pair of polygons is similar. Write a proportion to find each missing measure. Then solve.

9.

10.



11. **COOKING** Denise is making recipe for a large group in which she needs 12 kilograms of ground beef. How many pounds of ground beef does she need? (1 lb ≈ 0.4536 kg)

12. **TRAVEL** On a map, 1 inch = 7.5 miles. How many miles does 2.5 inches represent?

BUSINESS For Exercises 13 and 14, use the table that shows the number of new customers in The Lucky Diner at different times during one day.

Time	New Customers
12 P.M.	30
2 P.M.	6
4 P.M.	15
5 P.M.	32

13. Find the rate of change in new customers per hour between 4 P.M. and 5 P.M.

14. Find the rate of change in new customers per hour between 12 P.M. and 2 P.M. Then interpret its meaning.

15. **STANDARDS PRACTICE** A flag is being made that has an area of six square feet. Approximately how many square meters of fabric is this? (1 ft ≈ 0.3048 m)

   F 0.58 m^2 **H** 19.69 m^2
   G 1.83 m^2 **J** 64.58 m^2

16. **MEASUREMENT** Is the relationship between the weight and number of months linear? If so, find the constant rate of change. If not, explain your reasoning.

Number of Months	Weight (lb)
4	14
6	18
8	20
10	22

CHAPTER 4

California Standards Practice

Cumulative, Chapters 1–4

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1 A jar contains 25% green buttons, 32% yellow buttons, 20% brown buttons, and 23% white buttons. There are 300 buttons in the jar altogether. Which proportion can be used to find w, the total number of white buttons in the jar?

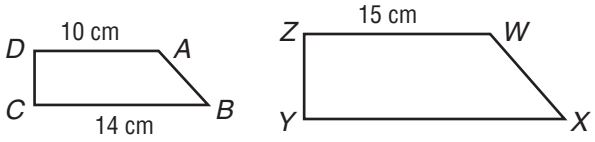
A $\frac{23}{100} = \frac{300}{w}$

B $\frac{23}{w} = \frac{300}{100}$

C $\frac{23}{100} = \frac{w}{300}$

D $\frac{23}{300} = \frac{w}{100}$

2 Trapezoid $ABCD$ is similar to trapezoid $WXYZ$. Find the length of $\overline{XY}$.



F 20 cm

G 21 cm

H 24 cm

J 27 cm

TEST-TAKING TIP

Question 2 This problem involves similar figures. If two polygons are similar, then you can use a scale factor or a proportion to find the missing measure(s).

3 How many seconds are in $1\frac{1}{2}$ hours?

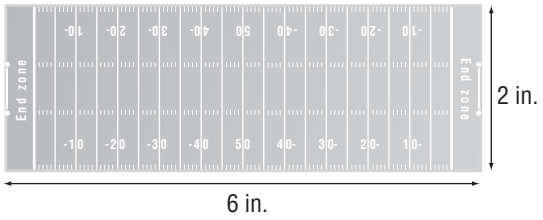
A 90

B 540

C 3,600

D 5,400

4 The scale drawing of a football field was made using a scale of 1 inch = 20 yards.



What is the length, in yards, of the football field, including the end zones?

F 100 yd

G 120 yd

H 130 yd

J 150 yd

5 Sixty-five miles per hour is the same rate as which of the following?

A 1.08 miles per second

B 3,900 miles per second

C 6.5 miles per minute

D 1.08 miles per minute

6 Between which two whole numbers is $\sqrt{66}$ located on a number line?

F 6 and 7

G 7 and 8

H 8 and 9

J 9 and 10

7 Which operation results in the same answer regardless of the numbers involved?

A divide by one

B multiply by one

C add zero

D multiply by zero

MathOnline **California Standards Practice at** ca.gr7math.com

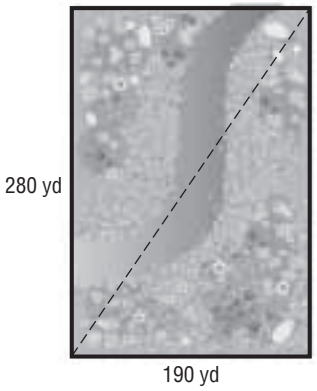
8 Rebekah is $1\frac{1}{2}$ meters tall. About how tall is she in feet and inches?
(1 meter ≈ 39 inches)

F 3 feet 3 inches
G 4 feet 0 inches
H 4 feet 9 inches
J 4 feet 8 inches

9 During a 3-hour period, 2,292 people rode the rollercoaster at an amusement park. Which proportion can be used to find x, the number of people who rode the coaster during a 12-hour period if the rate is the same?

A $\frac{3}{2{,}292} = \frac{x}{12}$
B $\frac{3}{2{,}292} = \frac{12}{x}$
C $\frac{3}{x} = \frac{12}{2{,}292}$
D $\frac{x}{3} = \frac{12}{2{,}292}$

10 A park is shaped like a rectangle with the dimensions shown below. Which of the following is closest to the length of a diagonal of the park?



F 165 yd
G 290 yd
H 340 yd
J 405 yd

11 A teacher plans to buy 5 pencils for each student in her class. If pencils come in packages of 18 and cost $1.99 per package, what other information is needed to find the cost of the pencils?

A the cost of erasers
B the number of students in the whole school
C the number of students in her class
D the name of the store where she is buying the pencils

12 Which fraction is between $\frac{2}{3}$ and $\frac{9}{10}$?

F $\frac{1}{5}$
G $\frac{1}{2}$
H $\frac{3}{4}$
J $\frac{11}{9}$

Pre-AP

Record your answers on a sheet of paper. Show your work.

13 The table shows how much Susan earns when she works at a fast-food restaurant.

Time (h)	Wages ($)
2	9
4	18
6	27
8	36

a. Graph the data from the table and connect the points with a line.
b. Find the slope of the line.
c. What is Susan's rate of pay?
d. If Susan continues to be paid at this rate, how much money will she make for working 10 hours?

NEED EXTRA HELP?													
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson...	4-3	4-5	4-6	4-8	4-6	3-2	1-2	4-6	4-3	3-6	1-1	2-2	4-10
For Help with Standard...	AF4.2	6NS1.3	MG1.1	MG1.2	MG1.3	NS2.4	AF1.3	MG1.1	AF4.2	MG3.3	MR1.1	NS1.1	AF3.4