

Unit 3

Geometry and Measurement

Focus

Demonstrate an understanding of geometry, spatial reasoning, and measurement.

CHAPTER 6 Geometry and Spatial Reasoning

BIG Idea Understand plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures.

CHAPTER 7 Measurement: Area and Volume

BIG Idea Compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. Know how perimeter, area, and volume are affected by changes of scale.



Cross-Curricular Project

Math and Architecture

Under Construction You've been selected to head the architectural and construction teams on a house of your own design. You'll create the uniquely-shaped floor plan, research different floor coverings, and finally research different loans to cover the cost of purchasing these floor coverings. So grab a hammer and some nails, and don't forget your geometry and measurement tool kits. You're about to construct a cool adventure!

Math  **online** Log on to ca.gr7math.com to begin.



CHAPTER

6

Geometry and Spatial Reasoning



BIG Idea

- **Standard 7MG3.0** Deepen understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures.

Key Vocabulary

congruent polygons (p. 320)

reflection (p. 332)

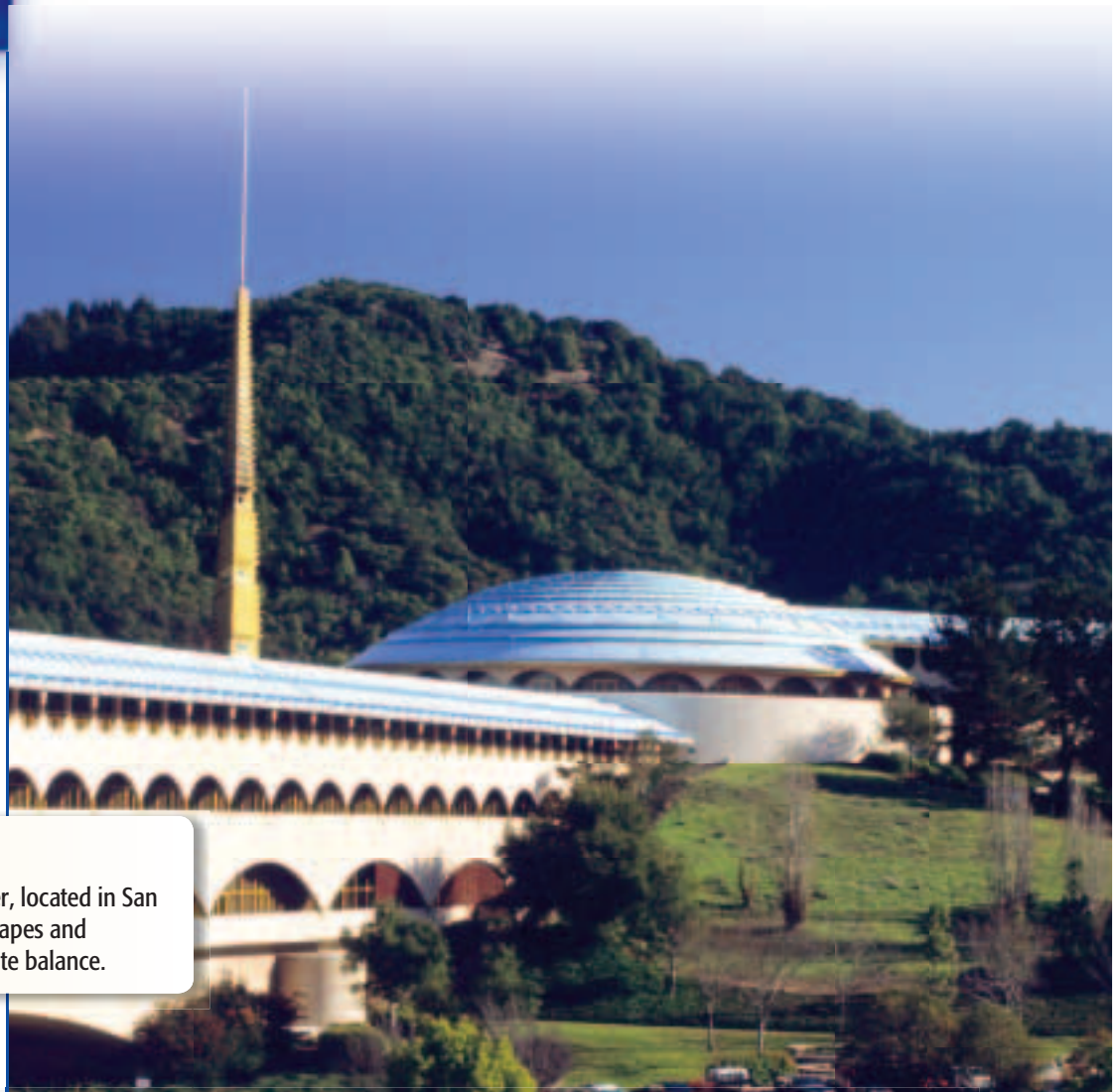
transformation (p. 332)

translation (p. 337)



Real-World Link

Architecture The Marin Civic Center, located in San Rafael, California, uses geometric shapes and properties such as symmetry to create balance.

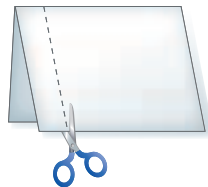


FOLDABLES

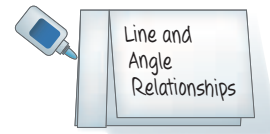
Study Organizer

Geometry and Spatial Reasoning Make this Foldable to help you organize your notes. Begin with 7 sheets of plain $8\frac{1}{2}'' \times 11''$ paper.

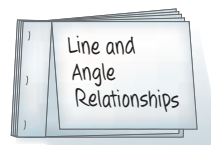
- 1** **Fold** a sheet of paper in half lengthwise. Cut a 1'' tab along the left edge through one thickness.



- 2** **Glue** the 1'' tab down. Write the title of the lesson on the front tab.



- 3** **Repeat** Steps 1–2 for the remaining sheets of paper. Staple together to form a booklet.



GET READY for Chapter 6

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2



Take the Online Readiness Quiz at ca.gr7math.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

Solve each equation. (Lesson 1-9)

1. $49 + b + 45 = 180$

2. $t + 98 + 55 = 180$

3. $15 + 67 + k = 180$

4. **LAWNS** Lawrence made \$60 on Monday and \$48 on Tuesday mowing lawns. How much did he make on Wednesday if his three-day total was \$180? (Lesson 1-9)

Evaluate each expression. (Lesson 1-2)

5. $(3 - 2)180$

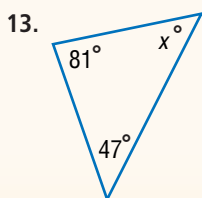
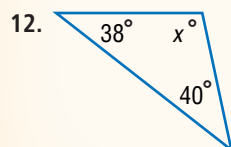
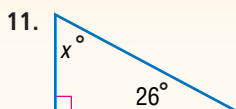
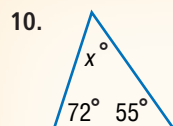
6. $(7 - 2)180$

7. $(9 - 2)180$

8. $(11 - 2)180$

9. **NUMBER SENSE** Find the product of the difference of 5 and 2 and 180. (Lesson 1-2)

Find the value of x in each triangle. (Lesson 1-9)



QUICK Review

Example 1

Solve $82 + g + 41 = 180$.

$$82 + g + 41 = 180$$

$$123 + g = 180$$

$$\underline{-123} \quad \underline{-123}$$

$$g = 57$$

Write the equation.

Add 82 and 41.

Subtract 123 from each side.

Example 2

Evaluate $(8 - 2)180$.

$$(8 - 2)180 = (6)180$$

$$= 1,080$$

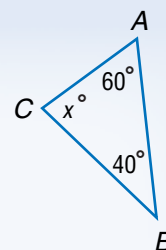
Subtract 2 from 8.

Multiply.

Example 3

Find the value of x in $\triangle ABC$.

The sum of the measures of the angles of a triangle is 180° .



$$60 + 40 + x = 180$$

$$100 + x = 180$$

$$\underline{-100} \quad \underline{-100}$$

$$x = 80$$

$$m\angle A = 60, m\angle B = 40$$

Add.

Subtract 100 from each side.

6-1

Line and Angle Relationships

Main IDEA

Identify special pairs of angles and relationships of angles formed by two parallel lines cut by a transversal.



Standard 7MG3.1
Identify and construct
basic elements of

geometric figures (e.g., altitudes, midpoints, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.

NEW Vocabulary

congruent angles

point

line

collinear

plane

ray

angle

vertical angles

complementary angles

supplementary angles

READING

in the Content Area

For strategies in reading this lesson, visit ca.gr7math.com.

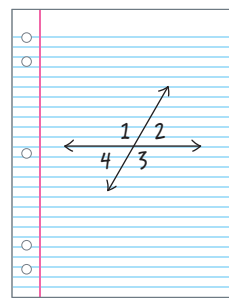
MINI Lab

STEP 1

Draw a horizontal line on notebook paper and a line that intersects the horizontal line at one point.

STEP 2

Label the angles formed as shown.



- Suppose that the measures of angles 2 and 4 are each 60° . Using angle relationships you have previously learned or a protractor, find and record the measure of each numbered angle. Explain your reasoning.
- Congruent angles** are angles that have the same measure. Describe the pairs of angles that appear to be congruent.
- What do you notice about the measures of angles that are side by side?

In the Mini Lab, you drew *lines* and *points*. You have also used these words in everyday language. In mathematics, they have very specific definitions.

- A **point** is simply a location.
- A **line** is made up of points, has no thickness or width, and has infinite length. Points on the same line are said to be **collinear**.
- A **plane** is a flat surface made up of points. A plane has no depth and extends infinitely in all directions.

KEY CONCEPT

Points, Lines, and Planes

	Point	Line	Plane
Model			
Symbols	point P	line n , line $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$, line AB or BA	plane T , plane XYZ , plane XZY , plane YXZ , plane YZX , plane ZXY , plane ZYX

STUDY TIP

Lines and Planes

There is exactly one line through any two points. There is exactly one plane through any three noncollinear points.

EXAMPLES

Name Lines and Planes

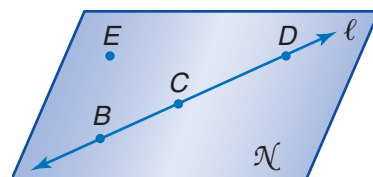
Use the figure to name each of the following.

1 a line containing point B

There are three points on the line. Any two of the points can be used to name the line.

$\overleftrightarrow{BC}$ $\overleftrightarrow{CB}$ $\overleftrightarrow{BD}$ $\overleftrightarrow{DB}$ $\overleftrightarrow{CD}$ $\overleftrightarrow{DC}$

The line can also be named as line ℓ .



2 a plane containing point C

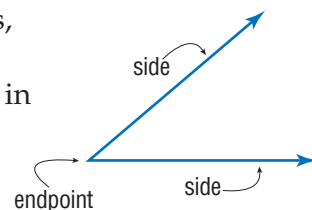
The plane can be named as plane N. You can also use the letters of any three *noncollinear* points to name the plane.

plane BCE plane BDE plane CDE

CHECK Your Progress

a. Use the figure to name a plane containing points B and D.

Other geometry terms are related to points, lines, and planes. For example, a **ray** is a part of a line having one endpoint and extending indefinitely in one direction. An **angle** is made up of two noncollinear rays sharing a common endpoint. The rays are called *sides* of the angle.



Pairs of angles can be classified by their relationship to each other.

CONCEPT Summary

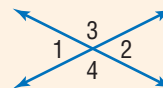
Special Pairs of Angles

Vertical angles are opposite angles formed by intersecting lines.

$\angle 1$ and $\angle 2$ are vertical angles.

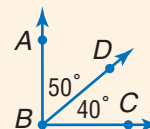
$\angle 3$ and $\angle 4$ are vertical angles.

Vertical angles are congruent.



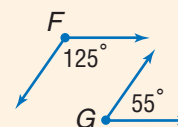
The sum of the measures of **complementary angles** is 90° .

$\angle ABD$ and $\angle DBC$ are complementary angles.



The sum of the measures of **supplementary angles** is 180° .

$\angle F$ and $\angle G$ are supplementary angles.



READING Math

Naming Angles Angles are named using the endpoint as the middle letter and a point from each side, such as $\angle ABD$. If there is no confusion, they can be named using only the endpoint, such as $\angle F$.

Angle Measure The measure of $\angle ABD$ is 50° . In symbols, $m\angle ABD = 50^\circ$.



Real-World Career . . .

How Does a Carpenter Use Math?

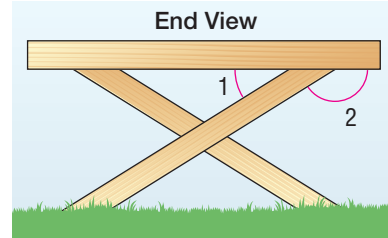
Carpenters use angle relationships when cutting lumber to build anything from furniture to houses.



For more information, go to ca.gr7math.com.

Real-World EXAMPLE

- 3 CARPENTRY** You are building a bench for a picnic table. Classify the relationship between $\angle 1$ and $\angle 2$. If $m\angle 1 = 32^\circ$, find $m\angle 2$. Justify your method.



$\angle 1$ and $\angle 2$ are supplementary.

So, the sum of their measures is 180° .

$$m\angle 1 + m\angle 2 = 180 \quad \text{Write an equation.}$$

$$32 + x = 180 \quad m\angle 1 = 32 \text{ and } m\angle 2 = x$$

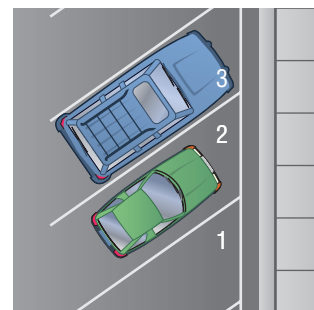
$$\underline{-32} \quad \underline{-32} \quad \text{Subtract 32 from each side.}$$

$$x = 148 \quad \text{Simplify.}$$

So, $m\angle 1 = 148^\circ$.

CHECK Your Progress

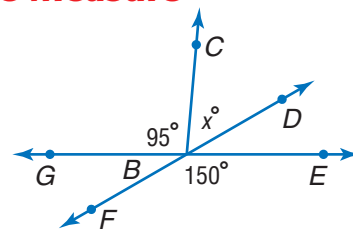
- b. PARKING** Engineers angled the parking spaces along a downtown street so that cars could park and back out easily. All of the lines marking the parking spaces are parallel. If $\angle 1 \cong \angle 2$ and $m\angle 1 = 55^\circ$, find $m\angle 3$. Explain your reasoning.



EXAMPLE Find a Missing Angle Measure

- 4** Find the value of x in the figure.

Angles GBD and FBE are vertical angles. So, the angles are congruent. Congruent angles have equal measures.



$$m\angle GBD = m\angle FBE \quad \text{Write an equation.}$$

$$m\angle GBC + m\angle CBD = m\angle FBE$$

$$95 + x = 150$$

$$m\angle GBC = 95 \text{ and } m\angle CBD = x$$

$$\underline{-95} \quad \underline{-95}$$

$$\text{Subtract 95 from each side.}$$

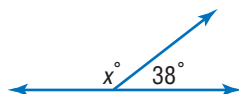
$$x = 55$$

$$\text{Simplify.}$$

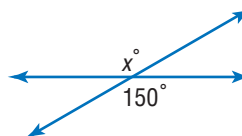
CHECK Your Progress

Find the value of x in each figure.

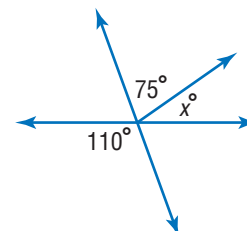
c.



d.



e.



READING Math

Congruent and Equals

The $\cong$ symbol is used to show that two angles are congruent.

$$\angle GBD \cong \angle FBE$$

The $=$ symbol is used to show that two measures are equal.

$$m\angle GBD = m\angle FBE$$



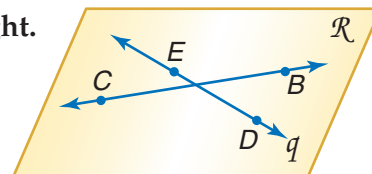
Personal Tutor at ca.gr7math.com

CHECK Your Understanding

Examples 1, 2
(p. 307)

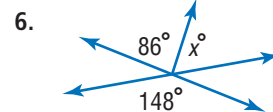
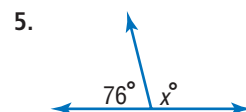
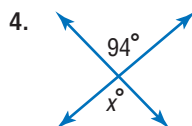
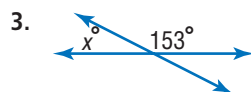
For Exercises 1 and 2, refer to the figure at the right.

1. Name a line containing point E .
2. Name a plane containing points B and D .

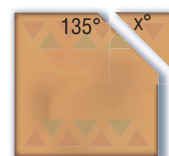


Examples 3, 4
(p. 308)

Find the value of x in each figure.



7. **CONSTRUCTION** Jack cuts a piece of tile at a 135° angle. What is the measure of the other angle formed by the cut?



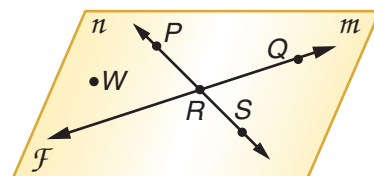
Exercises

HOMEWORK HELP

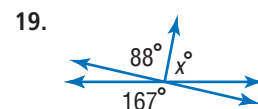
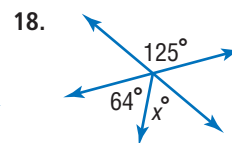
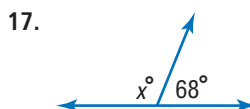
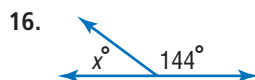
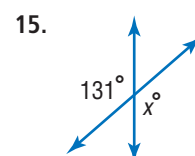
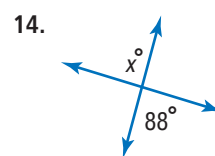
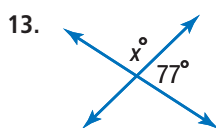
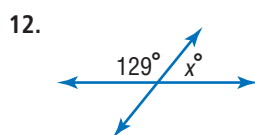
For Exercises	See Examples
8–11	1
12–19	2, 3
20–23	4

For Exercises 8–11, refer to the figure at the right.

8. Name a line that contains point S .
9. Name the plane containing lines n and m .
10. Name a point *not* contained in lines ℓ , m , or n .
11. What is another name for line m ?

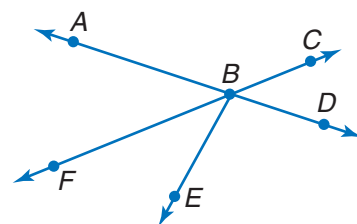


Find the value of x in each figure.



For Exercises 20–23, refer to the figure at the right.

20. Classify the relationship between $\angle CBD$ and $\angle ABF$.
21. Classify the relationship between $\angle ABF$ and $\angle ABC$.
22. If $m\angle ABC = 145^\circ$, find $m\angle CBD$.
23. If $m\angle ABF = 35^\circ$, find $m\angle CBD$.



Real-World Link
The Leaning Tower of Pisa is located in the town of Pisa, Italy.
Source: NOVA Online

24. **ARCHITECTURE** Refer to the image at the left. If $m\angle 1 = 84.5^\circ$ and $\angle 1 \cong \angle 3$, classify the relationship between $\angle 2$ and $\angle 3$. Then find $m\angle 2$.

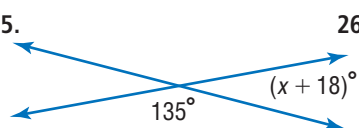
EXTRAPRACTICE

See pages 691, 713.

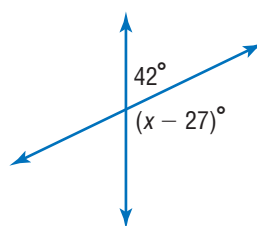
Math online

Self-Check Quiz at
ca.gr7math.comFind the value of x in each figure.

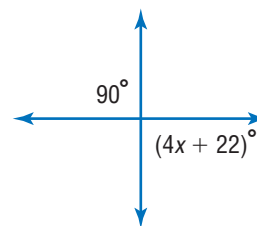
25.



26.



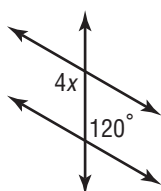
27.

**H.O.T. Problems**

28. **OPEN ENDED** Draw and label three collinear points on a plane. Then draw three noncollinear points on the same plane.
29. **REASONING** How many planes are determined by three collinear points? Justify your response.
30. **WRITING IN MATH** Describe a real-world example of a plane containing points, lines, and angles.

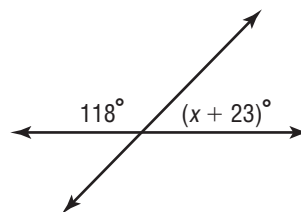
STANDARDS PRACTICE

31. In the figure below, the two angles are congruent. Find the value of x .



- A 30 C 116
B 40 D 124

32. Find the value of x .



- F 39° H 141°
G 62° J 157°

Spiral Review

33. **BANKING** A savings account starts with \$560. If the simple interest rate is 3%, find the total amount after 18 months. (Lesson 5-9)

Find each percent of change. Round to the nearest tenth if necessary. State whether the percent of change is an *increase* or a *decrease*. (Lesson 5-8)

34. original: 20 35. original: 45 36. original: 620
new: 27 new: 18 new: 31
37. **ARCHAEOLOGY** Two artifacts were found at a dig. On a coordinate plane, one artifact was found at (1, 5), and the other artifact was found at (3, 1). How far apart were the two artifacts? Round to the nearest tenth if necessary. (Lesson 3-7)

GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate each expression using the given value.

Round to the nearest tenth if necessary. (Lesson 1-2)

38. $110n - 250$; $n = 4$

39. $(n - 1)40$; $n = 10$

40. $\frac{(n + 1)25}{n}$; $n = 8$

Extend 6-1

Geometry Lab Constructions

Main IDEA

Construct basic elements of geometric figures using a compass and straightedge.



Standard 7MG3.1

Identify and **construct** basic elements of geometric figures (e.g., altitudes, midpoints, diagonals, **angle bisectors**, and **perpendicular bisectors**; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.
Standard 7MR2.4 Make and test **conjectures** using **inductive** and deductive reasoning.

NEW Vocabulary

line segment
perpendicular lines
perpendicular bisector
midpoint
angle bisector
altitude

READING Math

Segments The symbol for line segment JK is $\overline{JK}$.

You can use a compass and a straightedge to construct basic elements of geometric figures. For example, a **line segment** is a straight path between two endpoints. Line segments that have the same length are called *congruent segments*.

ACTIVITY

Congruent Segments

1

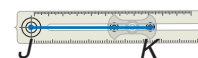
STEP 1

Draw $\overline{JK}$. Then use a straightedge to draw a line segment longer than $\overline{JK}$. Label it $\overline{LM}$.



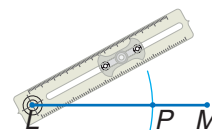
STEP 2

Place the compass at J and adjust the compass setting so you can place the pencil tip on K . The compass setting equals the length of $\overline{JK}$.



STEP 3

Using this setting, place the compass tip at L . Draw an arc to intersect $\overline{LM}$. Label the intersection P .



$\overline{LP}$ is congruent to $\overline{JK}$.



CHECK Your Progress

- Draw a line segment. Construct a line segment congruent to the one drawn. Use a ruler to verify the segments are congruent.

Perpendicular lines are lines that form right angles. A **perpendicular bisector** is a perpendicular line that divides a line segment into two congruent segments at the **midpoint**.

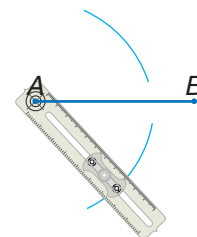
ACTIVITY

Perpendicular Bisectors

2

STEP 1

Draw $\overline{AB}$. Then place the compass at point A . Using a setting greater than one half the length of $\overline{AB}$, draw an arc above and below $\overline{AB}$.



(continued on the next page)

STUDY TIP

Segment Bisectors

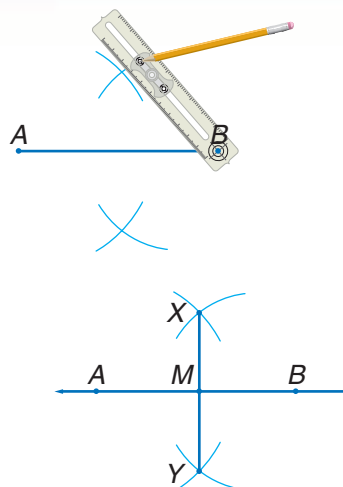
There can be an infinite number of *bisectors* of a line segment, but only one perpendicular bisector.

STEP 2 Using this setting, place the compass at point B . Draw another set of arcs above and below $\overline{AB}$ as shown.

STEP 3 Label the intersection of these arcs X and Y as shown.

STEP 4 Draw $\overline{XY}$. Label the intersection of $\overline{AB}$ and this new line M .

$\overline{XY}$ is the perpendicular bisector of $\overline{AB}$.



CHECK Your Progress

- Draw a line segment. Construct the perpendicular bisector of the segment.

An **angle bisector** is a ray that divides an angle into two congruent angles.

ACTIVITY Angle Bisectors

3 STEP 1 Draw $\angle JKL$.

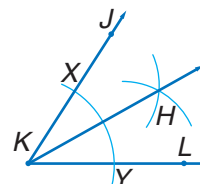
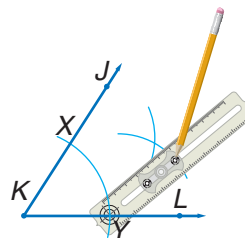
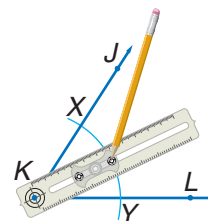
STEP 2 Place the compass at point K and draw an arc that intersects both sides of the angle. Label the intersections X and Y .

STEP 3 With the compass at point X , draw an arc in the interior of $\angle JKL$.

STEP 4 Using this setting, place the compass at point Y . Draw another arc.

STEP 5 Label the intersection of these arcs H . Then draw $\overrightarrow{KH}$.

$\overrightarrow{KH}$ is the angle bisector of $\angle JKL$.



CHECK Your Progress

- c. Draw an obtuse angle. Then bisect the angle.

An **altitude** of a triangle is a segment from one vertex to the line containing the opposite side and perpendicular to that side.

READING Math

Segments The height of a triangle is the length of the altitude.

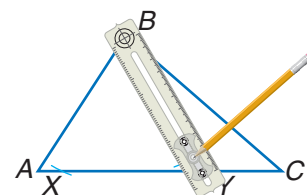
ACTIVITY

Construct an Altitude

4

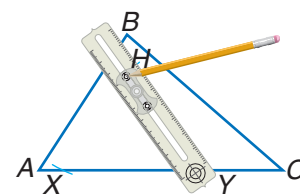
STEP 1

Place the compass at vertex B and draw two arcs intersecting $\overleftrightarrow{AC}$. Label the points where the arcs intersect the side X and Y .



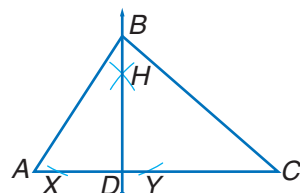
STEP 2

Adjust the compass to an opening greater than $\frac{1}{2}XY$. Place the compass on point X and draw an arc above $\overleftrightarrow{AC}$. Using the same setting, place the compass on point Y and draw another arc above $\overleftrightarrow{AC}$. Label the point of intersection of the arcs H .



STEP 3

Use a straightedge to draw $\overleftrightarrow{BH}$. Label the point where $\overleftrightarrow{BH}$ intersects $\overleftrightarrow{AC}$ as D .



$\overline{BD}$ is an altitude of $\triangle ABC$.

CHECK Your Progress

- d. Construct the altitudes to $\overline{AB}$ and $\overline{BC}$. (Hint: You may need to extend the lines containing the sides of your triangle.) What do you notice about the altitudes of the triangle?

ANALYZE THE RESULTS

1. Draw two different line segments. Then construct a right triangle in which the given segments are the legs.
2. Draw a large acute triangle. Construct the perpendicular bisectors for each side. What do you notice about the perpendicular bisectors?
3. Repeat Exercise 2, constructing the angle bisectors for each angle. What do you notice?
4. **MAKE A CONJECTURE** Predict a relationship involving the altitudes, perpendicular bisectors, and angle bisectors for any kind of triangle.



6-2

Problem-Solving Investigation

MAIN IDEA: Solve problems by using the logical reasoning strategy.



Standard 7MR1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed. **Standard 7NS1.3** Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

P.S.I. TEAM +

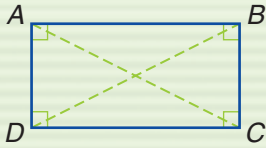
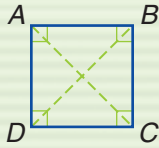
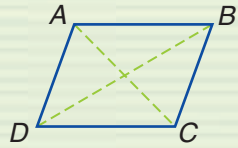
e-Mail: USE LOGICAL REASONING

YOUR MISSION: Use logical reasoning to solve the problem.

THE PROBLEM: How can Zach find another property that is true for rectangles, but not parallelograms?

Zach: I know that a rectangle is a parallelogram with four right angles. Both parallelograms and rectangles have opposite sides that are congruent and parallel.



EXPLORE	Try investigating the diagonals of rectangles and parallelograms to see whether there is any pattern.
PLAN	Draw several different rectangles and parallelograms, measure the diagonals, and look for a pattern
SOLVE	 $AC = BD$  $AC = BD$  $AC \neq BD$ It appears that the diagonals of a rectangle are congruent, but the diagonals of a parallelogram are not.
CHECK	You can try several more examples to see whether your conjecture appears to be true. But at this point, it is just a conjecture, not an actual proof.

Analyze The Strategy

- Inductive reasoning** is the process of making a conjecture after observing several examples. Determine where Zach used inductive reasoning. Explain.
- WRITING IN MATH** Write about a situation in which you use inductive reasoning.

Mixed Problem Solving

For Exercises 3–5, solve each problem using logical reasoning.

3. **GEOMETRY** Draw several parallelograms and measure their angles. What seems to be true about opposite angles of parallelograms?
4. **MEASUREMENT** You need to measure 2 pints of juice for a punch recipe. You have a large container of pineapple juice, an empty 5-pint container, and an empty 4-pint container. Explain how you can use only these containers to measure 2 pints of juice.



5. **NUMBER SENSE** Write each fraction in the table as a decimal. Then use logical reasoning to write the decimal equivalents for the fractions $\frac{3}{11}$, $\frac{6}{11}$, and $\frac{9}{11}$.

Fraction	Decimal
$\frac{1}{11}$	
$\frac{4}{11}$	
$\frac{8}{11}$	

Use any strategy to solve Exercises 6–9. Some strategies are shown below.

PROBLEM-SOLVING STRATEGIES

- Use the four-step plan.
- Look for a pattern.
- Draw a diagram.
- Use logical reasoning.

6. **GEOMETRY** Some pentagons are arranged according to the pattern below. If the sides of the pentagons shown are each 1 unit long, find the perimeter of the pattern formed by 8 pentagons.



7. **MONEY** After a trip to the mall, Alex and Marcus counted their money to see how much they had left. Alex said, "If I had \$4 more, I would have as much as you." Marcus replied, "If I had \$4 more, I would have twice as much as you." How much does each boy have?
8. **LAUNDRY** You need two clothespins to hang one towel on a clothesline. One clothespin can be used on a corner of one towel and a corner of the towel next to it. What is the least number of clothespins you need to hang 8 towels?
9. **SPORTS** Nuna, Brianna, Mackenzie, Evelina, and Bianca were the first five finishers of a race. From the given clues, give the order in which they finished.
 - Nuna passed Mackenzie just before the finish line.
 - Bianca finished 5 seconds ahead of Nuna.
 - Brianna crossed the finish line after Mackenzie.
 - Evelina was fifth at the finish line.

Select the Operation

For Exercises 10 and 11, select an appropriate operation(s) to solve the problem. Justify your selection(s) and solve the problem.

10. **MEASUREMENT** The circumference of Earth around the equator is 24,901.55 miles. The circumference through the North and South Poles is 24,859.82 miles. How much greater is the circumference of Earth around the equator than through the poles?
11. **BIRDS** The arctic tern has the longest migration of any bird. Each year, it flies over 21,750 miles. If the average lifespan of an arctic tern is 20 years, on average, how many miles will it have flown in the course of its life?

6-3

Polygons and Angles

Main IDEA

Find the sum of the angle measures of a polygon and the measure of an interior angle of a regular polygon.



Standard 7MR3.3 Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.

Standard 7AF1.1 Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g. three less than a number, half as large as area A .)

MINI Lab

Copy and complete the table. The sum of the angle measures of a triangle is 180° .

- Predict the number of triangles and the sum of the angle measures in a polygon with 8 sides.

Number of Sides	Sketch of Figure	Number of Triangles	Sum of Angle Measures
3		1	$1(180^\circ) = 180^\circ$
4		2	$2(180^\circ) = 360^\circ$
5			
6			

- Write an algebraic expression that could represent the number of triangles in an n -sided polygon. Then write an expression to represent the sum of the angle measures in an n -sided polygon.

In the Mini Lab, you used the sum of the angle measures of a triangle to find the sum of the interior angle measures of various polygons. An **interior angle** is an angle that lies inside a polygon.

NEW Vocabulary

interior angle
equilateral
equiangular
regular polygon

KEY CONCEPT

Interior Angle Sum of a Polygon

Words The sum of the measures of the interior angles of a polygon is $(n - 2)180$, where n represents the number of sides.

Symbols $S = (n - 2)180$.

EXAMPLE

Find the Sum of Interior Angle Measures

- ALGEBRA** Find the sum of the measures of the interior angles of a decagon.

$$S = (n - 2)180$$

Write an equation.

$$S = (10 - 2)180$$

A decagon has 10 sides. Replace n with 10.

$$S = (8)180 \text{ or } 1,440$$

Simplify.

The sum of the measures of the interior angles of a decagon is $1,440^\circ$.



CHECK Your Progress

Find the sum of the angle measures of each polygon.

a. hexagon

b. octagon

c. 15-gon

Naming Polygons

Polygons are named by the number of sides.

5 sides: pentagon

6 sides: hexagon

7 sides: heptagon

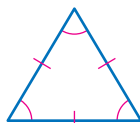
8 sides: octagon

9 sides: nonagon

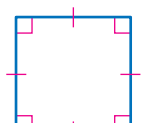
10 sides: decagon



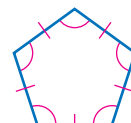
A polygon that is **equilateral** (all sides congruent) and **equiangular** (all angles congruent) is called a **regular polygon**. Since all the angles of a regular polygon are congruent, their measures are equal.



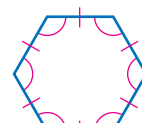
equilateral triangle



square



regular pentagon



regular hexagon



Real-World Link

Frank Lloyd Wright designed 1,141 buildings during his 70-year career. He used obtuse angles and circles, creating unusually shaped structures such as the spiral Guggenheim Museum in New York City.

Source: architecture.about.com



Real-World EXAMPLE

2

ARCHITECTURE The Ennis-Brown House in Los Angeles, California, shown at the right was designed by architect Frank Lloyd Wright. The exterior of the house consists of repeating regular quadrilaterals. Find the measure of an interior angle of a quadrilateral.



Step 1

Find the sum of the measures of the angles.

$$S = (n - 2)180 \quad \text{Write an equation.}$$

$$S = (4 - 2)180 \quad \text{Replace } n \text{ with } 4.$$

$$S = (2)180 \text{ or } 360 \quad \text{Simplify.}$$

The sum of the measures of the interior angles is 360.

Step 2

Divide 360 by 4, the number of interior angles, to find the measure of one interior angle. So, the measure of one interior angle of a regular quadrilateral is $360^\circ \div 4$ or 90° .



CHECK Your Progress

Find the measure of one interior angle in each regular polygon. Round to the nearest tenth if necessary.

d. octagon

e. heptagon

f. 20-gon



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CHECK Your Understanding

Example 1
(p. 316)

Find the sum of the angle measures of each polygon.

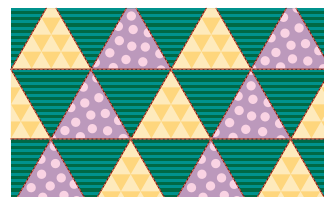
1. quadrilateral

2. nonagon

3. 12-gon

Example 2
(p. 317)

4. **QUILTING** The quilt pattern shown is made of repeating equilateral triangles. What is the measure of one interior angle of a triangle?



Extra Examples at ca.gr7math.com

(l) Roger Wood/CORBIS, (r) Bill Aron/PhotoEdit



Exercises

HOMEWORK HELP

For Exercises	See Examples
5–10	1
11–16	2

Find the sum of the measures of the interior angles of each polygon.

5. pentagon
6. heptagon
7. 11-gon
8. 14-gon
9. 19-gon
10. 24-gon

11. **ART** The sculpture below consists of repeating regular pentagons and hexagons. Find the measure of one interior angle of a pentagon.



12. **NATURE** Each chamber of a bee honeycomb is a regular hexagon. What is the measure of an interior angle in the honeycomb?



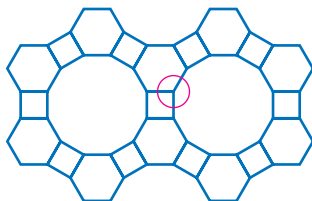
Find the measure of one interior angle in each regular polygon. Round to the nearest tenth if necessary.

13. nonagon
14. decagon
15. 13-gon
16. 16-gon

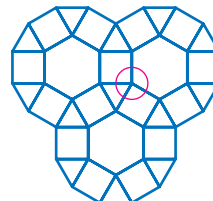
ART For Exercises 17 and 18, use the following information.

A tessellation is a repetitive pattern of polygons that fit together without overlapping and without gaps between them. For each tessellation, find the measure of each angle at the circled vertex. Then find the sum of the angles.

17.



18.



19. **ARCHITECTURE** The surface of the dome of Spaceship Earth in Orlando, Florida, consists of repeating equilateral triangles as shown. Find the measure of each angle in each outlined triangle. Then make a conjecture about the interior angle measures in equilateral triangles of different sizes.



EXTRA PRACTICE

See pages 692, 713.

Math online

Self-Check Quiz at
ca.gr7math.com

H.O.T. Problems

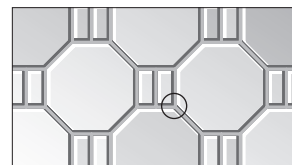
20. **CHALLENGE** How many sides does a regular polygon have if the measure of an interior angle is 157.5° ? Justify your answer.
21. **WRITING IN MATH** Explain the relationship between the number of sides of a regular polygon and the measure of each interior angle.

22. The following statements are true about $\triangle GHK$.
- $m\angle G = m\angle H + m\angle K$.
 - $\angle H$ and $m\angle K$ are complementary.
 - The measure of each angle is evenly divisible by 15.

Which choice does *not* fit all three statements for angles G , H , and K ?

- | | |
|---------------------------------|---------------------------------|
| A $m\angle G = 90^\circ$ | C $m\angle G = 90^\circ$ |
| $m\angle H = 45^\circ$ | $m\angle H = 50^\circ$ |
| $m\angle K = 45^\circ$ | $m\angle K = 40^\circ$ |
| B $m\angle G = 90^\circ$ | D $m\angle G = 90^\circ$ |
| $m\angle H = 75^\circ$ | $m\angle H = 60^\circ$ |
| $m\angle K = 15^\circ$ | $m\angle K = 30^\circ$ |

23. Which statement is *not* true about the pattern of repeating regular octagons and rectangles?

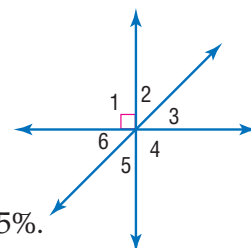


- F** The sum of the angles in each rectangle is 360° .
- G** The sum of the angles in each octagon is $1,080^\circ$.
- H** The sum of the angles at the circled vertex is 270° .
- J** The measure of each interior angle of an octagon is 135° .

Spiral Review

Classify each pair of angles as *complementary*, *supplementary*, or *vertical*. (Lesson 6-1)

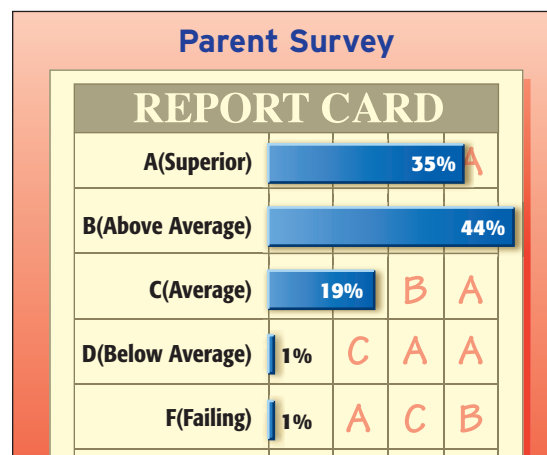
- | | |
|-------------------------------|-------------------------------|
| 24. $\angle 3$ and $\angle 6$ | 25. $\angle 2$ and $\angle 3$ |
| 26. $\angle 5$ and $\angle 6$ | 27. $\angle 1$ and $\angle 4$ |
28. Marisa put \$580 in a savings account at a simple interest rate of 5.5%. How much interest will the account earn in 3 years? (Lesson 5-9)



SCHOOL For Exercises 29 and 30, use the following information.

A recent survey asked parents to grade themselves based on their involvement in their children's education. The results are shown at the right. (Lesson 5-2)

29. Write the percent of parents who gave themselves an "A" as a decimal and as a fraction in simplest form.
30. Did more or less than $\frac{2}{5}$ of the parents give themselves a "B"?



GET READY for the Next Lesson

PREREQUISITE SKILL Decide whether the figures are congruent. Write *yes* or *no* and explain your reasoning. (Lesson 4-4)



6-4

Congruent Polygons

Main IDEA

Identify congruent polygons.



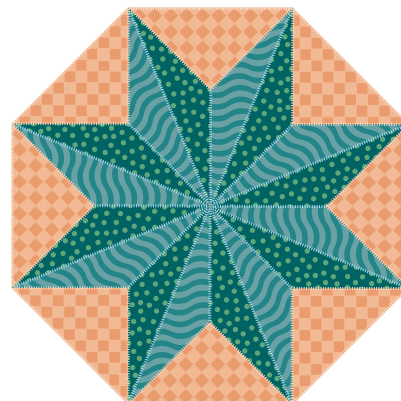
Standard
7MG3.4

Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and **what congruence means about the relationships between the sides and angles of the two figures.**

GET READY for the Lesson

QUILTING A template, or pattern, for a quilt block contains the minimum number of shapes needed to create the pattern.

1. How many different kinds of triangles are shown in the *Winter Stars* quilt at the right? Explain your reasoning and draw each triangle.
2. Copy the quilt and label all matching triangles with the same number, starting with 1.



NEW Vocabulary

congruent polygon

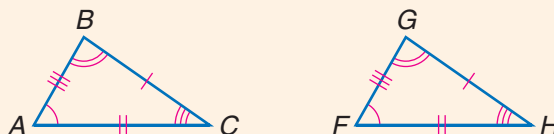
Polygons that have the same size and shape are called **congruent polygons**.

KEY CONCEPT

Congruent Polygons

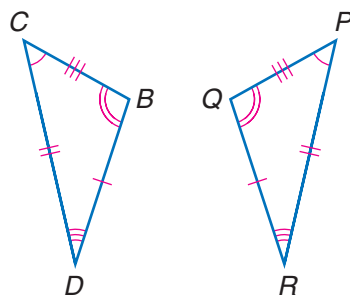
Words If two polygons are congruent, their corresponding sides are congruent and their corresponding angles are congruent.

Model



Symbols Congruent angles: $\angle A \cong \angle F$, $\angle B \cong \angle G$, $\angle C \cong \angle H$
Congruent sides: $\overline{BC} \cong \overline{GH}$, $\overline{AC} \cong \overline{FH}$, $\overline{AB} \cong \overline{FG}$

In a congruence statement, the letters identifying each polygon are written so that corresponding vertices appear in the same order. For example, for the diagram below, write $\triangle CBD \cong \triangle PQR$.



$$\triangle CBD \cong \triangle PQR$$

Vertex C corresponds to vertex P.
Vertex B corresponds to vertex Q.
Vertex D corresponds to vertex R.

Two polygons are congruent if all pairs of corresponding angles are congruent and all pairs of corresponding sides are congruent.



STUDY TIP

Congruence Statements

Other possible congruence statements for Example 1 are $\triangle YZX \cong \triangle NLM$, $\triangle ZXY \cong \triangle LMN$, $\triangle YXZ \cong \triangle NML$, $\triangle XZY \cong \triangle MLN$, and $\triangle ZYX \cong \triangle LNM$.



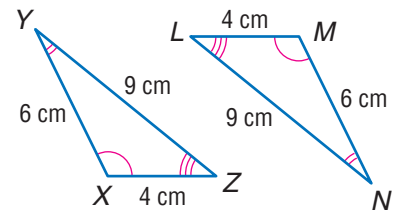
EXAMPLE Identify Congruent Polygons

- 1 Determine whether the triangles shown are congruent. If so, name the corresponding parts and write a congruence statement.

The arcs indicate that $\angle X \cong \angle M$, $\angle Y \cong \angle N$, and $\angle Z \cong \angle L$.

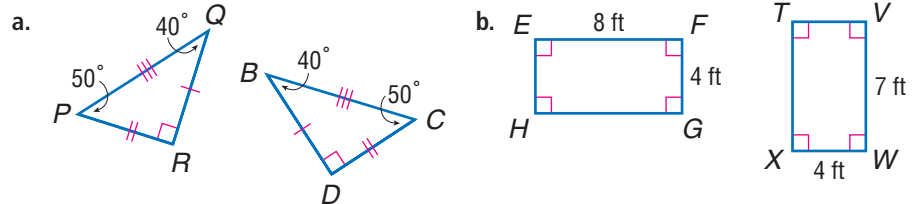
The side measures indicate that $\overline{XY} \cong \overline{MN}$, $\overline{YZ} \cong \overline{NL}$, and $\overline{XZ} \cong \overline{ML}$.

Since all pairs of corresponding angles and sides are congruent, the two triangles are congruent. One congruence statement is $\triangle XYZ \cong \triangle MNL$.



CHECK Your Progress

Determine whether the polygons shown are congruent. If so, name the corresponding parts and write a congruence statement.



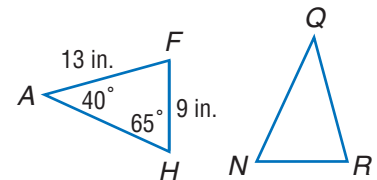
Online Personal Tutor at ca.gr7math.com

EXAMPLES Find Missing Measures

In the figure, $\triangle AFH \cong \triangle QRN$.

- 2 Find $m\angle Q$.

According to the congruence statement, $\angle A$ and $\angle Q$ are corresponding angles. So, $\angle A \cong \angle Q$. Since $m\angle A = 40^\circ$, $m\angle Q = 40^\circ$.



- 3 Find RN .

$\overline{FH}$ corresponds to $\overline{RN}$. So, $\overline{FH} \cong \overline{RN}$. Since $FH = 9$ inches, $RN = 9$ inches.

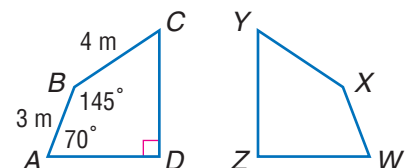
READING Math

Recall that symbols like RN refer to the measure of the segment with those endpoints.

CHECK Your Progress

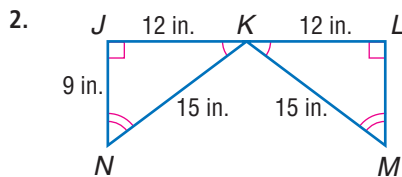
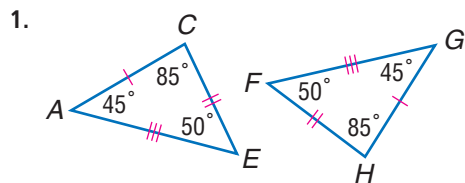
In the figure, quadrilateral $ABCD$ is congruent to quadrilateral $WXYZ$. Find each measure.

- c. $m\angle X$
d. YX
e. $m\angle Y$



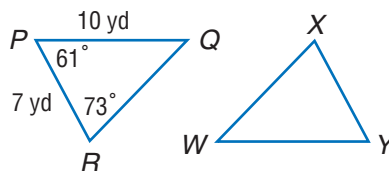
CHECK Your Understanding

Example 1 Determine whether the polygons shown are congruent. If so, name the corresponding parts and write a congruence statement.
(p. 321)



Examples 2, 3 In the figure, $\triangle PQR \cong \triangle YWX$. Find each measure.
(p. 321)

3. $m\angle X$
4. YW
5. XY
6. $m\angle W$

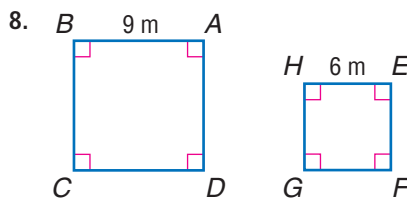
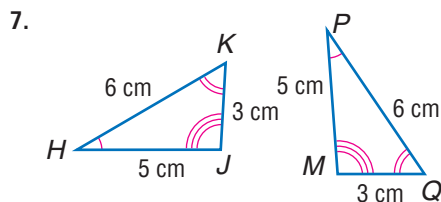


Exercises

HOMEWORK HELP

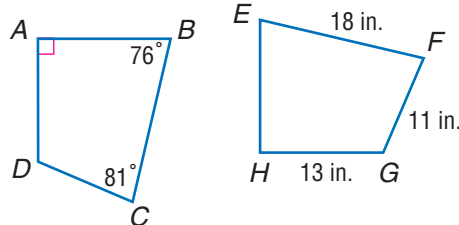
For Exercises	See Examples
7–8	1
9–12	2, 3

Determine whether the polygons shown are congruent. If so, name the corresponding parts and write a congruence statement.

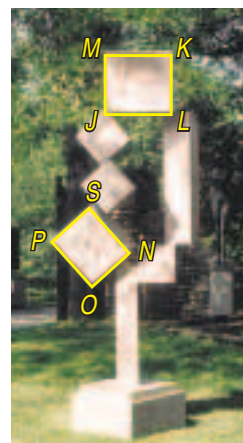


In the figure, quadrilateral $ABCD$ is congruent to quadrilateral $HEFG$. Find each measure.

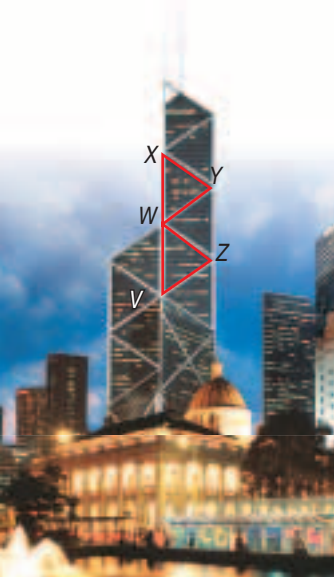
9. AD
10. DC
11. $m\angle G$
12. $m\angle H$



13. **ART** The structure shown at the right, *Cubi XII*, created by David Smith, is located at the Hirshhorn Museum and Sculpture Garden in Washington, D.C. If quadrilaterals $JMKL$ and $PSNO$ are both squares, write one statement you would need to know in order to show that the quadrilaterals are congruent. Explain your reasoning.



14. **ARCHITECTURE** The Bank of China Tower shown at the left was designed by architect I.M. Pei and consists of congruent glass triangles. If $\triangle WXY \cong \triangle VWZ$, and $m\angle V = 60^\circ$, and $m\angle VWZ = 50^\circ$, find $m\angle Y$.



Real-World Link

The Bank of China Tower uses triangular bracing as protection against high winds caused by typhoons.

Source: greatbuildings.com



EXTRAPRACTICE

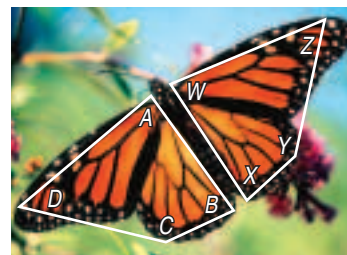
See pages 692, 713.

Math online

Self-Check Quiz at
ca.gr7math.com

H.O.T. Problems

15. **INSECTS** The wings of a monarch butterfly are shaped as congruent quadrilaterals. Write a congruence statement. Then find $m\angle A$ if $m\angle Z = 45^\circ$, $m\angle Y = 145^\circ$, and $m\angle X = 90^\circ$.



16. **CHALLENGE** State whether the following statement is *sometimes*, *always* or *never* true. Explain your reasoning.

If the perimeters of two triangles are equal, then the triangles are congruent.

17. **WRITING IN MATH** Explain how you could determine whether two similar polygons were also congruent.



STANDARDS PRACTICE

18. Which statement must be true if $\triangle PQR \cong \triangle TUV$?

- A $\overline{PQ} \cong \overline{UV}$
- B $\overline{QR} \cong \overline{TV}$
- C $\angle P \cong \angle T$
- D $\angle R \cong \angle U$

19. In the scaffolding below, $\triangle ABC \cong \triangle DCB$, $\overline{AC}$ is 2.5 meters long, $\overline{BC}$ is 1 meter long, and $\overline{AB}$ is 2.7 meters long. What is the length of $\overline{BD}$?



- F 1 meter
- G 2 meters
- H 2.5 meters
- J 2.7 meters

Spiral Review

ALGEBRA Find the measure of one interior angle in each regular polygon. Round to the nearest tenth if necessary. (Lesson 6-3)

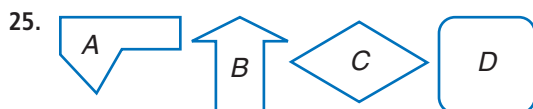
20. triangle 21. pentagon 22. heptagon 23. nonagon

24. **CITY SERVICES** The street maintenance vehicles for the city of Centerburg cannot safely make turns less than 70° . Should the proposed site of the new maintenance garage at the northeast corner of Park and Main be approved? Explain. (Lesson 6-1)



GET READY for the Next Lesson

PREREQUISITE SKILL Which figure *cannot* be folded so one half matches the other half?



Geometry Lab

Investigating Congruent Triangles

Main IDEA

Investigate which three pairs of corresponding parts can be used to show that two triangles are congruent.



Standard
7MG3.4

Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.

Standard 7MR2.4 Make and test conjectures using inductive and deductive reasoning.

In this lab, you will investigate whether it is possible to show that two triangles are congruent without showing that all six pairs of corresponding parts are congruent.

ACTIVITY

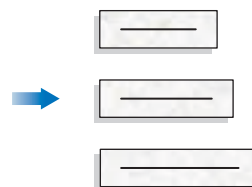
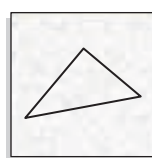
Concepts in Motion

Animation ca.gr7math.com

1

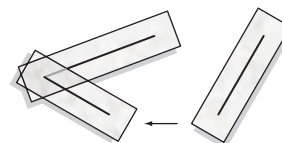
STEP 1

Draw a triangle on a piece of patty paper. Copy the sides onto another piece of patty paper and cut them out.



STEP 2

Arrange and tape the pieces together so that they form a triangle.



ANALYZE THE RESULTS

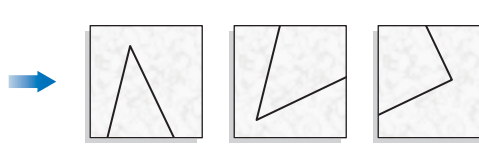
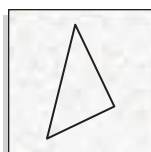
1. Is the triangle you formed congruent to the original triangle? Explain.
2. Try to form another triangle with the given sides. Is it congruent to the original triangle?
3. **MAKE A CONJECTURE** Based on this activity, can three pairs of congruent sides be used to show that two triangles are congruent?

ACTIVITY

2

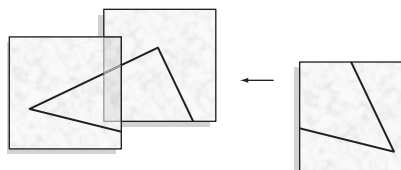
STEP 1

Draw a triangle on a piece of patty paper. Copy each angle of the triangle onto separate pieces of patty paper. Extend each ray of each angle to the edge of the patty paper.



STEP 2

Arrange and tape the pieces together so that they form a triangle.



ANALYZE THE RESULTS

4. Is the triangle you formed congruent to the original triangle? Explain.
5. Try to form another triangle with the given angles. Is it congruent to the original triangle?
6. **MAKE A CONJECTURE** Based on this activity, can three pairs of congruent angles be used to show that two triangles are congruent?

ACTIVITY

3

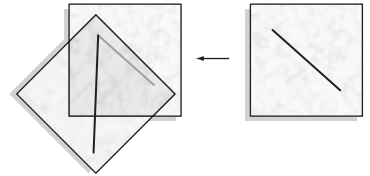
STEP 1

Draw a triangle on a piece of patty paper. Copy two sides of the triangle and the angle between them onto separate pieces of patty paper and cut them out.



STEP 2

Arrange and tape the pieces together so that the two sides are joined to form the rays of the angle. Then tape these joined pieces onto a piece of construction paper and connect the two rays to form a triangle.



ANALYZE THE RESULTS

7. Is the triangle you formed congruent to the original triangle? Explain.
8. Try to form another triangle with the given sides and angle. Is it congruent to the original triangle?
9. **MAKE A CONJECTURE** Based on this activity, can two pairs of congruent sides and the pair of congruent angles between them be used to show that two triangles are congruent?
10. **EXTENSION** Use patty paper to investigate and make a conjecture about whether each of these additional cases can be used to show that two triangles are congruent.
 - Case 4** two pairs of congruent sides and a pair of congruent angles *not* between them
 - Case 5** two pairs of congruent angles and the pair of congruent sides between them
 - Case 6** two pairs of congruent angles and a pair of congruent sides *not* between them

Mid-Chapter Quiz

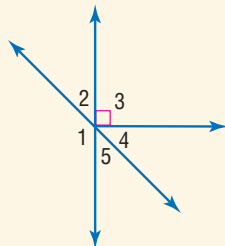
Lessons 6-1 through 6-4

Find the value of x in each figure. (Lesson 6-1)

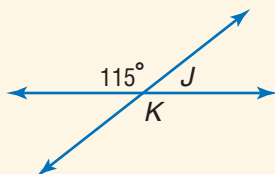
-
-
-
-

For Exercises 5–8, use the figure. Classify each pair of angles as *complementary*, *supplementary*, or *vertical*. (Lesson 6-1)

- $\angle 1$ and $\angle 2$
- $\angle 2$ and $\angle 5$
- $\angle 4$ and $\angle 5$
- $\angle 1$ and $\angle 5$



- LINES** Refer to the figure below. Classify the relationship between $\angle J$ and $\angle K$. Then find the measure of $\angle J$.



- NUMBERS** Consider the following pattern.

$$1^2 = 1$$

$$11^2 = 121$$

$$111^2 = 12,321$$

Use logical reasoning to find the next equation. Explain your reasoning. (Lesson 6-2)

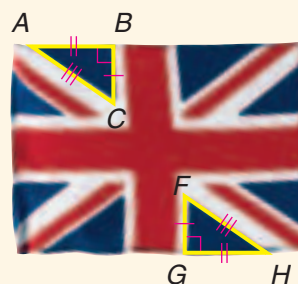
ALGEBRA Find the sum of the measures of the interior angles of each polygon. (Lesson 6-3)

- pentagon
- 20-gon
- 15-gon

- STANDARDS PRACTICE** Mrs. Lytle's kitchen tile is made up of a pattern of repeating regular octagons and squares. Which statement is true concerning the pattern? (Lesson 6-3)

- The sum of the angle measures in each square is 180° .
- The sum of the angle measures at each vertex is $1,080^\circ$.
- The measure of the angle at each vertex is 90° .
- The measure of each interior angle of a octagon is 135° .

- FLAGS** The blue portions of the flag below are triangular. Determine whether the indicated triangles are congruent. If so, name the corresponding parts and write a congruence statement. (Lesson 6-4)



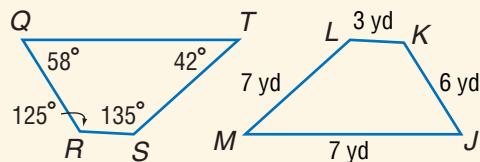
In the figure, quadrilateral $QRST$ is congruent to quadrilateral $JKLM$. Find each measure. (Lesson 6-4)

16. QT

17. QR

18. $m\angle M$

19. $m\angle K$



- STANDARDS PRACTICE** Which statement is *not* true if $\triangle ABC \cong \triangle DEF$? (Lesson 6-4)

F $\overline{BC} \cong \overline{EF}$

H $\angle F \cong \angle B$

G $\overline{AB} \cong \overline{DE}$

J $\angle A \cong \angle D$

6-5

Symmetry

Main IDEA

Identify line symmetry and rotational symmetry.



Standard 7MG3.2

Understand and use coordinate graphs to

plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.

NEW Vocabulary

line symmetry

line of symmetry

rotational symmetry

angle of rotation

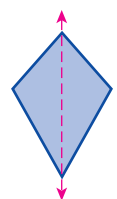
MINI Lab

ARCHITECTURE The Pentagon is the headquarters of the United States Department of Defense and is located in Washington, D.C. Trace the outline of the Pentagon onto both a piece of tracing paper and a transparency.

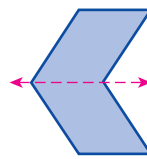


1. Draw a line through the center of the Pentagon. Then fold your paper across this line. What do you notice about the two halves?
2. Are there other lines you can draw that will produce the same result? If so, how many?
3. Place the transparency over the outline on your tracing paper. Use your pencil point at the center of the Pentagon to hold the transparency in place. How many times can you rotate the transparency from its original position so that the two figures match?
4. Find the first angle of rotation by dividing 360° by the total number of times the figures matched.
5. List the other angles of rotation by adding the measure of the first angle of rotation to the previous angle measure. Stop when you reach 360° .

A figure has **line symmetry** if it can be folded over a line so that one half of the figure matches the other half. This fold line is called the **line of symmetry**.



vertical line of symmetry

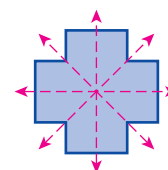


horizontal line of symmetry



no line of symmetry

Some figures, such as the Pentagon in the Mini Lab above, have more than one line of symmetry. The figure at the right has multiple lines of symmetry: one vertical, one horizontal, and two diagonal.

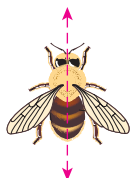


EXAMPLES

Identify Line Symmetry

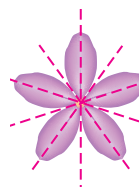
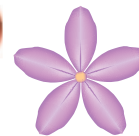
Determine whether each figure has line symmetry. If it does, trace the figure and draw all lines of symmetry. If not, write *none*.

1



This figure has one vertical line of symmetry.

2



The figure has five lines of symmetry.



CHECK Your Progress

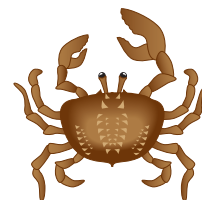
a.



b.



c.



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A figure has **rotational symmetry** if it can be rotated or turned less than 360° about its center so that the figure looks exactly as it does in its original position. The degree measure of the angle through which the figure is rotated is called the **angle of rotation**. Some figures have just one angle of rotation, while others, like the Pentagon, have several.

EXAMPLE

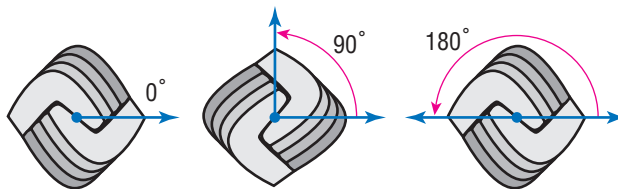
Identify Rotational Symmetry

3

LOGOS Determine whether the figure has rotational symmetry. Write *yes* or *no*. If *yes*, name its angle(s) of rotation.



Yes, this figure has rotational symmetry. It will match itself after being rotated 180° .



CHECK Your Progress

d.



e.



f.



Real-World Link

Many companies and nonprofit groups, such as the American Red Cross, use a logo so people can easily identify their products or services. They often design their logo to have line or rotational symmetry.



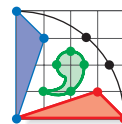
Real-World Link

The Pennsylvania Dutch, or Pennsylvania Germans, created signs that were painted on the sides of barns or houses. Many feature designs that have rotational symmetry.

Source: folkart.com

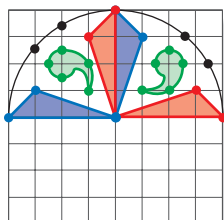
EXAMPLE Use a Rotation

- 4 FOLK ART** Copy and complete the barn sign shown so that the completed figure has rotational symmetry with 90° , 180° , and 270° as its angles of rotation.

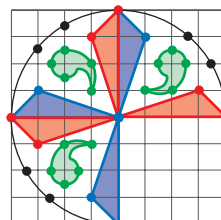


Use the procedure described above and the points indicated to rotate the figure 90° , 180° , and 270° counterclockwise. A 90° rotation clockwise produces the same rotation as a 270° rotation counterclockwise.

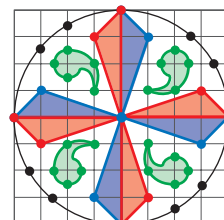
90° counterclockwise



180° counterclockwise

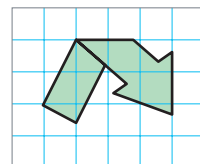


90° clockwise



CHECK Your Progress

- g. **SYMBOLS** Copy and complete the symbol for recycling shown so that the completed figure has rotational symmetry with 120° and 240° as its angles of rotation.



CHECK Your Understanding



SPORTS For Exercises 1 and 2, complete parts a and b for each figure.

Examples 1, 2
(p. 328)

Example 3
(p. 328)

- Determine whether the logo has line symmetry. If it does, trace the figure and draw all lines of symmetry. If not, write *none*.
- Determine whether the logo has rotational symmetry. Write *yes* or *no*. If *yes*, name its angle(s) of rotation.

1.

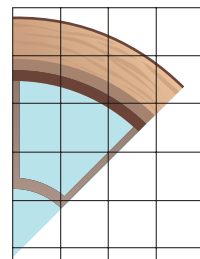


2.



Example 4
(p. 329)

- ARCHITECTURE** Copy and complete the window for the partial window shown so that the completed window has rotational symmetry with 45° , 90° , 135° , 180° , 225° , 270° , and 315° as its angles of rotation.



Exercises



HOMEWORK HELP

For Exercises	See Examples
4a–7a, 8	1, 2
4b–7b, 9	3
10, 11	4

JAPANESE FAMILY CRESTS For Exercises 4–7, complete parts a and b for each family crest.

- Determine whether the crest has line symmetry. If it does, trace the crest and draw all lines of symmetry. If not, write *none*.
- Determine whether the crest has rotational symmetry. Write *yes* or *no*. If *yes*, name its angle(s) of rotation.

4.



5.



6.



7.



ROAD SIGNS For Exercises 8 and 9, use the diagrams below.

a.



b.



c.

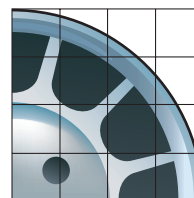


d.

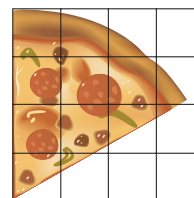


- Determine whether each sign has line symmetry. If it does, trace the sign and draw all lines of symmetry. If not, write *none*.
- Which of the signs above could be rotated and still look the same? If any, name the angle(s) of rotation.

- HUBCAPS** A partial hubcap is shown. Copy and complete the figure so that the completed hubcap has rotational symmetry of 90° , 180° , and 270° .



- PIZZA** A piece of pizza is shown. Copy and complete the figure so that the entire pizza has rotational symmetry of 60° , 120° , 180° , 240° , and 300° . How many slices are needed to complete the pizza?



- ARCHITECTURE** Determine whether the Taj Mahal in Agra, India, has line symmetry. If it does, state the number of lines of symmetry and describe each one. If not, write *none*.



Taj Mahal

- ART** Describe the kind(s) of symmetry shown in the stained glass window at the left.



Real-World Link

The stained glass window of the Notre Dame Cathedral in Paris, France, has a diameter of 10 meters. The Cathedral itself is 35 meters high, 48 meters wide, and 130 meters long.
Source: parisdigest.com

**EXTRAPRACTICE**

See pages 692, 713.

Math onlineSelf-Check Quiz at
ca.gr7math.com**H.O.T. Problems**

14. **ALPHABET** Which capital letters of the alphabet produce the same letter after being rotated 180° ?
15. **TRIANGLES** Which types of triangles—*scalene*, *isosceles*, *equilateral*—have line symmetry? Which have rotational symmetry?

CHALLENGE For Exercises 16 and 17, determine whether each statement is *true* or *false*. If *false*, give a counterexample.

16. If a figure has one horizontal and one vertical line of symmetry, then it also has rotational symmetry.
17. If a figure has rotational symmetry, it also has line symmetry.
18. **WRITING IN MATH** Explain the difference between line symmetry and rotational symmetry.

**STANDARDS PRACTICE**

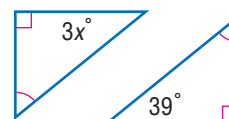
19. The figures below have a repeating pattern.



Which shows a 180° rotation of the 17th figure in the pattern?

**Spiral Review**

20. **ALGEBRA** Find the value of x in the two congruent triangles. (Lesson 6-4)



ALGEBRA Find the sum of the measures of the interior angles of each polygon. (Lesson 6-3)

21. hexagon 22. octagon 23. 14-gon 24. 20-gon

25. **SHOPPING** A pair of boots cost \$130 and are on sale for 15% off this price. Find the amount of the discount. (Lesson 5-3)

GET READY for the Next Lesson

PREREQUISITE SKILL Find the coordinates of the vertices of polygon $H'J'K'L'$ after polygon $HJKL$ is dilated using the given scale factor. Then graph polygon $HJKL$ and its dilation. (Lesson 4-5)

26. $H(-6, 2)$, $J(4, 4)$, $K(7, -2)$, $L(-2, -4)$; scale factor $\frac{1}{2}$

Main IDEA

Graph reflections on a coordinate plane.



Standard 7MG3.2
Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.

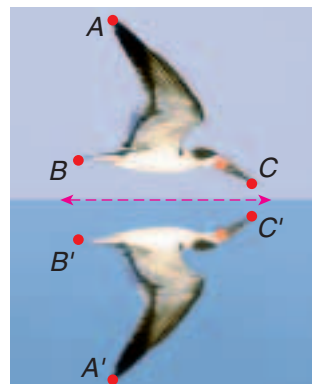
NEW Vocabulary

reflection
line of reflection
transformation
image

GET READY for the Lesson

PHOTOGRAPHY The undisturbed surface of a pond acts like a mirror and can provide the subject for beautiful photographs.

1. Compare the shape and size of the bird to its image in the water.
2. Compare the perpendicular distance from the water line to each of the points shown. What do you observe?
3. The points A , B , and C appear *counterclockwise* on the bird. How are these points oriented on the bird's image?



The mirror image produced by flipping a figure over a line is called a **reflection**. This line is called the **line of reflection**. A reflection is one type of **transformation** or mapping of a geometric figure. In mathematics, an **image** is the position of a figure after a transformation.

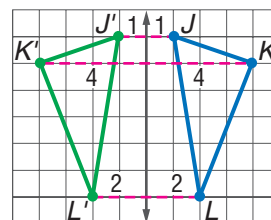
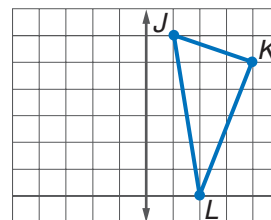
EXAMPLE Draw a Reflection

- 1** Copy $\triangle JKL$ at the right on graph paper. Then draw the image of the figure after a reflection over the given line.

Step 1 Count the number of units between each vertex and the line of reflection.

Step 2 For each vertex, plot a point the same distance away from the line on the other side.

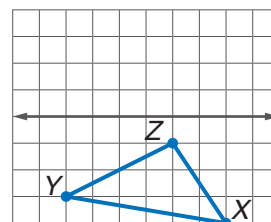
Step 3 Connect the new vertices to form the image of $\triangle JKL$, $\triangle J'K'L'$.

**READING Math**

Notation Read J' as J prime. It is the image of point J .

CHECK Your Progress

- a. Copy the figure on a piece of graph paper. Then draw the image of the figure after a reflection over the given line.

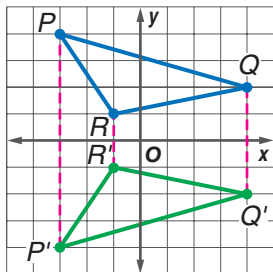


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**EXAMPLES****Reflect a Figure Over an Axis****REVIEW Vocabulary**

vertex the point where two sides of a figure intersect;
Example: point P in $\triangle PQR$ is formed by the intersection of PQ and PR .

- 2** Graph $\triangle PQR$ with vertices $P(-3, 4)$, $Q(4, 2)$, and $R(-1, 1)$. Then graph the image of $\triangle PQR$ after a reflection over the x -axis, and write the coordinates of its vertices.

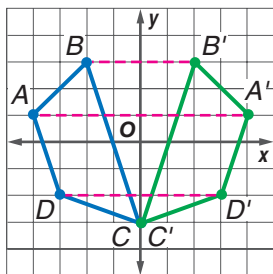


The coordinates of the vertices of the image are $P'(-3, -4)$, $Q'(4, -2)$, and $R'(-1, -1)$. Examine the relationship between the coordinates of each figure.

	same	
	opposites	
$P(-3, 4)$	→	$P'(-3, -4)$
$Q(4, 2)$	→	$Q'(4, -2)$
$R(-1, 1)$	→	$R'(-1, -1)$

Notice that the y -coordinate of a point reflected over the x -axis is the opposite of the y -coordinate of the original point.

- 3** Graph quadrilateral $ABCD$ with vertices $A(-4, 1)$, $B(-2, 3)$, $C(0, -3)$, and $D(-3, -2)$. Then graph the image of $ABCD$ after a reflection over the y -axis, and write the coordinates of its vertices.



The coordinates of the vertices of the image are $A'(4, 1)$, $B'(2, 3)$, $C'(0, -3)$, and $D'(3, -2)$. Examine the relationship between the coordinates of each figure.

	opposites	
	same	
$A(-4, 1)$	→	$A'(4, 1)$
$B(-2, 3)$	→	$B'(2, 3)$
$C(0, -3)$	→	$C'(0, -3)$
$D(-3, -2)$	→	$D'(3, -2)$

Notice that the x -coordinate of a point reflected over the y -axis is the opposite of the x -coordinate of the original point.

STUDY TIP

Points on Line of Reflection Notice that if a point lies on the line of reflection, the image of that point has the same coordinates as those of the point on the original figure.

CHECK Your Progress

Graph $\triangle FGH$ with vertices $F(1, -1)$, $G(5, -3)$, and $H(2, -4)$. Then graph the image of $\triangle FGH$ after a reflection over the given axis, and write the coordinates of its vertices.

b. x -axis

c. y -axis

If a figure touches the line of reflection as it does in Example 3, then the figure and its image together form a new figure that has line symmetry. The line of reflection is then also a line of symmetry.



Real-World Link

Many cultures use masks in rituals as well as theatrical performances. Many masks were created to look like human or animal faces, which display vertical line symmetry.

Source: The History Channel

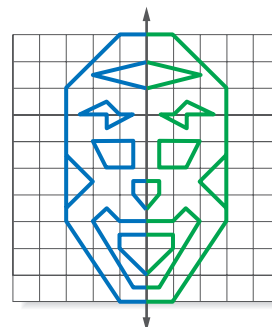
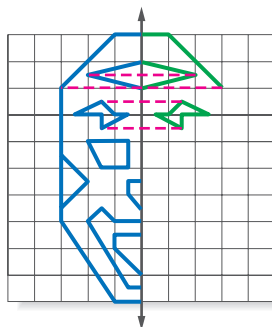
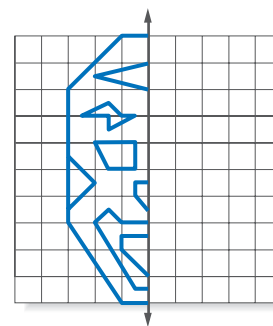
EXAMPLE Use a Reflection

- 4 **MASKS** Copy and complete the mask shown so that the completed figure has a vertical line of symmetry.

You can reflect the half of the mask shown over the indicated vertical line.

Find the distance from each vertex on the figure to the line of reflection.

Then plot a point that same distance away on the opposite side of the line. Connect vertices as appropriate.



CHECK Your Progress

- d. **ART** Copy and complete the portion of the animal shown so that the completed picture has horizontal line symmetry. What is the animal?



CHECK Your Understanding

Examples 1–2 (pp. 332–333)

Graph the figure with the given vertices. Then graph the image of the figure after a reflection over the x -axis and y -axis and write the coordinates of the image's vertices.

1. $\triangle ABC$ with vertices $A(3, 5)$, $B(4, 1)$, and $C(1, 2)$
2. $\triangle WXY$ with vertices $W(-1, -2)$, $X(0, -4)$, and $Y(-3, -5)$

Example 4 (p. 334)

3. **ART** Copy and complete the pattern shown so that the completed figure has vertical line symmetry.

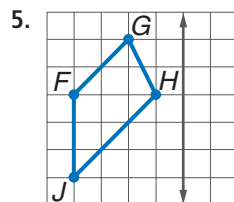
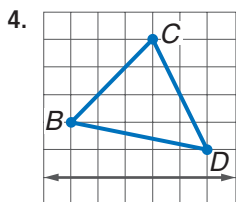


Exercises

HOMEWORK HELP

For Exercises	See Examples
4, 5	1
6–9	2, 3
10, 11	4

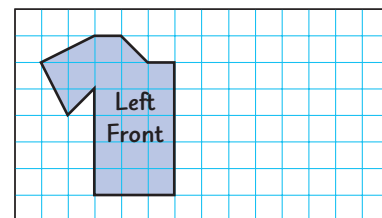
Copy each figure onto graph paper. Then draw the image of the figure after a reflection over the given line.



Graph the figure with the given vertices. Then graph the image of the figure after a reflection over the given axis, and write the coordinates of the image's vertices.

- triangle ABC with vertices $A(-1, -1)$, $B(-2, -4)$, and $C(-4, -1)$; x -axis
- triangle FGH with vertices $F(3, 3)$, $G(4, -3)$, and $H(2, 1)$; y -axis
- square $JKLM$ with vertices $J(-2, 0)$, $K(-1, -2)$, $L(-3, -3)$, and $M(-4, -1)$; y -axis
- quadrilateral $PQRS$ with vertices $P(1, 3)$, $Q(3, 5)$, $R(5, 2)$, and $S(3, 1)$; x -axis

10. **PATTERNS** The drawing shows the pattern for the left half of the front of the shirt. Copy the pattern onto grid paper. Then draw the outline of the pattern after it has been flipped over a vertical line. Label it "Right Front".



11. **ART** The top half of a Ukrainian decorative egg is shown. Copy the figure onto a piece of paper. Then draw the egg design after it has been reflected over a horizontal line.



12. **ARCHITECTURE** Describe in what ways the symmetry of the Fogong Monastery, shown at the left, is similar to that of the Eiffel Tower in Paris, France, shown at the right.

13. Triangle XYZ has vertices $X(-1, 3)$, $Y(2, 5)$, and $Z(3, -2)$. Find the coordinates of the image after a reflection over the x -axis and then the y -axis.

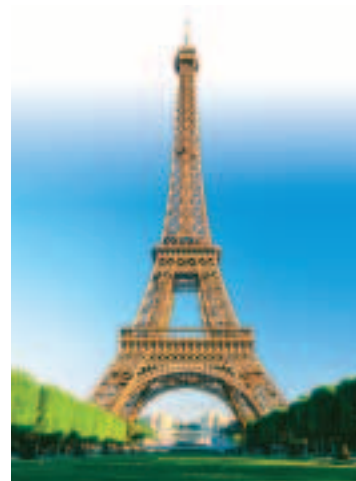
14. **FIND THE DATA** Refer to the California Data File on pages 16–19. Choose an image that illustrates a reflection.



Real-World Link

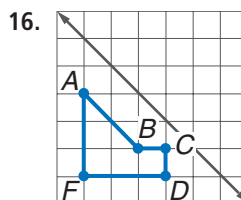
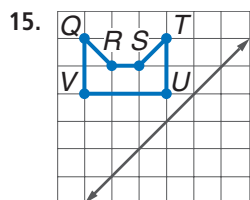
The Fogong Monastery in Yingxian, China, is an example of a *pagoda*, a popular style of traditional Chinese architecture. The monastery has five stories and each story is octagonal.

Source: chinaknowledge.org



Copy each figure onto graph paper. Then draw the image of the figure after a reflection over the given line.

EXTRA PRACTICE
See pages 693, 713.
Math online
Self-Check Quiz at
ca.gr7math.com



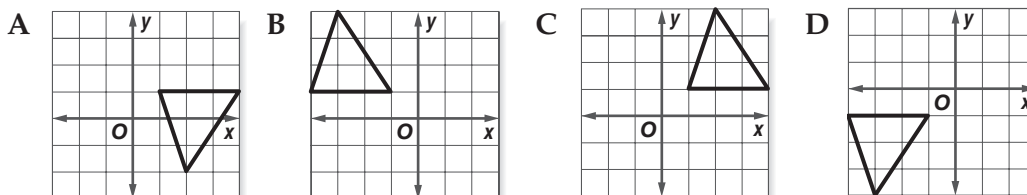
H.O.T. Problems

17. **OPEN ENDED** Draw a right triangle ABC in the first quadrant of a coordinate plane. Then draw the image after a reflection over the x -axis.
18. **CHALLENGE** Suppose point P with coordinates $(-4, 5)$ is reflected so that the coordinates of its image are $(-4, -5)$. Without graphing, which axis was this point reflected over? Explain your reasoning.
19. **WRITING IN MATH** Find the coordinates of the point (x, y) after it has been reflected over the x -axis. Then find the coordinates of the point (x, y) after it has been reflected over the y -axis. Explain your reasoning.



STANDARDS PRACTICE

20. Which of the following is the reflection of $\triangle ABC$ with vertices $A(1, -1)$, $B(4, -1)$, and $C(2, -4)$ over the x -axis?

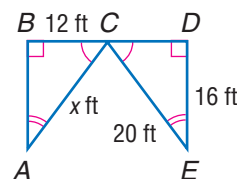


Spiral Review

Determine whether each regular polygon has rotational symmetry. Write *yes* or *no*. If *yes*, name its angle(s) of rotation. (Lesson 6-5)



24. **ALGEBRA** Find the value of x if the triangles at the right are congruent. (Lesson 6-4)



GET READY for the Next Lesson

PREREQUISITE SKILL Add. (Lesson 1-4)

25. $-4 + (-1)$

26. $-5 + 3$

27. $-1 + 4$

Main IDEA

Graph translations on a coordinate plane.



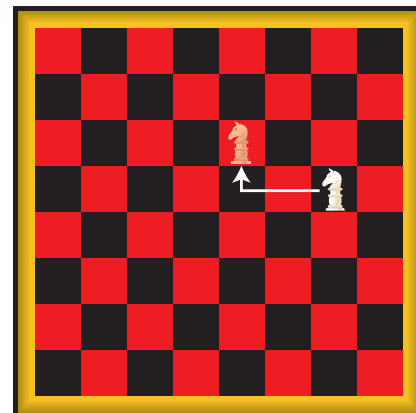
Standard 7MG3.2
Understand and use
coordinate graphs to

plot simple figures,
determine lengths and areas
related to them, and
determine their image
under translations and
reflections.

GET READY for the Lesson

CHESS In chess, there are rules governing how many spaces and in what direction each game piece can be moved during a player's turn. The diagram at the right shows one legal move of a knight.

1. Describe the motion involved in moving the knight.
2. Compare the shape, size, and orientation of the knight in its original position to that of the knight in its new position.

**NEW Vocabulary**

translation

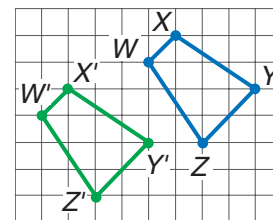
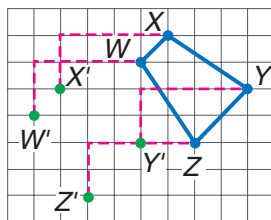
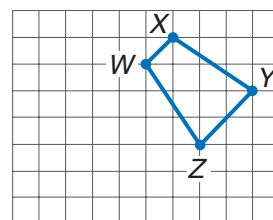
A **translation** (sometimes called a *slide*) is the movement of a figure from one position to another without turning it.

EXAMPLE Draw a Translation

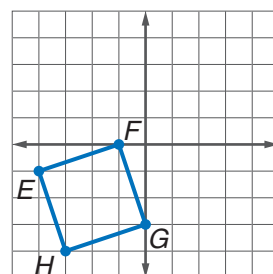
- 1 Copy trapezoid WXYZ at the right on graph paper. Then draw the image of the figure after a translation 4 units left and 2 units down.

Step 1 Move each vertex of the trapezoid 4 units left and 2 units down.

Step 2 Connect the new vertices to form the image.

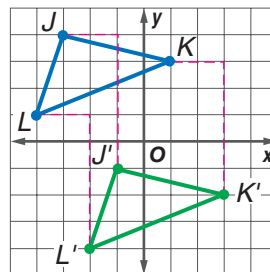
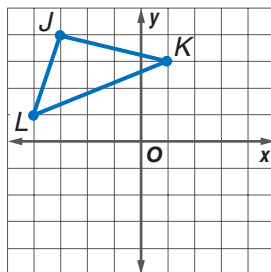
**CHECK Your Progress**

- a. Copy square EFGH at the right on graph paper. Then draw the image of the figure after a translation 5 units right and 3 units up.



EXAMPLE Translation in the Coordinate Plane

- 2** Graph $\triangle JKL$ with vertices $J(-3, 4)$, $K(1, 3)$, and $L(-4, 1)$. Then graph the image of $\triangle JKL$ after a translation 2 units right and 5 units down. Write the coordinates of its vertices.



The coordinates of the vertices of the image are $J'(-1, -1)$, $K'(3, -2)$, and $L'(-2, -4)$. Notice that these vertices can also be found by adding 2 to the x -coordinates and -5 to the y -coordinates, or $(2, -5)$.

Original		Add $(2, -5)$		Image
$J(-3, 4)$	$\rightarrow$	$(-3 + 2, 4 + (-5))$	$\rightarrow$	$J'(-1, -1)$
$K(1, 3)$	$\rightarrow$	$(1 + 2, 3 + (-5))$	$\rightarrow$	$K'(3, -2)$
$L(-4, 1)$	$\rightarrow$	$(-4 + 2, 1 + (-5))$	$\rightarrow$	$L'(-2, -4)$

CHECK Your Progress

Graph $\triangle ABC$ with vertices $A(4, -3)$, $B(0, 2)$, and $C(5, 1)$. Then graph its image after each translation, and write the coordinates of its vertices.

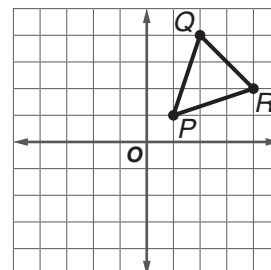
b. 2 units down

c. 4 units left and 3 units up

STANDARDS EXAMPLE

- 3** If triangle PQR is translated 2 units left and 3 units down, what are the coordinates of point R' ?

- A $(2, 2)$ C $(4, 2)$
B $(4, -1)$ D $(2, -1)$



Read the Item

You are asked to determine the coordinates of point R' after the original figure has been translated 2 units left and 3 units down.

Solve the Item

You can answer this question without translating the entire triangle.

STUDY TIP

Translations In the coordinate plane, a translation can be described using an ordered pair. A translation *up* or to the *right* is positive. A translation *down* or to the *left* is negative. $(2, -5)$ means a translation 2 units right and 5 units down.

**Test-Taking Tip**

Eliminate the Possibilities Since point R was translated 2 units left, the x -coordinate of point R' is $4 - 2$ or 2. This eliminates choices B and C.

The coordinates of point R are $(4, 2)$.

The x -coordinate of R is 4, so the x -coordinate of R' is $4 - 2$ or 2.

The y -coordinate of R is 2, so the y -coordinate of R' is $2 - 3$ or -1 .

The coordinates of R' are $(2, -1)$.

The answer is D.

Original Figure

Translating 2 units left is the same as subtracting 2 from the x -coordinate.

Translating 3 units down is the same as subtracting 3 from the y -coordinate.

CHECK Your Progress

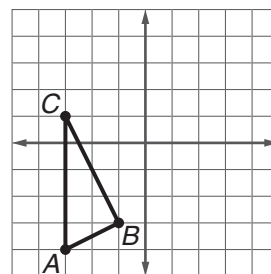
- d. If $\triangle ABC$ with vertices $A(-3, -4)$, $B(-1, -3)$, and $C(-3, 1)$ is translated 3 units to the right and 4 units up, what are the coordinates of B' ?

F $(2, 1)$

H $(-4, 1)$

G $(-4, -7)$

J $(2, -7)$

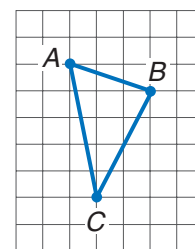


Online Personal Tutor at ca.gr7math.com

CHECK Your Understanding

Example 1 (p. 337) For Exercises 1 and 2, copy the figure at the right.

1. Draw the image of $\triangle ABC$ after a translation 4 units left and 1 unit up.
2. Draw the image of $\triangle ABC$ after a translation 2 units right and 3 units down.



Example 2 (p. 338) Graph $\triangle XYZ$ with vertices $X(-4, -4)$, $Y(-3, -1)$, and $Z(2, -2)$. Then graph the image of $\triangle XYZ$ after each translation, and write the coordinates of its vertices.

3. 3 units right and 4 units up
4. 2 units left and 3 units down

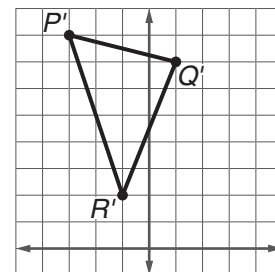
Example 3 (p. 338) **STANDARDS PRACTICE** Triangle PQR is translated 5 units left and 3 units down. If the coordinates of P' are $(-3, 8)$, find the coordinates of P .

A $(-8, 11)$

C $(-8, 5)$

B $(-6, 3)$

D $(2, 5)$



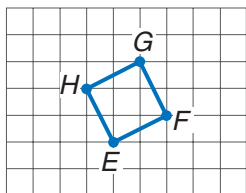
Exercises

HOMEWORK HELP

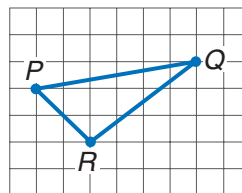
For Exercises	See Examples
6–9	1
10–11	2
19, 20	3

Copy each figure onto graph paper. Then draw the image of the figure after the indicated translation.

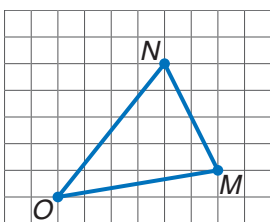
6. 5 units right and 3 units up



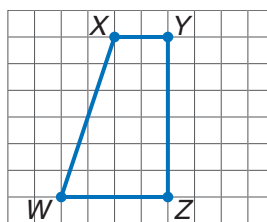
7. 3 units right and 4 units down



8. 2 units left and 5 units down



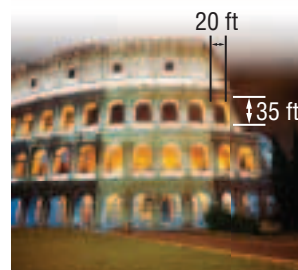
9. 1 unit left and 2 units up



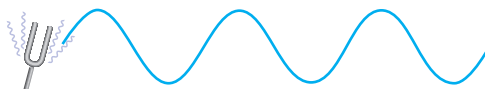
Graph the figure with the given vertices. Then graph the image of the figure after the indicated translation, and write the coordinates of its vertices.

10. $\triangle ABC$ with vertices $A(1, 2)$, $B(3, 1)$, and $C(3, 4)$ translated 2 units left and 1 unit up
11. rectangle $JKLM$ with vertices $J(-3, 2)$, $K(3, 5)$, $L(4, 3)$, and $M(-2, 0)$ translated by 1 unit right and 4 units down

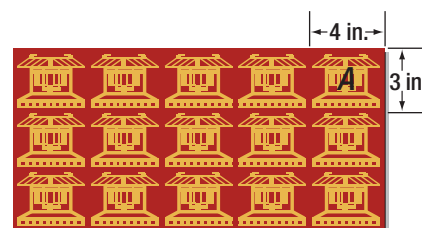
12. **ARCHITECTURE** The arches in the first three stories of the Coliseum in Rome, Italy, are translations of one another. Describe the minimum number of translations of the indicated arch needed to create the section shown in the photo at the right.



13. **MUSIC** The sound wave of a tuning fork is shown below. Look for a pattern in the sound wave. Copy the sound wave and indicate where this pattern repeats or is translated. Find how many translations of the original pattern are shown in the diagram.



14. **WALLPAPER** The wallpaper design at the right is a traditional Japanese design. Describe the minimum number of translations of the original pattern, A , needed to create the section shown.



EXTRA PRACTICE

See pages 693, 713.

Math online

Self-Check Quiz at
ca.gr7math.com



15. **GEOMETRY** Triangle RST has vertices $R(4, 2)$, $S(-8, 0)$, and $T(6, 7)$. When translated, R' has coordinates $(-2, 4)$. Find the coordinates of S' and T' . Then describe the translation of triangle RST .

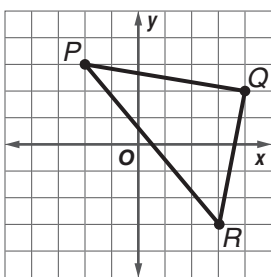
H.O.T. Problems

16. **REASONING** A figure is translated by $(-5, 7)$. Then the result is translated by $(5, -7)$. Without graphing, what is the final position of the figure? Explain your reasoning.
17. **CHALLENGE** What are the coordinates of the point (x, y) after being translated m units left and n units up?
18. **WRITING IN MATH** Write a real-world problem in which you would need to translate a figure. Then solve your problem.



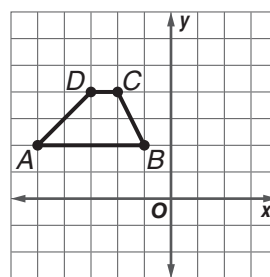
STANDARDS PRACTICE

19. If $\triangle PQR$ is translated 4 units right and 3 units up, what are the coordinates of R' ?



- A $(-1, -6)$ C $(-1, 0)$
B $(7, 0)$ D $(7, -6)$

20. Find the coordinates of C' of trapezoid $ABCD$ after a translation 3 units right and 7 units down.



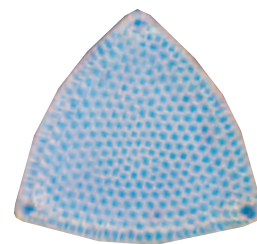
- F $(1, 3)$ H $(1, -3)$
G $(5, 7)$ J $(-9, 1)$

Spiral Review

21. Graph polygon $ABCDE$ with vertices $A(-5, -3)$, $B(-2, 1)$, $C(-3, 4)$, $D(0, 2)$, and $E(0, -3)$. Then graph the image of the figure after a reflection over the y -axis, and write the coordinates of its vertices. (Lesson 6-6)

LIFE SCIENCE A *diatom* is a microscopic algae. For Exercises 22 and 23, use the diagram of the diatom at the right. (Lesson 6-5)

22. Does the diatom have line symmetry? If so, trace the figure and draw any lines of symmetry. If not, write *none*.
23. Does the diatom have rotational symmetry? Write *yes* or *no*. If *yes*, name its angle(s) of rotation.



Order each set of numbers from least to greatest. (Lesson 5-2)

24. 16% , $\frac{1}{6}$, 1.6 , $\frac{1}{16}$

25. $\frac{3}{8}$, 0.65 , 38% , $\frac{5}{8}$

26. 0.44 , $\frac{4}{5}$, $\frac{4}{9}$, 88%

Study Guide and Review



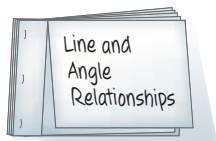
Download Vocabulary
Review from ca.gr7math.com

FOLDABLES

Study Organizer

GET READY to Study

Be sure the following
Key Concepts are noted
in your Foldable.



Key Concepts

Angle Relationships (Lesson 6-1)

- The sum of the measures of complementary angles is 90° .
- The sum of the measures of supplementary angles is 180° .
- Vertical angles are opposite angles formed by intersecting lines and are congruent.

Polygons (Lessons 6-3 and 6-4)

- The sum of the measures of the interior angles of a polygon is $(n - 2)(180)$.
- In congruent polygons, corresponding sides and angles are congruent.

Symmetry (Lesson 6-5)

- A figure with line symmetry can be folded over a line so that the two halves match.
- A figure with rotational symmetry can be rotated about its center so that it looks exactly as it does in its original position.

Transformations (Lessons 6-6 and 6-7)

- In a reflection, the image is congruent to the original figure, but the orientation of the image is different from that of the original figure.
- In a translation, the image is congruent to the original figure, and the orientation of the image is the same as that of the original figure.

Key Vocabulary

angle (p. 307)	line symmetry (p. 327)
collinear (p. 306)	plane (p. 306)
complementary angles (p. 307)	point (p. 306)
congruent angles (p. 306)	ray (p. 307)
image (p. 332)	reflection (p. 332)
inductive reasoning (p. 314)	rotational symmetry (p. 328)
interior angles (p. 316)	supplementary angles (p. 307)
line (p. 306)	transformation (p. 332)
line of reflection (p. 332)	translation (p. 337)
line of symmetry (p. 327)	vertical angles (p. 307)

Vocabulary Check

State whether each sentence is *true* or *false*.
If *false*, replace the underlined word or
number to make a true sentence.

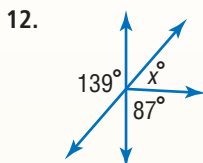
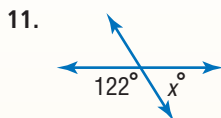
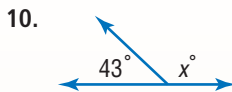
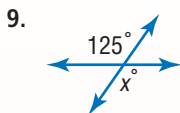
1. $m\angle 1$ is read as the measure of $\angle 1$.
2. A translation of $(3, -2)$ means a translation 3 units left and 2 units down.
3. If $\triangle ABC \cong \triangle DEF$, then $\angle C \cong \angle E$.
4. Inductive reasoning is the process of making a rule after observing several examples and using that rule to make a decision.
5. A polygon whose angles are all congruent is said to be equilateral.
6. A rectangle will match itself after being rotated 90° , 180° , and 270° .
7. P' , the image of point P , is read as P prime.
8. When a transformation occurs, the resulting figure is called a line of reflection.

Lesson-by-Lesson Review

6-1

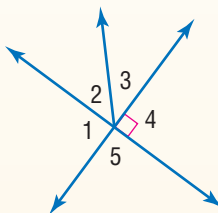
Line and Angle Relationships (pp. 306–310)

Find the value of x in each figure.

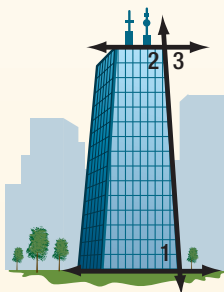


Refer to the figure below. Classify each pair of angles as *complementary*, *supplementary*, or *vertical*.

13. $\angle 1$ and $\angle 4$
14. $\angle 2$ and $\angle 3$
15. $\angle 1$ and $\angle 5$
16. $\angle 4$ and $\angle 5$



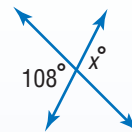
17. **ARCHITECTURE** On the skyscraper at the right, $m\angle 1 = 86^\circ$ and $\angle 1 \cong \angle 3$. Classify the relationship between $\angle 2$ and $\angle 3$. Then find $m\angle 2$.



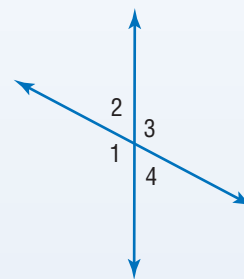
Example 1 Find the value of x in the figure.

Since the angle labeled x° and the angle labeled 108° are vertical angles, they are congruent.

Therefore, $x = 108$.



Example 2 Classify $\angle 2$ and $\angle 3$ as *complementary*, *supplementary*, or *vertical*.



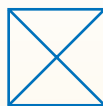
Since $\angle 2$ and $\angle 3$ form a straight line, they are supplementary angles.

6-2

PSI: Use Logical Reasoning (pp. 314–315)

Solve each problem using logical reasoning.

18. **GEOMETRY** Draw several squares and connect the opposite vertices. Then measure the four angles that are formed by the intersecting diagonals on each square. What seems to be true about the diagonals of a square?



Example 3

Use logical reasoning to find the next number.

3, 5, 8, 12, 17, ...

Since the numbers increase by 2, 3, 4, and 5, the next number will increase by 6. The next number is 23.

6-3

Polygons and Angles (pp. 316–319)

Find the sum of the measures of the interior angles of each polygon.

19. decagon

20. 32-gon

Find the measure of one interior angle in each regular polygon. Round to the nearest tenth if necessary.

21. heptagon 22. pentagon

23. **RUGS** Find the measure of an interior angle of a rug shaped like a regular octagon.

Example 4 Find the measure of one interior angle of a regular hexagon.

Find the sum of the measures of the angles.

$$S = (n - 2)180 \quad \text{Write an equation.}$$

$$S = (6 - 2)180 \quad \text{Replace } n \text{ with } 6.$$

$$S = (4)180 \quad \text{Subtract.}$$

$$S = 720 \quad \text{Multiply.}$$

The sum of the measures of the interior angles is 720° .

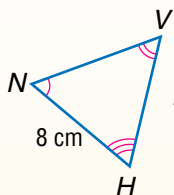
Divide 720° by 6, the number of interior angles. So, the measure of one interior angle of a regular hexagon is $720^\circ \div 6$ or 120° .

6-4

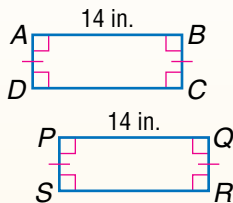
Congruent Polygons (pp. 320–323)

Determine whether the polygons shown are congruent. If so, name the corresponding parts and write a congruence statement.

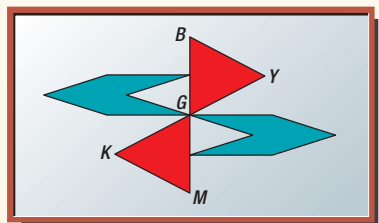
24.



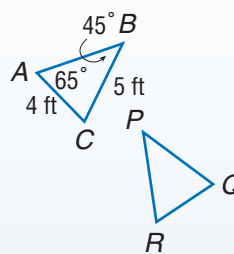
25.



26. **ART** $\triangle BGY \cong \triangle MGK$ in the art design below. If $m\angle Y = 55^\circ$, find $m\angle K$.



Example 5 In the figure below, $\triangle ABC \cong \triangle RPQ$. Find PQ .



$\overline{PQ}$ corresponds to $\overline{BC}$.

Since $BC = 5$ feet, $PQ = 5$ feet.

6-5

Symmetry (pp. 327–331)

BOATING Determine whether each signal flag has line symmetry. If it does, trace the figure and draw all lines of symmetry. If not, write *none*.



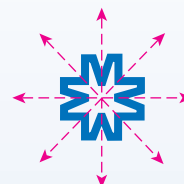
30. Which of the figures above has rotational symmetry? Name the angle(s) of rotation.
31. **ART** Determine whether the plate design below has rotational symmetry. If it does, name the angle(s) of rotation.



Example 6 Determine whether the logo below has line symmetry. If it does, trace the figure and draw all lines of symmetry. If not, write *none*.



The logo has line symmetry.



Example 7 Determine whether the logo above has rotational symmetry. If it does, name its angle(s) of rotation.



The logo has rotational symmetry. Its angles of rotation are 90° , 180° , and 270° .

6-6

Reflections (pp. 332–336)

Graph parallelogram $QRST$ with vertices $Q(2, 5)$, $R(4, 5)$, $S(3, 1)$, and $T(1, 1)$. Then graph its image after a reflection over the given axis, and write the coordinates of its vertices.

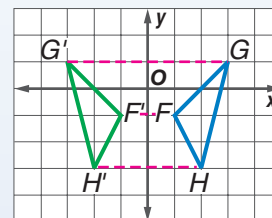
32. x -axis 33. y -axis

34. **ANIMALS** Copy and complete the starfish shown so that the completed figure has a vertical line of symmetry.



Example 8 Graph $\triangle FGH$ with vertices $F(1, -1)$, $G(3, 1)$, and $H(2, -3)$ and its image after a reflection over the y -axis.

The x -coordinate of a point reflected over the y -axis is the opposite of the x -coordinate of the original point. So, the coordinates of the vertices of the image are $F'(-1, -1)$, $G'(-3, 1)$, and $H'(-2, -3)$.



6-7

Translations (pp. 337–341)

Copy the figure at the right onto graph paper. Then draw the image of the figure after the indicated translation.



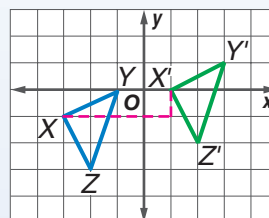
35. 4 units left and 2 units up
36. 3 units right and 1 unit down

Graph $\triangle ABC$ with vertices $A(2, 2)$, $B(3, 5)$, and $C(5, 3)$. Then graph its image after the indicated translation, and write the coordinates of its vertices.

37. 1 unit right and 4 units down
38. 2 units left and 3 units up
39. **HIKING** From her car, Marjorie hiked 2 miles north and 3 miles west before she decided to stop and rest. If her starting point can be represented by the point $P(1, 4)$, what are the coordinates of her resting point?

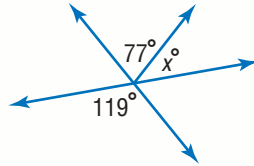
Example 9 Graph $\triangle XYZ$ with vertices $X(-3, -1)$, $Y(-1, 0)$, and $Z(-2, -3)$ and its image after a translation 4 units right and 1 unit up.

The coordinates of the vertices of the image can be found by adding 4 to the x -coordinates and 1 to the y -coordinates. The coordinates of the image are $X'(1, 0)$, $Y'(3, 1)$, and $Z'(2, -2)$.



Practice Test

1. **ALGEBRA** Find the value of x .

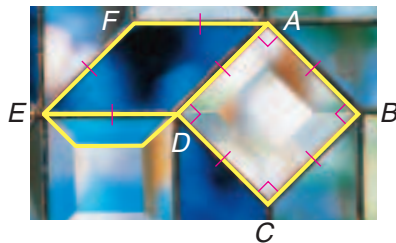


2. **ALGEBRA** Angles P and Q are supplementary. Find $m\angle P$ if $m\angle Q = 139^\circ$.

ALGEBRA Find the sum of the measures of the interior angles of each regular polygon. Then, find the measure of one interior angle.

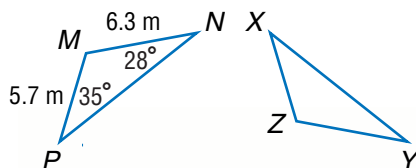
3. octagon
4. 15-gon

5. **STANDARDS PRACTICE** Which of the following statements is *not* true concerning the quadrilaterals in the stained glass window?



- A The sum of the angle measures in quadrilateral $ADEF$ is 360° .
B Quadrilateral $ABCD$ is a regular polygon.
C The quadrilaterals are congruent.
D The sum of the angle measures in quadrilateral $ABCD$ is 360° .

In the figure below, $\triangle MNP \cong \triangle ZYX$. Find each measure.



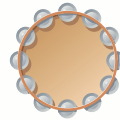
6. ZY
7. $\angle X$
8. $\angle Z$

MUSIC Determine whether each figure has line symmetry. If it does, trace the figure and draw all lines of symmetry. If not, write *none*.

9.



10.

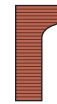


11.

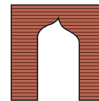


12. **MUSIC** Which figure above has rotational symmetry? Name its angle(s) of rotation.

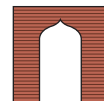
13. **STANDARDS PRACTICE** A portion of an archway is shown. Which of the following shows the completed archway with vertical line symmetry?



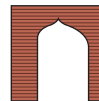
F



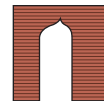
H



G



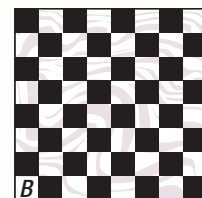
J



Graph $\triangle JKL$ with vertices $J(2, 3)$, $K(-1, 4)$, and $L(-3, -5)$. Then graph its image and write the coordinates of its vertices after each transformation.

14. reflection over the x -axis
15. translation 2 units left and 5 units up

16. **CHESS** Describe the minimum number of translations needed to create the pattern of the chess board shown from the original square B if each square has a side length of 1 inch.



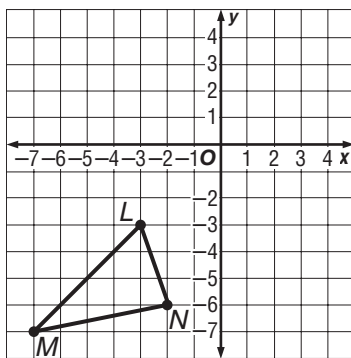
California Standards Practice

Cumulative, Chapters 1–6



Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

- 1 If $\triangle LMN$ is translated 5 units up and 7 units to the right, what are the coordinates of point L' ?

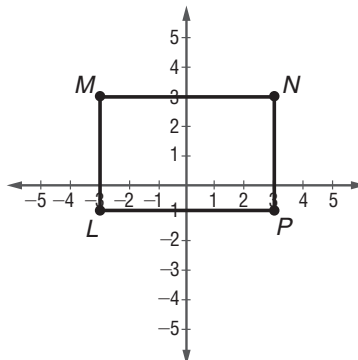


- A $(-10, 2)$ C $(2, 5)$
 B $(4, 2)$ D $(4, -3)$
- 2 A refrigerator costs \$560. If the refrigerator is on sale for 30% off the regular price, how much is the discount?
- F \$392 H \$175
 G \$260 J \$168
- 3 A microscope slide shows 35 red blood cells out of 60 blood cells. How many red blood cells would be expected in a sample of the same blood that has 840 blood cells?
- A 2.5 C 510
 B 490 D 1,440

TEST-TAKING TIP

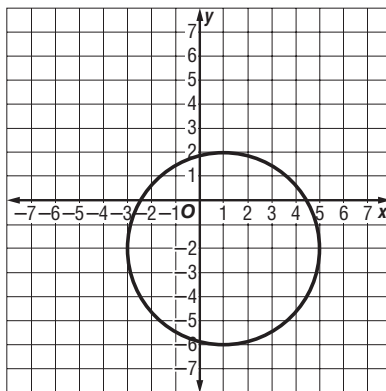
Question 3 The ratio of the number of red blood cells to the total blood cells on the first slide is the same as the ratio on the second slide. Use a proportion.

- 4 The graph of rectangle $LMNP$ is shown below.



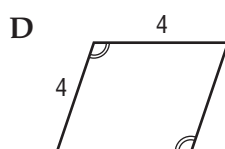
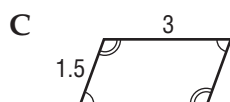
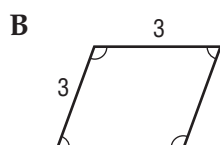
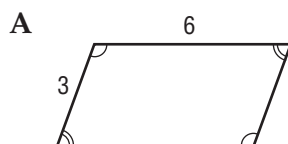
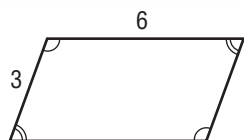
What is the area, in square units, of rectangle $LMNP$?

- F 24 H 12
 G 18 J 9
- 5 Find $-18 - (-7)$.
- A -25 C -9
 B -11 D 25
- 6 A circle with a radius of 4 units has its center at $(1, -2)$ on a coordinate grid. If the circle is translated 5 units up and 4 units left, what will be the coordinates of the new center?



- F $(-5, 2)$ H $(-3, 3)$
 G $(-4, 2)$ J $(5, 3)$

- 7 Which figure is congruent to the figure below?



- 8 Jesse purchased a new digital camera for \$499 and a printer for \$299 including tax. If he plans to pay the total amount in 6 equal monthly payments, what is a reasonable estimate of the amount he will pay each month?

F \$66.50
G \$133.00
H \$155.00
J \$165.00

- 9 Dannie can make 3 bracelets in 55 minutes. At this rate, how many hours will it take her to make 18 bracelets?

A 3.3 C 9.17
B 5.5 D 330

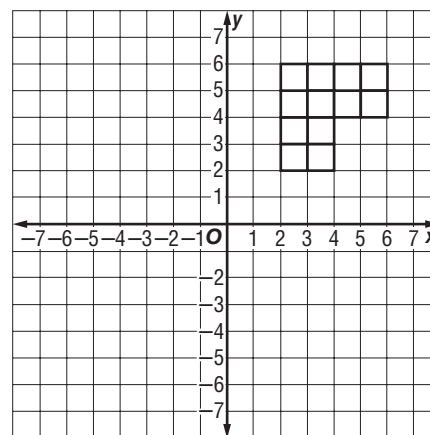
- 10 Stu saved \$19.75 when he purchased shoes. If the sale price was 25% off the regular price, what was the original price?

F \$79 H \$25
G \$35 J \$20

Pre-AP

Record your answers on a sheet of paper.
Show your work.

- 11 Use the figure in the coordinate grid.



- Graph the figure after a reflection over the x -axis.
- Graph the figure after a reflection over the y -axis.
- Graph the figure if it is reflected over the line $y = 2$ and then over the line $y = -2$. What transformation is this the same as?

NEED EXTRA HELP?

If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11
Go to Lesson...	6-7	5-8	4-3	3-7	1-5	6-7	6-4	1-1	4-1	5-8	6-6
For Help with Standard...	MG3.2	NS1.7	AF4.2	MG3.2	NS1.2	MG3.2	MG3.4	MR2.1	AF4.2	NS1.7	MG3.2