



CHAPTER

7

Measurement: Area and Volume



- **Standard 7MG2.0**
Compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. Know how perimeter, area, and volume are affected by changes of scale.

Key Vocabulary

cone (p. 381)

cylinder (p. 374)

pyramid (p. 369)

prism (p. 369)

Real-World Link

Caverns Stalactites and stalagmites are cone-shaped formations found in caverns. If you know the diameter of the base and the height of the formation, you can determine the volume of rocks and minerals in the formation.

FOLDABLES Study Organizer

Measurement: Area and Volume Make this Foldable to help you organize your notes. Begin with a piece of $11'' \times 17''$ paper.

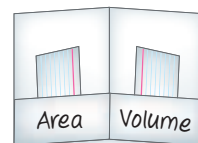
- 1** Fold in half widthwise.



- 2** Open and fold the bottom to form a pocket. Glue edges.



- 3** Label each pocket. Place several index cards in each pocket.



GET READY for Chapter 7

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2



Take the Online Readiness Quiz at ca.gr7math.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

Multiply. (Lessons 2-3 and 2-9)

1. $\frac{1}{3} \cdot 8 \cdot 12$ 2. $\frac{1}{3} \cdot 4 \cdot 9^2$

3. **RUNNING** Julian runs 4 miles a day for 6 days each week. If he decides to run $\frac{1}{3}$ of this distance, how many miles will he run in one week? (Lesson 2-3)

Evaluate $2ab + 2bc + 2ac$ for the values of the variables indicated.

(Lesson 1-2)

4. $a = 5, b = 4, c = 8$
5. $a = 2, b = 3, c = 9$
6. $a = 5.4, b = 2.9, c = 7.1$
7. $a = 2.6, b = 6.4, c = 10.8$

Find the value of each expression.
Use $\pi \approx 3.14$. Round to the nearest tenth. (Prior Grade)

8. $\pi \cdot 15$ 9. $2 \cdot \pi \cdot 3.2$
10. $\pi \cdot 7^2$ 11. $\pi \cdot (19 \div 2)^2$

12. **PIZZA** The distance, in inches, around a circular pizza with diameter 14 inches is given by the expression $\pi \cdot 14$. Evaluate this expression. Round to the nearest tenth. (Prior Grade)

QUICK Review

Example 1

Multiply $\frac{1}{3} \cdot 5 \cdot 6^2$.

$$\begin{aligned}\frac{1}{3} \cdot 5 \cdot 6^2 &= \frac{1}{3} \cdot 5 \cdot 36 && \text{Evaluate } 6^2. \\ &= \frac{1}{3} \cdot 180 && \text{Multiply 5 by 36.} \\ &= 60 && \text{Multiply } \frac{1}{3} \text{ by 180.}\end{aligned}$$

Example 2

Evaluate $2ab + 2bc + 2ac$ if $a = 7, b = 4$, and $c = 2$.

$$\begin{aligned}2ab + 2bc + 2ac &= 2(7)(4) + 2(4)(2) + 2(7)(2) && \text{Substitute } a = 7, b = 4, \text{ and } c = 2. \\ &= 56 + 16 + 28 && \text{Multiply.} \\ &= 100 && \text{Add.}\end{aligned}$$

Example 3

Evaluate $\pi \cdot 16^2$. Use $\pi \approx 3.14$. Round to the nearest tenth.

$$\begin{aligned}\pi \cdot 16^2 &\approx 3.14 \cdot 256 && \text{Evaluate } 16^2. \\ &\approx 803.8 && \text{Multiply 3.14 by 256.}\end{aligned}$$

7-1

Circumference and Area of Circles

Main IDEA

Find the circumference and area of circles.



Standard 7MG2.1

Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

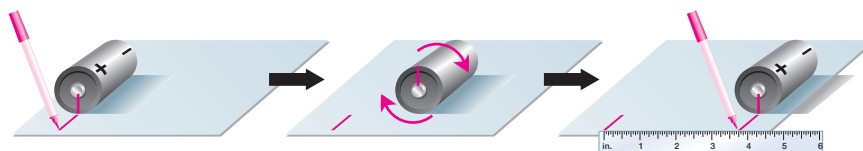
Standard 7MG3.1 Identify and construct basic elements of geometric figures (e.g., altitudes, midpoints, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.

NEW Vocabulary

circle
center
radius
chord
diameter
circumference
pi

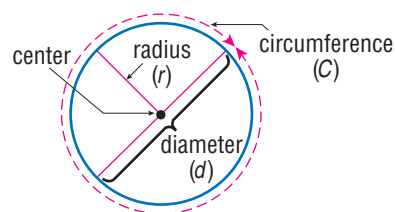
MINI Lab

- STEP 1** Measure and record the distance d across the circular part of an object, such as a battery or a can, through its center.
- STEP 2** Place the object on a piece of paper. Mark the point where the object touches the paper on both the object and on the paper.
- STEP 3** Carefully roll the object so that it makes one complete rotation. Then mark the paper again.
- STEP 4** Finally, measure the distance C between the marks.



1. What distance does C represent?
2. Find the ratio $\frac{C}{d}$ for this object.
3. Repeat the steps above for at least two other circular objects and compare the ratios of C to d . What do you observe?
4. Graph the data you collected as ordered pairs, (d, C) . Then describe the graph.

A **circle** is a set of points in a plane that are the same distance from a given point in the plane, called the **center**. The segment from the center to any point on the circle is called the **radius**. A **chord** is any segment with both endpoints on the circle. A **diameter** is a chord that passes through the center. It is the longest chord.



The diameter of a circle is twice its radius or $d = 2r$.

The distance around the circle is called the **circumference**. The ratio of the circumference of a circle to its diameter is always 3.1415926.... It is represented by the Greek letter **π (pi)**. The numbers 3.14 and $\frac{22}{7}$ are often used as approximations for π . So, $\frac{C}{d} = \pi$. This can also be written as $C = \pi d$ or $C \approx 3.14d$.



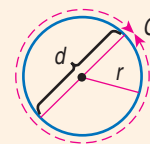
KEY CONCEPT

Circumference of a Circle

Words The circumference C of a circle is equal to its diameter d times π , or 2 times its radius r times π .

Symbols $C = \pi d$ or $C = 2\pi r$

Model



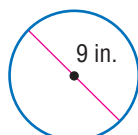
STUDY TIP

Calculating with π
Using 3.14 for π will result in a close approximation.

EXAMPLES

Find the Circumferences of Circles

1



$$C = \pi d$$

Circumference of a circle

$$C = \pi \cdot 9$$

Replace d with 9.

$$C = 9\pi$$

This is the *exact* circumference.

$$C \approx 9 \cdot 3.14 \text{ or } 28.3$$

Replace π with 3.14 and multiply.

The circumference is about 28.3 inches.

2



$$C = 2\pi r$$

Circumference of a circle

$$C \approx 2 \cdot 3.14 \cdot 7.2$$

Replace π with 3.14 and r with 7.2.

$$C \approx 45.2$$

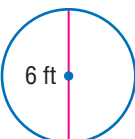
Multiply.

The circumference is about 45.2 centimeters.



CHECK Your Progress

a.



b.



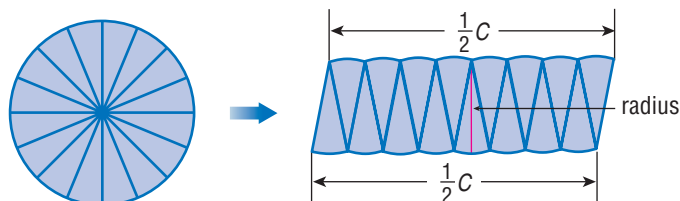
c.



Personal Tutor at ca.gr7math.com



A circle can be separated into congruent wedge-like pieces. Then the pieces can be rearranged to form a figure that resembles a parallelogram.



Since the circle has an area that is relatively close to the area of the parallelogram-shaped figure, you can use the formula for the area of a parallelogram to find the formula for the area of a circle.

$$A = bh$$

Area of a parallelogram

$$A = \left(\frac{1}{2} \cdot C\right)r$$

The base of the parallelogram is one-half the circumference and the height is the radius.

$$A = \left(\frac{1}{2} \cdot 2\pi r\right)r$$

Replace C with $2\pi r$.

$$A = \pi \cdot r \cdot r \text{ or } \pi r^2$$

Simplify.



Extra Examples at ca.gr7math.com



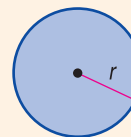
KEY CONCEPT

Area of a Circle

Words The area A of a circle is equal to π times the square of the radius r .

Symbols $A = \pi r^2$

Model



STUDY TIP

Estimation

To estimate the area of a circle, square the radius and then multiply by 3.

EXAMPLES

Find the Areas of Circles

Find the area of each circle. Round to the nearest tenth.

3



$$A = \pi r^2$$

Area of a circle

$$A \approx 3.14 \cdot 8^2$$

Replace π with 3.14 and r with 8.

$$A \approx 3.14 \cdot 64$$

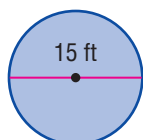
Evaluate 8^2 .

$$A \approx 201.0$$

Multiply.

The area is about 201.0 square kilometers.

4



$$A = \pi r^2$$

Area of a circle

$$A \approx 3.14 (7.5)^2$$

Replace π with 3.14 and r with half of 15 or 7.5.

$$A \approx 3.14 \cdot 56.25$$

Evaluate 7.5^2 .

$$A \approx 176.6$$

Multiply.

The area is about 176.6 square feet.



CHECK Your Progress

Find the area of each circle. Round to the nearest tenth.

d. The radius is 11 inches.

e. The diameter is 5 meters.



Real-World Link

The Sonic Pool, at the Huntington Botanical Gardens in San Marino, California, is a circular bowl filled with lake water. The water is vibrated to create wave patterns that visitors can reach in and touch.

Source: nedkahn.com



Real-World EXAMPLE

5

FOUNTAINS Refer to the information at the left. Suppose that you walk around the edge of the Sonic Pool and estimate its circumference to be 16 feet. Based on your estimate, what is the approximate diameter of the fountain?

$$C = \pi d$$

Circumference of a circle

$$16 \approx 3.14d$$

Replace π with 3.14 and C with 16.

$$\frac{16}{3.14} \approx d$$

Divide each side by 3.14.

$$5.1 \approx d$$

Divide.

The diameter of the fountain is about 5 feet.



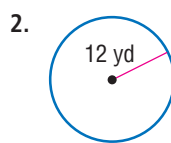
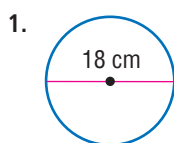
CHECK Your Progress

f. **HOME DECOR** A catalog states that a circular area rug covers 19.5 square feet. What is the approximate diameter of the rug?

CHECK Your Understanding

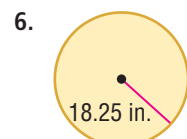
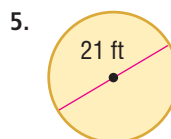
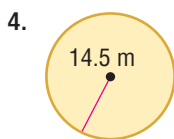
Find the circumference of each circle. Round to the nearest tenth.

Examples 1, 2
(p. 353)



Find the area of each circle. Round to the nearest tenth.

Examples 3, 4
(p. 354)



Example 5
(p. 354)

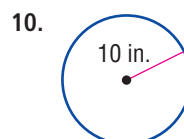
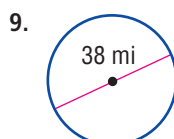
7. **BRACELETS** When Cammie finished making a friendship bracelet, it was 7.9 inches long. What was the diameter of the bracelet?

Exercises

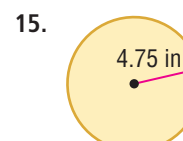
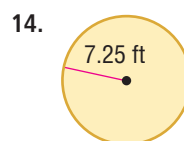
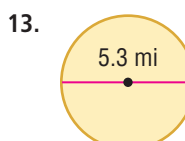
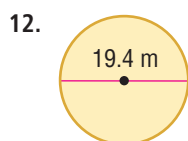
HOMEWORK *HELP*

For Exercises	See Examples
8–11	1, 2
12–15	3, 4
16–19	5

Find the circumference of each circle. Round to the nearest tenth.



Find the area of each circle. Round to the nearest tenth.



16. **CARS** If the tires on a car each have a diameter of 25 inches, how far will the car travel in 100 rotations of its tires?
17. **MEASUREMENT** A circular table top has a radius of $2\frac{1}{4}$ feet. A decorative trim is placed along the outside edge of the table. How long is the trim?
18. **SAFETY** A light in a parking lot illuminates a circular area 15 meters across. What is the area of the parking lot covered by the light?
19. **ANIMALS** A California ground squirrel usually stays within 150 yards of its burrow. Find the area of a California ground squirrel's world.

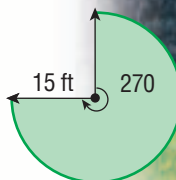
Find the circumference and area of each circle. Round to the nearest tenth.

20. The radius is 3.5 centimeters. 21. The diameter is 8.6 kilometers.
22. The diameter is 9 inches. 23. The radius is 0.6 mile.



24. Find the diameter of a circle if its area is 706.9 square millimeters.

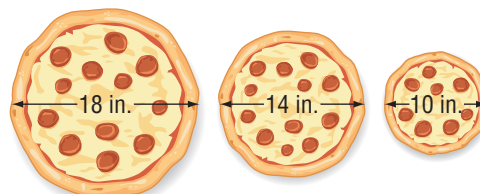
25. **LAWN CARE** The pattern of water distribution from a sprinkler is commonly a circle or part of a circle. A certain sprinkler is set to cover part of a circle measuring 270° . Find the area of the grass watered if the sprinkler reaches a distance of 15 feet.



Another approximate value for π is $\frac{22}{7}$. Use this value to find the circumference and area of each circle.

26. The diameter is 7 feet. 27. The radius is $2\frac{1}{3}$ inches.

28. **PIZZA** The pizzeria has a special that offers one large, two medium, or three small pizzas for \$12. Which offer is the best buy? Explain your reasoning.



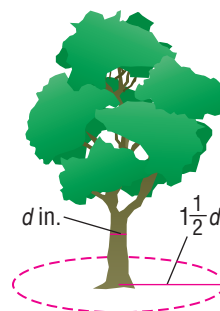
Large

Medium

Small

29. **SPORTS** Three tennis balls are packaged one on top of the other in a can. Which measure is greater, the can's height or circumference? Explain.

30. **TREES** During a construction project, barriers are placed around trees. For each inch of trunk diameter, the protection zone should have a radius of $1\frac{1}{2}$ feet. Find the area of this zone for a tree with a trunk circumference of 63 inches.



31. **GRAPHIC ARTS** Michael is painting a sign for a new coffee shop. On the sign, he drew a circle with a radius of 2 feet. He then drew another circle with a radius 1.5 times larger. How much greater is the area of the larger circle?

32. **FIND THE DATA** Refer to the California Data File on pages 16–19. Choose some data and write a real-world problem in which you would determine the area of a circle.

EXTRAPRACTICE

See pages 693, 714.

Math online

Self-Check Quiz at
ca.gr7math.com

H.O.T. Problems

33. **OPEN ENDED** Draw and label a circle that has a circumference between 10 and 20 centimeters. Justify your answer.
34. **NUMBER SENSE** If the radius of a circle is halved, how will this affect its circumference and its area? What happens to the circumference and area if the radius is doubled or tripled? Explain your reasoning. (*Hint:* Find the circumference and area for each circle and organize the data in a table.)



Real-World Link

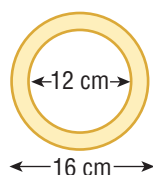
Trees should be planted so that they have plenty of room to grow. The planting site should have an area of at least 2 to 3 times the diameter of the circle the spreading roots of the maturing tree are expected to occupy.

Source: www.forestry.uga.edu

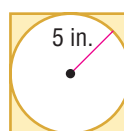


CHALLENGE Find the area of each shaded region.

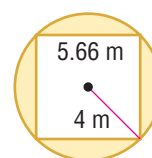
35.



36.



37.

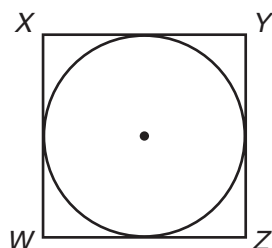


38. **WRITING IN MATH** Explain how to find the diameter of a circle if you know the measure of its area.



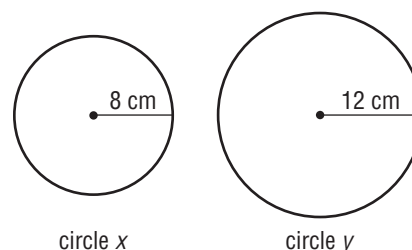
STANDARDS PRACTICE

39. In the figure below, the radius of the inscribed circle is 8 inches. What is the perimeter of square WXYZ?



- A 16π in.
- B 64 in.
- C 32 in.
- D 64π in.

40. Using the two circles shown below, what is $\frac{\text{circumference of circle } x}{\text{circumference of circle } y}$?



- F $\frac{3\pi}{4}$
- G $\frac{4\pi}{3}$
- H $\frac{3}{4}$
- J $\frac{4}{3}$

Spiral Review

GEOMETRY For Exercises 41 and 42, use $\triangle ABC$ with vertices $A(-2, -2)$, $B(-1, 2)$, and $C(1, 0)$.

- 41. **GEOMETRY** Graph $\triangle ABC$ and its image after it is translated 2 units right and 1 unit up. (Lesson 6-7)
- 42. **GEOMETRY** What are the coordinates of $\triangle A'B'C'$ when $\triangle ABC$ is reflected over the x -axis? (Lesson 6-6)
- 43. **ART** At an auction in New York City, a 2.55-square inch portrait of George Washington sold for \$1.2 million. About how much did the buyer pay per square inch for the portrait? (Lesson 4-1)

GET READY for the Next Lesson

44. **PREREQUISITE SKILL** The price of calculators has been decreasing. A calculator sold for \$125 in 1995. A similar calculator sold for \$89 in 2005. Use the *look for a pattern* strategy to determine the price of a similar calculator in 2025 if the price decrease continues at the same rate.

Extend 7-1

Geometry Lab Investigating Arcs and Angles

Main IDEA

Find measures of arcs and inscribed angles.



Standard 7MG3.1

Identify and construct basic elements of

geometric figures (e.g., altitudes, midpoints, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.

Standard 7MR2.4 Make and test conjectures using inductive and deductive reasoning.

NEW Vocabulary

central angle

arc

minor arc

major arc

semicircle

inscribed angle

In Lesson 6-1, you learned about angle relationships. Angles can also be placed in circles. A **central angle** is an angle that intersects a circle in two points and has its vertex at the center of the circle. It separates the circle into two parts, each of which is an **arc**.

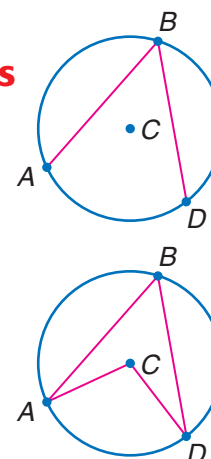
The measure of a central angle is equivalent to the measure of its corresponding arc. There are three types of arcs.

A minor arc measures less than 180° .	A major arc measures more than 180° .	A semicircle measures 180° .

An **inscribed angle** is an angle that has its vertex on the circle, and its sides contain chords of the circle.

ACTIVITY Measure of Inscribed Angles

- STEP 1** Use a compass to draw a circle and label the center C.
- STEP 2** Use a straightedge to draw chords BA and BD that do not go through the midpoint of the circle.
- STEP 3** Use a straightedge to draw $\overline{AC}$ and $\overline{CD}$.
- STEP 4** Measure $\angle ABD$ and $\angle ACD$.



ANALYZE THE RESULTS

- What seems to be the relationship between $m\angle ABD$ and $m\angle ACD$?
- Repeat Steps 1–4 with several different inscribed angles.
- MAKE A PREDICTION** Draw a circle with a radius of 3 inches. Then draw a central angle that measures 60° and an inscribed angle that intercepts the same arc. Without measuring, predict the measure of the inscribed angle. Then check your prediction by measuring.

READING Math

Arcs and Segments

The symbol $\widehat{AC}$ is read *arc AC*. The symbol $\overline{AC}$ is read *segment AC*.

ACTIVITY

Angles Inscribed in a Semicircle

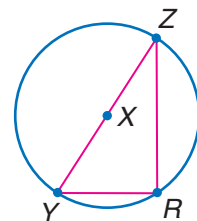
2

STEP 1

Use a compass to draw a circle with center X and diameter $\overline{YZ}$.

STEP 2

Draw and label point R on $\widehat{YZ}$. Use a straightedge to draw $\overline{RY}$ and $\overline{RZ}$.



Vocabulary Link

Inscribe

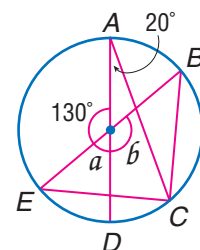
Everyday Use to write, engrave, or print characters on

Math Use to have its vertex (or vertices) on a circle and its sides contained in chords of the circle.

ANALYZE THE RESULTS

- What shape is formed by $\overline{RY}$, $\overline{RZ}$, and $\overline{YZ}$?
- Find $m\angle YRZ$. What kind of triangle is triangle YRZ ?
- Draw and label point T on $\widehat{YZ}$. Draw $\overline{TY}$ and $\overline{TZ}$. Find $m\angle YTZ$.
- MAKE A CONJECTURE** What is true about inscribed angles that intercept a semicircle?
- Find the measures of the missing angles and arcs in the figure at the right.

a. $\widehat{DB}$	b. $\angle a$	c. $\widehat{ECA}$	d. $\angle b$
e. $\angle ECB$	f. $\widehat{BA}$	g. $\widehat{DC}$	h. $\widehat{CB}$



ACTIVITY

Chords and Diameters

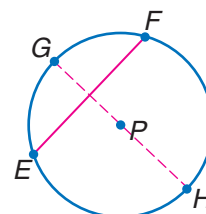
3

STEP 1

Use a compass to draw a circle and label the center P . Draw a chord that is not a diameter. Label it $\overline{EF}$.

STEP 2

Construct a line segment through P that is perpendicular to $\overline{EF}$ with endpoints on the circle. Label this as diameter $\overline{GH}$.



ANALYZE THE RESULTS

- Compare the lengths of $\widehat{EG}$ and $\widehat{FG}$. Then compare the lengths of $\widehat{EH}$ and $\widehat{FH}$.
- What is the relationship between diameter $\overline{GH}$ and chord $\overline{EF}$?
- MAKE A CONJECTURE** What is the relationship among a diameter, a chord, and its arc if the diameter is perpendicular to the chord?



7-2

Problem-Solving Investigation

MAIN IDEA: Solve a simpler problem.



Standard 7MR1.3 Determine when and how to break a problem into simpler parts. **Standard 7MR2.2** Apply strategies and results from simpler problems to more complex problems. **Standard 7AF4.2** Solve multistep problems involving rate, average speed, distance, and time or a direct variation.

P.S.I. TEAM +

e-Mail: SOLVE A SIMPLER PROBLEM

YOUR MISSION: Solve a problem by **solving a simpler problem**.

THE PROBLEM: What is the largest number of pieces that can be cut from one pizza using 5 straight cuts?

GINA: I have a circular pizza. A "cut" doesn't have to go through the center, just edge to edge.



EXPLORE	You know that a “cut” does not have to go through the center. Also, the pieces do not necessarily have to be the same size.														
PLAN	Solve a simpler problem using 1, 2, 3, and 4 cuts and then look for a pattern.														
SOLVE	 1 cut  2 cuts  3 cuts  4 cuts <table><tr><td>Cuts</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>Pieces</td><td>1</td><td>2</td><td>4</td><td>7</td><td>11</td><td>16</td></tr></table> </	Cuts	0	1	2	3	4	5	Pieces	1	2	4	7	11	16
Cuts	0	1	2	3	4	5									
Pieces	1	2	4	7	11	16									

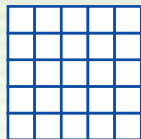
Analyze The Strategy

1. Explain why it was helpful for Gina to solve a simpler problem.
2. **WRITING IN MATH** Write about a situation in which you might need to solve a simpler problem in order to solve a more complicated problem. Then solve the problem.

Mixed Problem Solving

Use the *solve a simpler problem* strategy to solve Exercises 3–6.

3. **GEOMETRY** How many squares of any size are in the figure at the right?



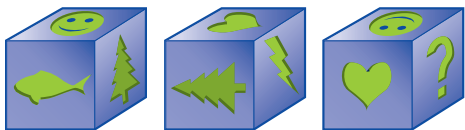
4. **TABLES** The school cafeteria has 15 square tables that can be pushed together to form one long table for class parties. Each square table can seat only one person on each side. How many people can be seated at the combined tables?
5. **PARTY SUPPLIES** Paper cups come in packages of 40 or 75. Monica needs 350 paper cups for the school party. How many packages of each size should she buy?
6. **GIFT WRAPPING** During the holidays, Tyler and Abigail earn extra money by wrapping gifts at a department store. Tyler wraps 8 packages an hour and Abigail wraps 10 packages an hour. Working together, about how long will it take them to wrap 40 packages?

Use any strategy to solve Exercises 7–11. Some strategies are shown below.

PROBLEM-SOLVING STRATEGIES

- Use the four-step plan.
- Look for a pattern.
- Use a Venn diagram.
- Solve a simpler problem.

7. **CUBES** Three different views of a cube are shown. If the fish is currently faceup, what figure is facedown?



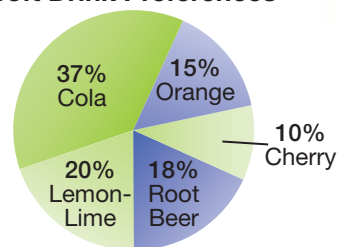
8. **NUMBER SENSE** Find the sum of all the whole numbers from 1 to 40, inclusive.

READING For Exercises 9 and 10, use the following information.

Carter Middle School has 487 fiction books and 675 nonfiction books. Of the nonfiction books, 84 are biographies.

9. Draw a Venn diagram of the situation.
10. How many books are *not* biographies?
11. **STATISTICS** The graph represents a survey of 400 students. Determine the difference in the number of students who prefer cola to lemon-lime soda.

Soft Drink Preferences



Select the Operation

For Exercises 12–14, select the appropriate operation(s) to solve the problem. Justify your selection(s) and solve the problem.

12. **TRAVEL** When Mrs. Lopez started her trip from Jackson, Mississippi, to Atlanta, Georgia, her odometer read 35,400 miles. When she reached Atlanta, her odometer read 35,782 miles. If the trip took $6\frac{1}{2}$ hours, what was her average speed?
13. **SCHOOL SUPPLIES** Ethan wishes to buy 4 pens, 1 ruler, and 8 folders at the school store. The prices are shown in the table below. If there is no tax, is \$11 enough to pay for Ethan's school supplies? Explain.
- | Item | Cost |
|--------|--------|
| Pens | \$1.75 |
| Ruler | \$1.09 |
| Folder | \$0.55 |
14. **HEALTH** A human heart beats an average of 72 times in one minute. Estimate the number of times a human heart beats in one year.

Explore 7-3

Measurement Lab Area of Irregular Figures

Main IDEA

Estimate areas of irregular figures.



Standard 7MG2.2

Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.

Standard 7MR2.2 Apply strategies and results from simpler problems to more complex problems.

ACTIVITY

Estimate the area of Idaho.

STEP 1 First, separate the figure into a triangle and a rectangle.

STEP 2 Area of triangle

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \cdot 200 \cdot 311 \\ &= 31,100 \end{aligned}$$

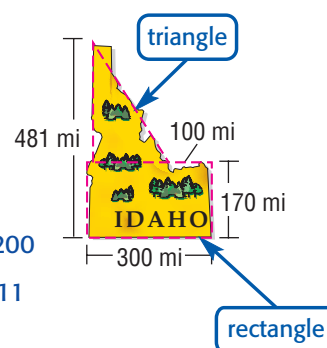
$b = 300 - 100$ or 200
 $h = 481 - 170$ or 311
Simplify.

Area of rectangle

$$\begin{aligned} A &= \ell w \\ &= 300 \cdot 170 \text{ or } 51,000 \quad \ell = 300 \text{ and } w = 170 \end{aligned}$$

The area of Idaho is about $31,100 + 51,000$ or $82,100$ square miles.

Check for Reasonableness Solve the problem another way. How does it compare to the answer in the activity?



ANALYZE THE RESULTS

- In the figure at the right, the area of California is separated into polygons. Explain how polygons can be used to estimate the total land area.
- Estimate the area of each region.
- Estimate the total area of California.
- RESEARCH** Use the Internet or another source to find the actual total land area of California. How does it compare to your answer in Exercise 3?
- RESEARCH** Estimate the area of another state. Use the Internet or another source to compare your estimate with the actual area.



7-3

Area of Complex Figures

Main IDEA

Find the area of complex figures.



Standard 7MG2.1

Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

Standard 7MG2.2 Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.

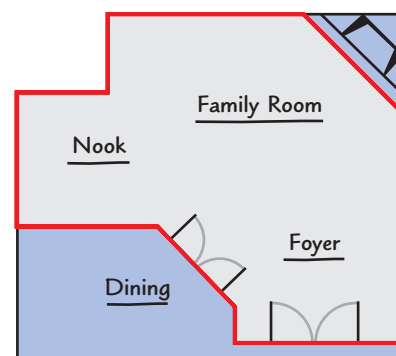
NEW Vocabulary

complex figure

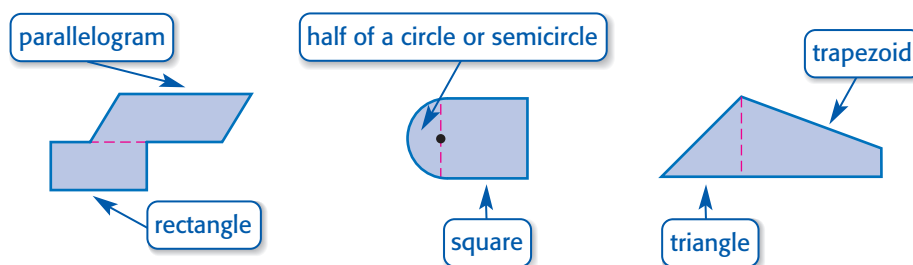
GET READY for the Lesson

CARPETING When carpeting, you must calculate the amount of floor space. Sometimes the space is made up of several shapes.

1. Identify some of the polygons that make up the family room, nook, and foyer area shown in this floor plan.
2. How can the polygons be used to find the total area that will be carpeted?



A **complex figure** is made up of two or more shapes.



To find the area of a complex figure, separate the figure into shapes whose areas you know how to find. Then find the sum of these areas.

The following is a review of area formulas.

KEY CONCEPT

Area Formulas

Shape	Words	Formula
Parallelogram	The area A of a parallelogram is the product of any base b and its height h .	$A = bh$
Triangle	The area A of a triangle is half the product of any base b and its height h .	$A = \frac{1}{2}bh$
Trapezoid	The area A of a trapezoid is half the product of the height h and the sum of the bases, b_1 and b_2 .	$A = \frac{1}{2}h(b_1 + b_2)$
Circle	The area A of a circle is equal to π times the square of the radius r .	$A = \pi r^2$

READING in the Content Area

For strategies in reading this lesson, visit ca.gr7math.com.

STUDY TIP

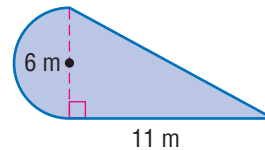
Semicircle Since a semicircle is half a circle, its area is $\frac{1}{2}\pi r^2$.

EXAMPLE

Find the Area of a Complex Figure

- 1 Find the area of the complex figure.

The figure can be separated into a semicircle and a triangle.



Area of semicircle

$$A = \frac{1}{2} \pi r^2$$

$$A \approx \frac{1}{2} \cdot 3.14 \cdot 6^2$$

$$A \approx 14.1$$

Area of triangle

$$A = \frac{1}{2}bh$$

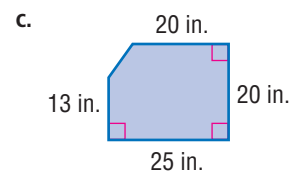
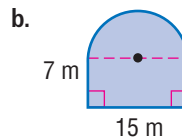
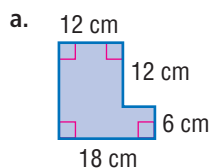
$$A = \frac{1}{2} \cdot 6 \cdot 11$$

$$A = 33$$

The area of the figure is about $14.1 + 33$ or 47.1 square meters.

CHECK Your Progress

Find the area of each figure. Round to the nearest tenth if necessary.



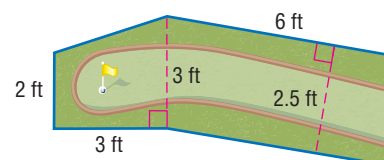
Real-World Link

There are 336 dimples on a regulation golf ball.

Source: mygolfrecord.com

Real-World EXAMPLE

- 2 **GOLF** The plan for one hole of a miniature golf course is shown. It is composed of a trapezoid and a parallelogram. How many square feet of turf will be needed to cover the putting green?



Area of trapezoid

$$A = \frac{1}{2}h(b_1 + b_2)$$

$$A = \frac{1}{2}(3)(2 + 3)$$

$$A = 7.5$$

Area of parallelogram

$$A = bh$$

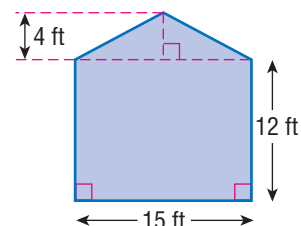
$$A = 6 \cdot 2.5$$

$$A = 15$$

So, $7.5 + 15$ or 22.5 square feet of turf will be needed.

CHECK Your Progress

- d. **SHEDS** Chloe's father is building a shed. How many square feet of wood are needed to build the back of the shed shown at the right?



Online Personal Tutor at ca.gr7math.com

STUDY TIP

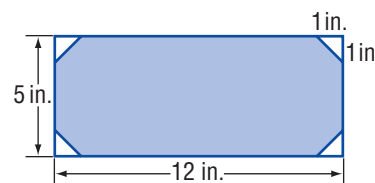
Congruent Triangles

Congruent triangles have corresponding sides and angles that are congruent.

EXAMPLE

Find the Area of a Shaded Region

- 3 In the figure at the right, four congruent triangles are cut from a rectangle. Find the area of the shaded region. Round to the nearest tenth if necessary.



Find the area of the rectangle and subtract the area of the four triangles.

Area of rectangle

$$A = \ell w$$

$$A = 12 \cdot 5 \quad \ell = 12, w = 5$$

$$A = 60 \quad \text{Simplify.}$$

Area of triangles

$$A = 4 \cdot \left(\frac{1}{2}bh\right)$$

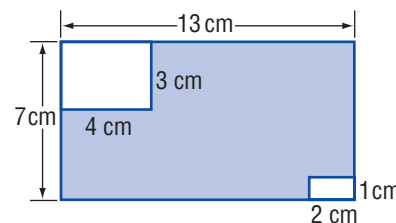
$$A = 4 \cdot \frac{1}{2} \cdot 1 \cdot 1 \quad b = 1, h = 1$$

$$A = 2 \quad \text{Simplify.}$$

The area of the shaded region is $60 - 2$ or 58 square inches.

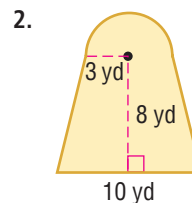
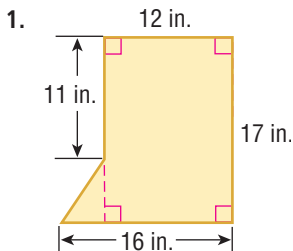
CHECK Your Progress

- e. Two rectangles are cut from a larger rectangle. Find the area of the shaded region. Round to the nearest tenth if necessary.



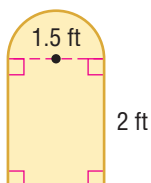
CHECK Your Understanding

Example 1 Find the area of each figure. Round to the nearest tenth if necessary.
(p. 364)

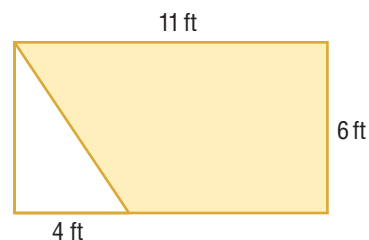


Examples 2, 3
(p. 364, 365)

3. **WINDOWS** The Lunas installed the window shown below. How many square feet is the window?



4. A triangle is cut from a rectangle. Find the area of the shaded region.

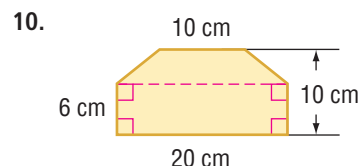
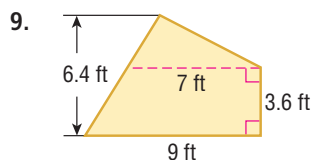
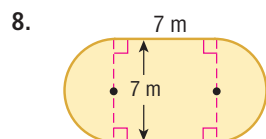
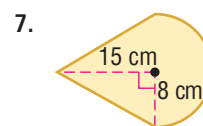
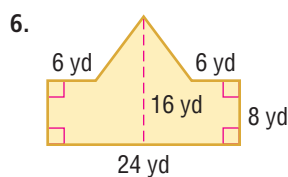
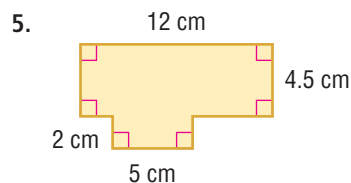


Exercises

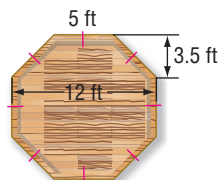
HOMESCHOOL HELP

For Exercises	See Examples
5–10	1
11, 12	2
13–16	3

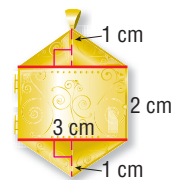
Find the area of each figure. Round to the nearest tenth if necessary.



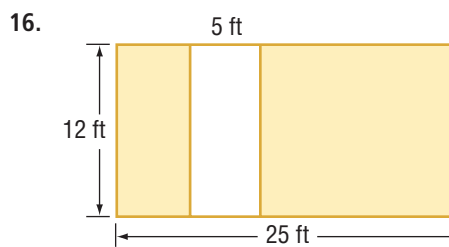
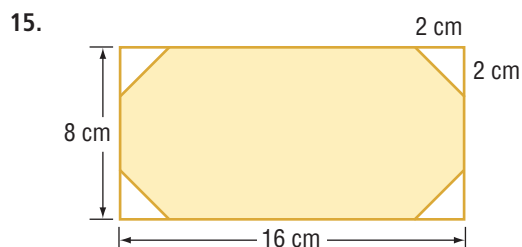
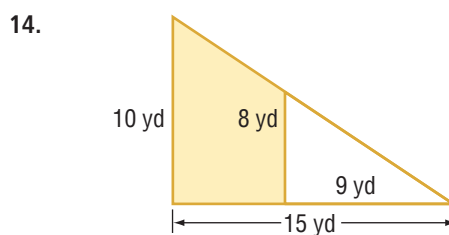
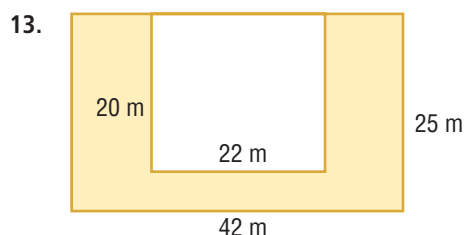
11. **CARPENTRY** Scott is constructing a deck like the one shown. What is the area of the deck?



12. **JEWELRY** A necklace comes with a gold pendant. What is the area of the pendant in square centimeters?



Find the area of the shaded region. Round to the nearest tenth if necessary.



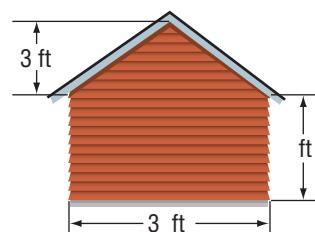
EXTRA PRACTICE

See pages 694, 714.

Math online

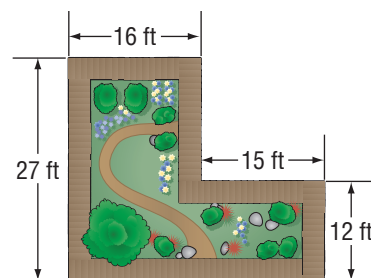
Self-Check Quiz at
ca.gr7math.com

17. **PAINTING** Suppose you are painting one side of a house. One gallon of paint covers 350 square feet and costs \$21.95. How much will it cost to buy enough paint if you apply one coat of paint?

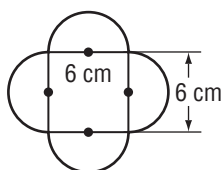


**H.O.T. Problems**

18. **CHALLENGE** In the diagram at the right, a 3-foot-wide wooden walkway surrounds a garden. What is the area of the walkway?
19. **WRITING IN MATH** Explain at least two different ways of finding the area of a hexagon. Include a drawing with your answer.

**STANDARDS PRACTICE**

20. What is the total area of the figure shown?



- A 92.5 cm^2
 B 64.3 cm^2
 C 56.5 cm^2
 D 36.0 cm^2

21. A rectangular vegetable garden that is 32 feet long and 28 feet wide is on a rectangular lot that is 310 feet long and 220 feet wide. The rest of the lot is grass. Approximately how many square feet is grass?



- F $8,688 \text{ ft}^2$ H $8,016 \text{ ft}^2$
 G $8,635 \text{ ft}^2$ J 282 ft^2

Spiral Review

22. **MODELS** Suppose you had 100 cubes. Use the *solve a simpler problem* strategy to determine the largest cube you could build with the cubes. (Lesson 7-2)
23. **MONUMENTS** Stonehenge is a circular array of giant stones in England. The diameter of Stonehenge is 30.5 meters. Find the approximate distance around Stonehenge. (Lesson 7-1)
24. **GEOMETRY** Graph rectangle $ABCD$ with vertices $A(-1, 3)$, $B(5, 3)$, $C(5, -2)$, and $D(-1, -2)$. Then graph its image after a translation 2 units right and 4 units down. (Lesson 6-7)

**GET READY for the Next Lesson**

PREREQUISITE SKILL Classify each polygon according to its number of sides.

25.



26.



27.



28.



Main IDEA

Identify and draw three-dimensional figures.



Standard 7MG3.6

Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, and the possible ways three planes might intersect).

NEW Vocabulary

coplanar
parallel
solid
polyhedron
edge
face
vertex
diagonal
skew lines
prism
base
pyramid

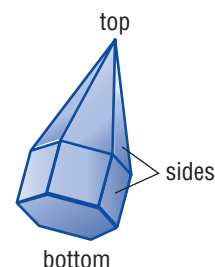
GET READY for the Lesson

CRYSTALS A two-dimensional figure has two dimensions, length and width. A three-dimensional figure, like the Amethyst crystal shown at the right, has three dimensions, length, width, and depth (or height).

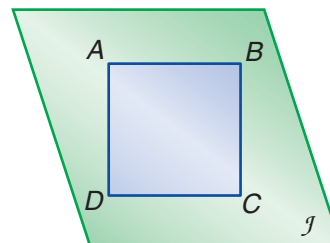
1. Name the two-dimensional shapes that make up the sides of this crystal.
2. If you observed the crystal from directly above, what two-dimensional figure would you see?
3. How are two- and three-dimensional figures related?



Amethyst

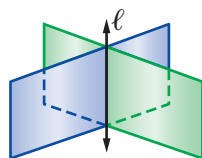


The figure at the right shows rectangle $ABCD$. Lines AB and DC are **coplanar** because they lie in the same plane. They are also **parallel** because they will never intersect, no matter how far they are extended.

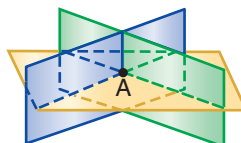


Just as two lines in a plane can intersect or be parallel, there are different ways that planes may be related in space.

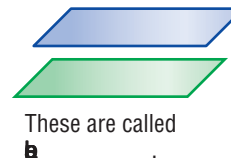
Intersect in a Line



Intersect at a Point



No Intersection



Intersecting planes can also form three-dimensional figures or **solids**. A **polyhedron** is a solid with flat surfaces that are polygons. Some terms associated with three-dimensional figures are edge, face, vertex, and diagonal.



Vocabulary Link

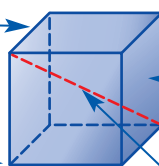
Intersection

Everyday Use the place where two roads cross

Math Use any point or points that figures have in common.

An **edge** is where two planes intersect in a line.

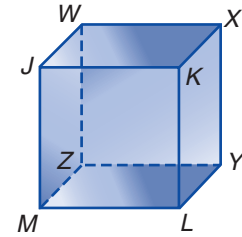
A **vertex** is where three or more planes intersect at a point.



A **face** is a flat surface.

A **diagonal** is a line segment whose endpoints are vertices that are neither adjacent nor on the same face.

Notice that in the figure at the right, $\overline{WX}$ and $\overline{KL}$ do not intersect. These segments are not parallel because they do not lie in the same plane. Lines that do not intersect and are not coplanar are called **skew lines**.



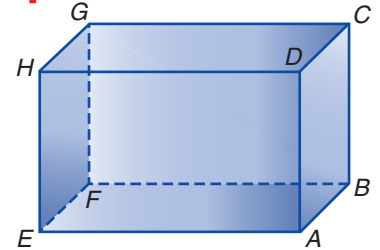
EXAMPLES

Identify Relationships

- 1 Name a plane that is parallel to plane ABC .
Plane EFG is parallel to plane ABC .

- 2 Identify a segment that is skew to $\overline{CG}$.
 $\overline{CG}$ and $\overline{EH}$ are skew.

- 3 Identify a set of points between which a diagonal can be drawn.
A segment between points A and G forms a diagonal.



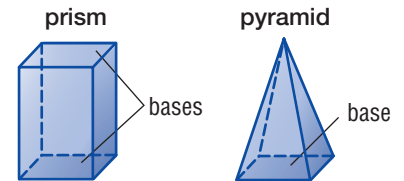
CHECK Your Progress

- a. Identify the intersection of planes ABC and CDH .

Prisms and pyramids are common solids. They are named by the shape of their bases.

A **prism** is a polyhedron with two parallel, congruent faces called **bases**.

A **pyramid** is a polyhedron with one base that is a polygon and faces that are triangles.



STUDY TIP

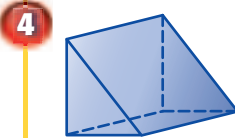
Common Error

In the drawing of a rectangular prism, the bases do not have to be on the top and bottom. Any two parallel rectangles are bases. In a triangular pyramid, any face is a base.

EXAMPLES

Identify Prisms and Pyramids

Identify each solid. Name the number and shapes of the faces. Then name the number of edges and vertices.



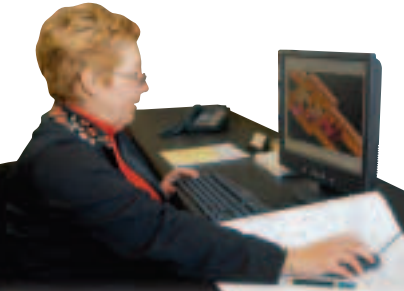
The figure has two parallel congruent bases that are triangles, so it is a triangular prism. The other three faces are rectangles. It has a total of 5 faces, 9 edges, and 6 vertices.



The figure has one base that is a pentagon, so it is a pentagonal pyramid. The other faces are triangles. It has a total of 6 faces, 10 edges, and 6 vertices.

CHECK Your Progress



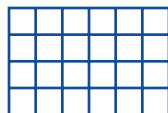


Real-World Link
Architects use computer-aided design and drafting technology to produce their drawings.

You can use three-dimensional drawings of objects to describe how different parts of the objects are related in space.

EXAMPLE Analyze Drawings

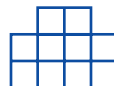
- 6 ARCHITECTURE** The drawing shows the plans for a new office building. Draw and label the top, front, and side views.



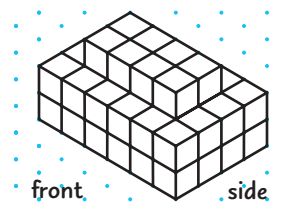
top view



front view

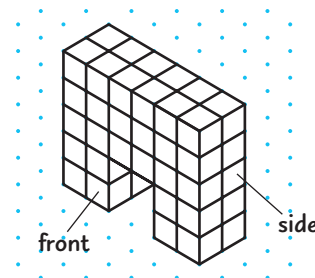


side view



CHECK Your Progress

- e. **CABINETS** Julian's brother drew plans for a cabinet as shown. Draw and label the top, front, and side views.



Personal Tutor at ca.gr7math.com

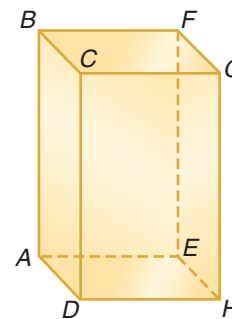


CHECK Your Understanding

Examples 1–3
(p. 369)

Use the figure at the right to identify the following points, lines, or planes.

- parallel planes
- skew lines
- two points that form a diagonal when connected
- intersecting planes



Examples 4, 5
(p. 369)

Identify each solid. Name the number and shapes of the faces. Then name the number of edges and vertices.

5.



6.



7.



Example 6
(p. 370)

8. **PETS** A pet lizard lives in an aquarium with a height of 2 units and a rectangular base 3 units long and 2 units wide. Draw and label the top, front, and side views.



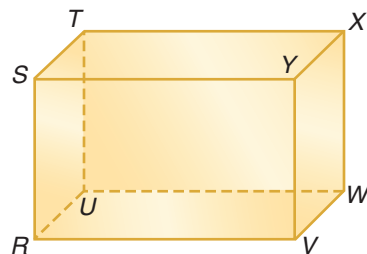
Exercises

HOMEWORK HELP

For Exercises	See Examples
9–12	1–3
13–16	4, 5
17–18	6

Use the figure at the right to identify the following points, lines, or planes.

9. parallel planes
10. skew lines
11. two points that form a diagonal when connected.
12. intersecting planes



Identify each solid. Name the number and shapes of the faces. Then name the number of edges and vertices.

13.



14.



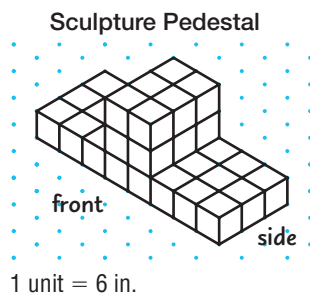
15.



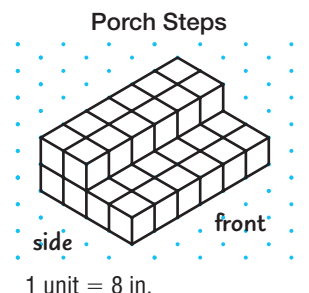
16.



17. **PEDESTALS** The plans for a sculpture pedestal are shown. Draw and label the top, front, and side views.



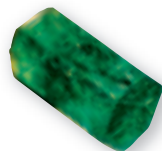
18. **STEPS** The Taros are planning to install the porch steps shown. Draw and label the top, front, and side views.



CRYSTALS For Exercises 19–21, complete parts a and b for each crystal.

- a. Identify the solid or solids that form the crystal.
- b. Draw and label the top and one side view of the crystal.

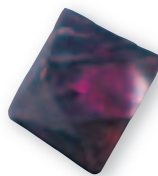
19.



Emerald



20.



Fluorite



21.



Quartz



22. State whether the following conjecture is *true* or *false*. If *false*, provide a counterexample.

Two planes in three-dimensional space can intersect at one point.

EXTRA PRACTICE

See pages 694, 714.

Math  online

Self-Check Quiz at
ca.gr7math.com



H.O.T. Problems

23. **OPEN ENDED** Choose a real-world object such as a chair or a desk. Draw the top, front, and side views of your object.

CHALLENGE Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

24. A prism has two congruent bases.

25. A pyramid has five vertices.

26. **WRITING IN MATH** Explain whether a top-front-side view diagram always provides enough information to draw a figure. If not, provide a counterexample.

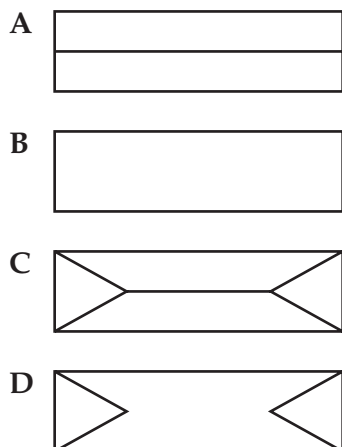


STANDARDS PRACTICE

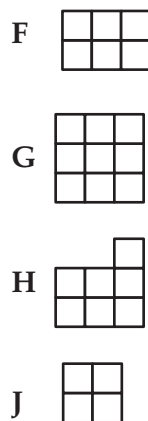
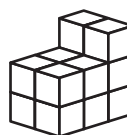
27. Benita received the gift box shown.



Which drawing best represents the top view of the gift box?



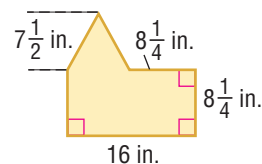
28. Which of the following represents a side view of the figure below?



Spiral Review

29. Find the area of the figure. Round to the nearest tenth. (Lesson 7-3)

30. **MANUFACTURING** The label that goes around a jar of peanut butter overlaps itself by $\frac{3}{8}$ inch. If the diameter of the jar is 2 inches, what is the length of the label? (Lesson 7-1)



GET READY for the Next Lesson

PREREQUISITE SKILL Find the area of each triangle described.

31. base, 3 in.; height, 10 in. 32. base, 8 ft; height, 7 ft 33. base, 5 cm; height, 11 cm

7-5

Volume of Prisms and Cylinders

Main IDEA

Find the volumes of prisms and cylinders.



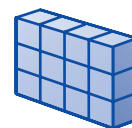
Standard 7MG2.1

Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

Standard 7MG2.2 Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.

MINI Lab

The rectangular prism at the right has a volume of 12 cubic units.



STEP 1

Model three other rectangular prisms with a volume of 12 cubic units.

STEP 2

Copy and complete the following table.

Prism	Length (units)	Width (units)	Height (units)	Area of Base (units ²)
A	4	1	3	4
B				
C				
D				

- Describe how the volume V of each prism is related to its length ℓ , width w , and height h .
- Describe how the area of the base B and the height h of each prism is related to its volume V .

NEW Vocabulary

volume
cylinder
complex solid

Volume is the measure of the space occupied by a solid. Standard measures of volume are cubic units such as cubic inches (in³) or cubic feet (ft³).

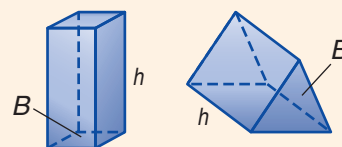
KEY CONCEPT

Volume of a Prism

Words

The volume V of a prism is the area of the base B times the height h .

Models



Symbols

$$V = Bh$$

STUDY TIP

Volume The formula for the volume of a rectangular prism is often written as $V = \ell wh$ since the area of base B of a rectangular prism is always equal to ℓw .

EXAMPLES

Find the Volumes of Prisms

- 1** Find the volume of the rectangular prism.

$$V = Bh$$

Volume of a prism

$$V = (\ell \cdot w)h$$

The base is a rectangle, so $B = \ell \cdot w$.

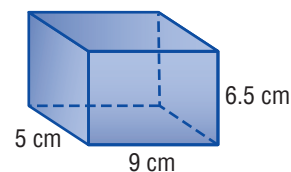
$$V = (9 \cdot 5)6.5$$

$$\ell = 9, w = 5, h = 6.5$$

$$V = 292.5$$

Simplify.

The volume is 292.5 cubic centimeters.



STUDY TIP

Common Error

Remember that the bases of a triangular prism are triangles. In Example 2, these bases are not on the top and bottom of the figure, but on its sides.

2 Find the volume of the triangular prism.

$$V = Bh$$

Volume of a prism

$$V = \left(\frac{1}{2} \cdot 6 \cdot 7\right)h$$

The base is a triangle, so $B = \frac{1}{2} \cdot 6 \cdot 7$.

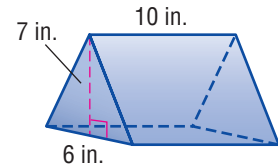
$$V = \left(\frac{1}{2} \cdot 6 \cdot 7\right)10$$

The height of the prism is 10.

$$V = 210$$

Simplify.

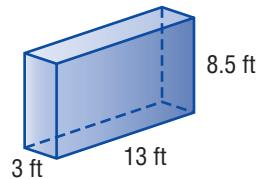
The volume is 210 cubic inches.



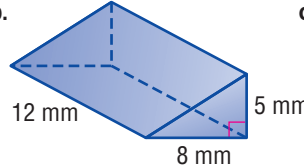
CHECK Your Progress

Find the volume of each prism.

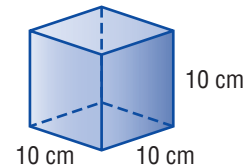
a.



b.



c.



A **cylinder** is a solid with bases that are congruent, parallel circles connected with a curved side. You can use the formula $V = Bh$ to find the volume of a cylinder, where the base is a circle.

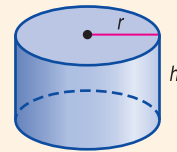
KEY CONCEPT

Volume of a Cylinder

Words

The volume V of a cylinder with radius r is the area of the base B times the height h .

Model



Symbols

$V = Bh$, where $B = \pi r^2$ or $V = \pi r^2 h$

STUDY TIP

Estimation

You can estimate the volume of the cylinder in Example 3 to be about $3 \cdot 7^2 \cdot 20$ or $2,940 \text{ ft}^3$ to check the reasonableness of your result.

EXAMPLE Find the Volume of a Cylinder

3 Find the volume of the cylinder. Round to the nearest tenth.

Since the diameter is 13 feet, the radius is 6.5 feet.

$$V = \pi r^2 h$$

Volume of a cylinder

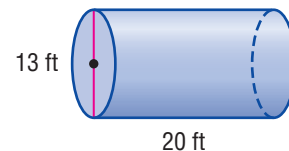
$$V \approx 3.14 \cdot 6.5^2 \cdot 20$$

Replace π with 3.14, r with 6.5, and h with 20.

$$V \approx 2,653.3$$

Simplify.

The volume is about 2,653.3 cubic feet.



CHECK Your Progress

Find the volume of each cylinder. Round to the nearest tenth.

d. radius, 2 in.; height 7 in.

e. diameter, 18 cm; height 5 cm



Objects that are made up of more than one type of solid are called **complex solids**. To find the volume of a complex solid, separate the figure into solids whose volumes you know how to find.

STUDY TIP

Estimation

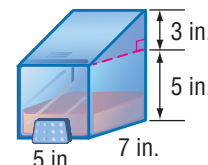
You can check the reasonableness of your result in Example 4 by estimating the volume. The volume should be between $5 \cdot 7 \cdot 5$ or 175 in^3 and $5 \cdot 7 \cdot 8$ or 280 in^3 .

EXAMPLE

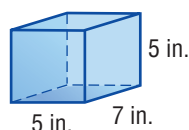
Find the Volume of a Complex Solid

- 4 DISPENSERS** Find the volume of the soap dispenser at the right.

The dispenser is made of one rectangular prism and one triangular prism. Find the volume of each prism.



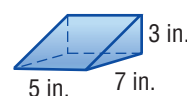
Rectangular Prism



$$V = Bh$$

$$V = (5 \cdot 7)5 \text{ or } 175$$

Triangular Prism



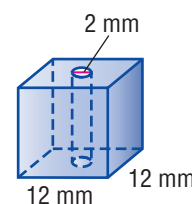
$$V = Bh$$

$$V = \left(\frac{1}{2} \cdot 7 \cdot 3\right)5 \text{ or } 52.5$$

The volume of the dispenser is $175 + 52.5$ or 227.5 cubic inches.

CHECK Your Progress

- f. **CRAFTS** Tanya uses cube beads similar to the one shown to make jewelry. Each bead has a circular hole through the middle. Find the volume of the bead.

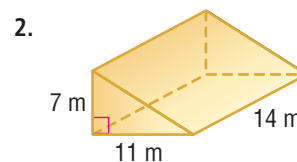
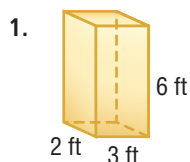


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CHECK Your Understanding

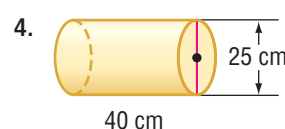
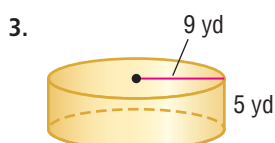
Examples 1, 2
(pp. 373–374)

Find the volume of each prism. Round to the nearest tenth if necessary.



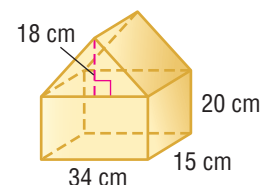
Example 3
(p. 374)

Find the volume of each cylinder. Round to the nearest tenth.



Example 4
(p. 375)

5. **TOYS** Gloria's younger sister received the toy house shown as a gift. What is the volume of the toy house?

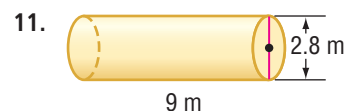
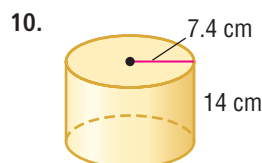
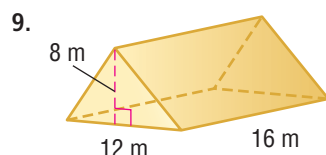
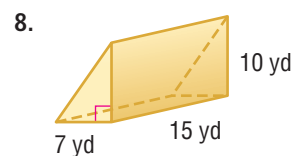
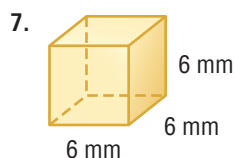
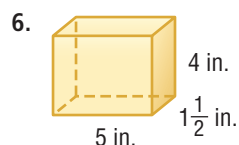


Exercises

HOMEWORK HELP

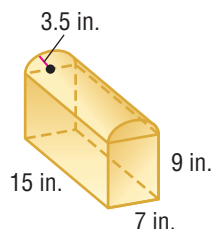
For Exercises	See Examples
6–9, 12, 13	1, 2
10, 11, 14, 15	3
16, 17	4

Find the volume of each solid. Round to the nearest tenth if necessary.

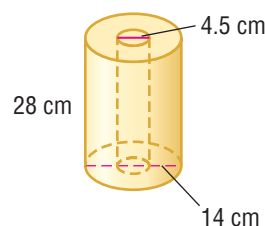


12. rectangular prism: length, 4 in.; width, 6 in.; height, 17 in.
 13. triangular prism: base of triangle, 5 ft; altitude, 14 ft; height of prism, $8\frac{1}{2}$ ft
 14. cylinder: radius, 25 m; height, 20 m
 15. cylinder: diameter, 7.2 cm; height, 5.8 cm

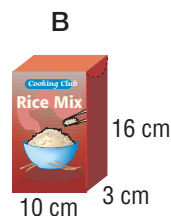
16. **MAILBOXES** The Francos have the mailbox shown below. Find the volume of the mailbox.



17. **TOWELS** An unused roll of paper towels has the dimensions shown. What is the volume of the unused roll?



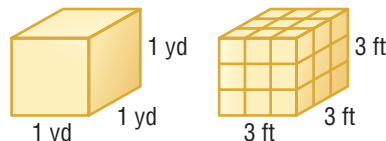
18. Find the height of a rectangular prism with a length of 6.8 meters, a width of 1.5 meters, and a volume of 91.8 cubic meters.
 19. Find the height of a cylinder with a radius of 4 inches and a volume of 301.6 cubic inches.
 20. **MEASUREMENT** A bar of soap in the shape of a rectangular prism has a volume of 16 cubic inches. After several uses, it measures $2\frac{1}{4}$ inches by 2 inches by $1\frac{1}{2}$ inches. How much soap was used?
 21. **PACKAGING** The Cooking Club is selling their own special blends of rice mixes. They can choose from the two containers below to package their product. Which container will hold more rice? Explain your reasoning.





22. **POOLS** A wading pool is to be 20 feet long, 11 feet wide, and 1.5 feet deep. The excavated dirt is to be hauled away by wheelbarrow. If the wheelbarrow holds 9 cubic feet of dirt, how many wheelbarrows of dirt must be hauled away from the site?

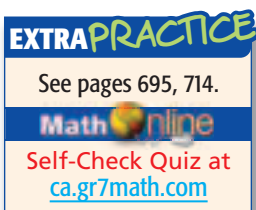
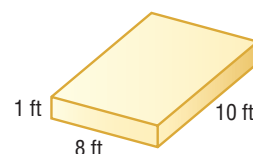
CONVERTING UNITS OF MEASURE For Exercises 23–25, use the cubes and the information below.



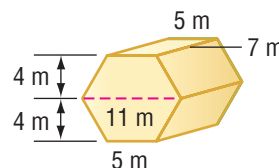
The volume of the left cube is 1 cubic yard. The right cube is the same size, but the unit of measure has been changed. So, 1 cubic yard = $(3)(3)(3)$ or 27 cubic feet. Use a similar process to convert each measurement.

23. $1 \text{ ft}^3 = \square \text{ in}^3$ 24. $1 \text{ cm}^3 = \square \text{ mm}^3$ 25. $1 \text{ m}^3 = \square \text{ cm}^3$

26. **GARDENING** Candice is making a vegetable garden with the dimensions shown. Each bag of planting soil she plans to use fills 0.5 cubic yard. How many bags of soil will she need to buy to fill her garden?



27. **GEOMETRY** Explain how you would find the volume of the hexagonal prism shown at the right. Then find its volume.



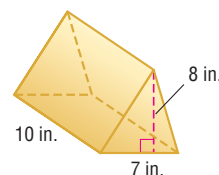
H.O.T. Problems

CHALLENGE For Exercises 28–31, describe how the volume of each solid is affected after the indicated change in its dimension(s).

28. You double one dimension of a rectangular prism.
29. You double two dimensions of a rectangular prism.
30. You double all three dimensions of a rectangular prism.
31. You double the radius of a cylinder.

32. **OPEN ENDED** Find the volume of a can or other cylindrical object, making sure to include appropriate units. Explain your method.

33. **FIND THE ERROR** Erin and Dulce are finding the volume of the prism shown at the right. Who is correct? Explain your reasoning.



$$\begin{aligned} A &= Bh \\ A &= (10 \cdot 7) \cdot 8 \\ A &= 560 \text{ in}^3 \end{aligned}$$

Erin

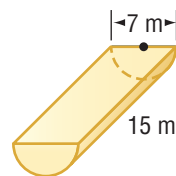
$$\begin{aligned} A &= Bh \\ A &= \left(\frac{1}{2} \cdot 7 \cdot 8\right) \cdot 10 \\ A &= 280 \text{ in}^3 \end{aligned}$$



Dulce



34. **SELECT A TOOL** Tyree needs to find the volume of the figure at the right. Which of the following tools might Tyree use to find the volume of the figure? Justify your selection(s). Then, use the tool(s) to solve the problem.



make a model

calculator

paper/pencil

35. **WRITING IN MATH** Write two formulas that you can use to find the volume of a rectangular prism. State the formula that you prefer to use and explain why.

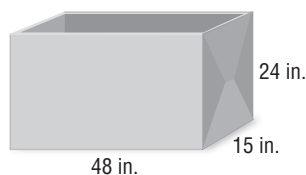


STANDARDS PRACTICE

36. A cylinder is 30 inches tall and has a diameter of 12 inches. Which is the closest to the volume of the cylinder in cubic feet?

A 1 ft^3
B 2 ft^3
C 3 ft^3
D 4 ft^3

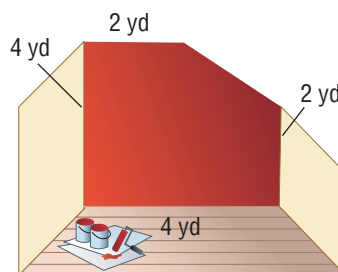
37. A cardboard box has the dimensions shown below. Which is the closest to the volume of the box in cubic feet?



F 8 ft^3 H 15.5 ft^3
G 10 ft^3 J 17 ft^3

Spiral Review

38. How many edges does an octagonal pyramid have? (Lesson 7-4)
39. **PAINTING** You are painting a wall of this room red. Find the area of the red wall to the nearest square foot. (Lesson 7-3)
40. **MEASUREMENT** The circumference of a circle is 16.5 feet. What is its area to the nearest tenth of a square foot? (Lesson 7-1)
41. **WOOL** Texas ranchers produce about 20% of U.S. wool. If 27.5 million pounds of wool are produced each year, how many pounds of wool are not produced in Texas? (Lesson 5-7)



Write each percent as a fraction or mixed number in simplest form. (Lesson 5-1)

42. 0.12% 43. 225% 44. 135% 45. $\frac{3}{8}\%$

GET READY for the Next Lesson

PREREQUISITE SKILL Multiply.

46. $\frac{1}{3} \cdot 6 \cdot 10$ 47. $\frac{1}{3} \cdot 7 \cdot 15$ 48. $\frac{1}{3} \cdot 4^2 \cdot 9$ 49. $\frac{1}{3} \cdot 6^2 \cdot 20$

Mid-Chapter Quiz

Lessons 7-1 through 7-5

Find the circumference and area of each circle. Round to the nearest tenth. (Lesson 7-1)

1.



2.



MEASUREMENT For Exercises 3 and 4, use the following information. Round to the nearest tenth. (Lesson 7-1)

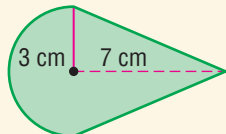
A shot-putter must stay inside the circle shown.



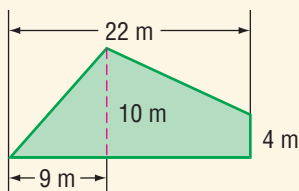
3. What is the area of the region in which the athlete is able to move?
4. What is the circumference of the circular region?
5. **DANCE** Balloons come in packages of 15 or 35. Julie needs 195 balloons for the spring dance. How many packages of each size should she buy? Use the *solve a simpler problem* strategy. (Lesson 7-2)

Find the area of each figure. Round to the nearest tenth if necessary. (Lesson 7-3)

6.



7.



8. **GEOMETRY** Draw and label the top view, a side view, and the front view of the figure. (Lesson 7-4)



9.

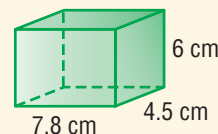


STANDARDS PRACTICE Juanita wants to sketch all of the faces of a triangular prism. What shapes will appear on her paper? (Lesson 7-4)

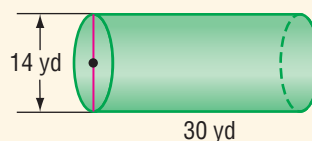
- A 2 squares, 2 triangles
- B 2 triangles, 3 rectangles
- C 3 triangles
- D 1 triangle, 3 rectangles

Find the volume of each solid. Round to the nearest tenth if necessary. (Lesson 7-5)

10.



11.



12.



STANDARDS PRACTICE Find the volume of a cube-shaped box with edges 15 inches in length. (Lesson 7-5)

- | | |
|----------------------|------------------------|
| F 225 in^3 | H $1,350 \text{ in}^3$ |
| G 900 in^3 | J $3,375 \text{ in}^3$ |

13. Find the width of a rectangular prism with a length of 7.6 meters, a height of 8 meters, and a volume of 88.4 cubic meters. Round to the nearest tenth. (Lesson 7-5)

7-6

Volume of Pyramids and Cones

Main IDEA

Find the volumes of pyramids and cones.



Standard 7MG2.1

Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

NEW Vocabulary

cone

MINI Lab

Concepts in Motion

Animation ca.gr7math.com

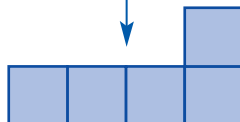
In this Mini Lab, you will investigate the relationship between the volume of a pyramid and the volume of a prism with the same base area and height.

Draw and cut out 5 squares.

2 in.



Tape together as shown.

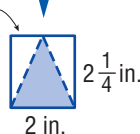


Fold and tape to form a cube with an open top.

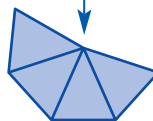


Draw and cut out 4 isosceles triangles.

1 in.



Tape together as shown.



Fold and tape to form an open square pyramid.



1. Compare the base areas and the heights of the two solids.
2. Fill the pyramid with rice, sliding a ruler across the top to level the amount. Pour the rice into the cube. Repeat until the prism is filled. How many times did you fill the pyramid in order to fill the cube?
3. What fraction of the cube's volume does one pyramid fill?

The volume of a pyramid is one-third the volume of a prism with the same base area and height.

KEY CONCEPT

Volume of a Pyramid

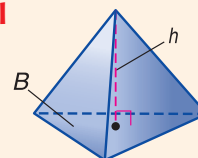
Words

The volume V of a pyramid is one-third the area of the base B times the height h .

Symbols

$$V = \frac{1}{3}Bh$$

Model



The height of a pyramid or cone is the distance from the vertex, perpendicular to the base.

STUDY TIP

Estimation You can estimate the volume of the pyramid in Example 1 to be about $\frac{1}{3}(\frac{1}{2} \cdot 8 \cdot 6)(11)$ or 88 m^3 . Since 95.04 m^3 is close to 88 m^3 , the answer is reasonable.

EXAMPLE

Find the Volume of a Pyramid

- 1 Find the volume of the pyramid. Round to the nearest tenth.

$$V = \frac{1}{3}Bh$$

Volume of a pyramid

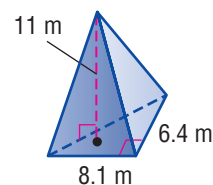
$$V = \frac{1}{3}\left(\frac{1}{2} \cdot 8.1 \cdot 6.4\right)11$$

$$B = \frac{1}{2} \cdot 8.1 \cdot 6.4, h = 11$$

$$V = 95.04$$

Simplify.

The volume is about 95.0 cubic meters.



CHECK Your Progress

- a. Find the volume of a pyramid that has a height of 5 yards and a square base with sides 2 yards long.



Real-World EXAMPLE

- 2 **ARCHITECTURE** The volume of the Pyramid Arena in Memphis, Tennessee, is about 38,520,000 cubic feet. If the height of the pyramid is 321 feet, find the area of the rectangular base.

$$V = \frac{1}{3}Bh$$

Volume of a pyramid

$$38,520,000 = \frac{1}{3} \cdot B \cdot 321$$

Replace V with 38,520,000 and h with 321.

$$38,520,000 = \frac{321}{3}B$$

Multiply.

$$\frac{3}{321} \cdot 38,520,000 = \frac{3}{321} \cdot \frac{321}{3}B$$

Multiply each side by $\frac{3}{321}$.

$$360,000 \approx B$$

Simplify.

The base of the pyramid is about 360,000 square feet.

CHECK Your Progress

- b. **CRAFTS** Nicco made a pyramid-shaped candle. The volume of the candle is 864 cubic centimeters and its base has an area of 144 square centimeters. How high is the candle?



Personal Tutor at ca.gr7math.com



Real-World Link

The Pyramid Arena's structure is appropriate as the city of Memphis gets its name from an Egyptian city, known for its ancient pyramids.

Source: pyramidarena.org

A **cone** is a three-dimensional figure with one circular base. A curved surface connects the base and the vertex. The volumes of a cone and a cylinder are related in the same way as those of a pyramid and prism.

KEY CONCEPT

Volume of a Cone

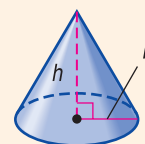
Words

The volume V of a cone with radius r is one-third the area of the base B times the height h .

Symbols

$$V = \frac{1}{3}Bh \text{ or } V = \frac{1}{3}\pi r^2h$$

Model



EXAMPLE Find the Volume of a Cone

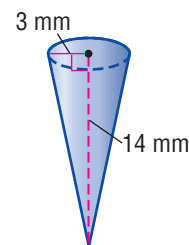
3 Find the volume of the cone.

$$V = \frac{1}{3}\pi r^2 h \quad \text{Volume of a cone}$$

$$V \approx \frac{1}{3} \cdot 3.14 \cdot 3^2 \cdot 14 \quad \text{Replace } \pi \text{ with } 3.14, r \text{ with } 3, \text{ and } h \text{ with } 14.$$

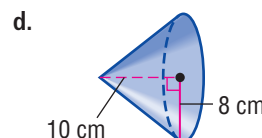
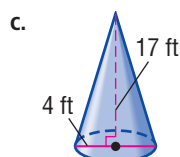
$$V \approx 131.9 \quad \text{Simplify.}$$

The volume is about 131.9 cubic millimeters.



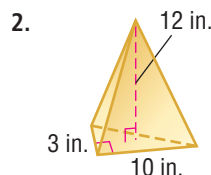
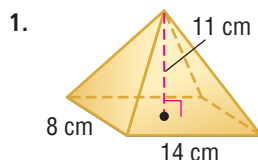
CHECK Your Progress

Find the volume of each cone. Round to the nearest tenth.



CHECK Your Understanding

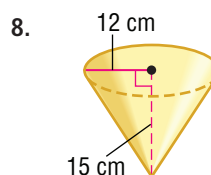
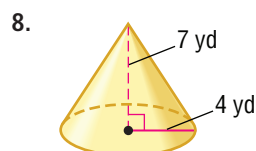
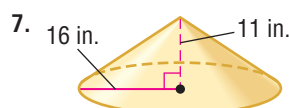
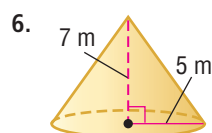
Example 1 Find the volume of each pyramid. Round to the nearest tenth.
(p. 381)



3. Find the volume of a pyramid that has a height of 125 centimeters and a square base with sides 95 centimeters long.
4. Find the volume of a pyramid that has a height of 17 feet and a square base with sides 22 feet long.

Example 2 5. **ARCHITECTURE** The Louvre Pyramid in Paris has a square base with sides 112 feet long. If the volume is 296,875 cubic feet, find the height of the pyramid.
(p. 381)

Example 3 Find the volume of each cone. Round to the nearest tenth.
(p. 382)

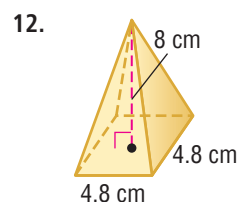
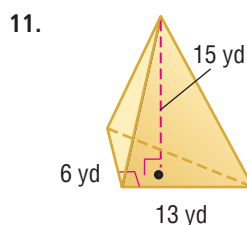
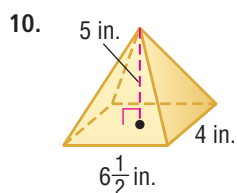


Exercises

HOMEWORK HELP

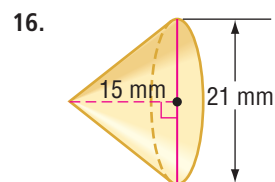
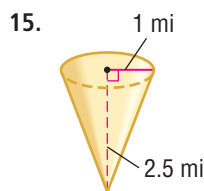
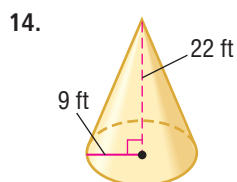
For Exercises	See Examples
10–13	1
18	2
14–17	3

Find the volume of each pyramid. Round to the nearest tenth.



13. triangular pyramid: triangle base, 10 cm; triangle height, 7 cm; pyramid height, 15 cm

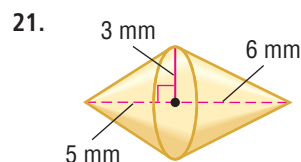
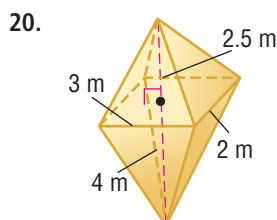
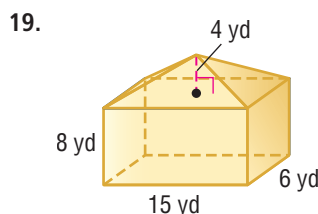
Find the volume of each cone. Round to the nearest tenth.



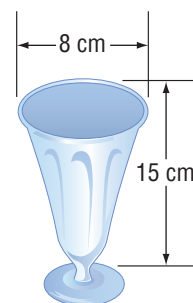
17. cone: diameter, 12 m; height, 5 m

18. **VOLCANO** A model of a volcano constructed for a science project is cone-shaped with a diameter of 10 inches. If the volume of the model is about 287 cubic inches, how tall is the model?

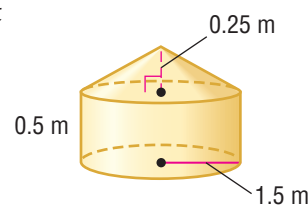
Find the volume of each solid. Round to the nearest tenth if necessary.



22. **FROZEN CUSTARD** You are filling cone-shaped glasses with frozen custard. Each glass has the dimensions shown. One gallon of custard is equivalent to 4,000 cubic centimeters. About how many glasses can you completely fill using one gallon of custard?



23. **IRRIGATION** A water tank like the one at the right is used to water flowers at a park. Water can be pumped from the tank at a rate of 25 liters per minute. How long will it take to use all of the water in a full tank? Round to the nearest minute. (Hint: 1 liter = 1,000 cm³)



EXTRAPRACTICE

See pages 695, 714.

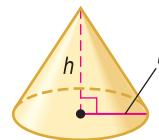
Math  Online

Self-Check Quiz at
ca.gr7math.com



H.O.T. Problems

24. **CHALLENGE** How could you change the height of a cone so that its volume would remain the same when its radius was tripled?
25. **OPEN ENDED** Draw and label a rectangular pyramid with a volume of 48 cubic centimeters.
26. **NUMBER SENSE** Which would have a greater effect on the volume of a cone, doubling its radius or doubling its height? Explain your reasoning.
27. **WRITING IN MATH** Write about a real-world situation that can be solved by finding the volume of a cone.

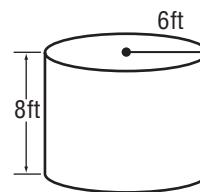


STANDARDS PRACTICE

28. A rectangular pyramid has a base 18 inches by 30 inches and a height of 36 inches. Which is closest to the volume of the pyramid in cubic feet?

A 2.5 ft^3
 B 3 ft^3
 C 4 ft^3
 D 5.5 ft^3

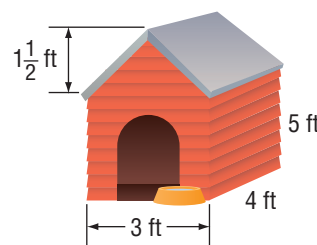
29. Find the volume of the cylinder.
 Round to the nearest tenth if necessary.



F 48 ft^3 H 288 ft^3
 G 150.3 ft^3 J 904.3 ft^3

Spiral Review

30. **MEASUREMENT** Find the volume of the doghouse at the right. (Lesson 7-5)
31. Name the number and shapes of the faces of a trapezoidal prism. Then name the number of edges and vertices. (Lesson 7-4)
32. **GEOMETRY** Graph triangle ABC with vertices $A(1, 2)$, $B(4, -1)$, and $C(2, -4)$. Then graph its image after a reflection over the y -axis, and write the coordinates of the image's vertices. (Lesson 6-5)
33. **SHOPPING** Etu saved \$90 when he purchased a DVD recorder on sale. If the sale price was 37.5% off the regular price, what was the regular price of the DVD recorder? (Lesson 5-4)



GET READY for the Next Lesson

PREREQUISITE SKILL Find the circumference of each circle. Round to the nearest tenth. (Lesson 7-1)

34. diameter, 9 in. 35. diameter, 5.5 ft 36. radius, 2 m 37. radius, 3.8 cm

Explore 7-7

Measurement Lab Surface Area of Cylinders

Main IDEA

Find the surface area of cylinders using models and nets.



Standard 7MG3.5 Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.

Standard 7MR2.5 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

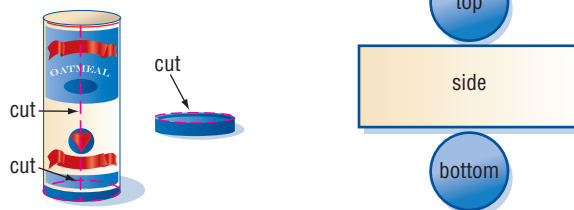
NEW Vocabulary

net

Nets are two-dimensional patterns of three-dimensional figures. You can use a net to find the area of each surface of a three-dimensional figure such as a cylinder.

ACTIVITY

- STEP 1** Use an empty cylinder-shaped container that has a lid. Measure and record the height of the container.
- STEP 2** Then label the lid and bottom face using a blue marker. Label the curved side using a red marker.
- STEP 3** Take off the lid of the container and make 2 cuts as shown. Next, cut off the sides of the lid. Finally, lay the lid, the curved side, and the bottom flat to form the net of the container.



ANALYZE THE RESULTS

1. Classify the two-dimensional shapes that make up the net of the container.
2. Find the area of each shape. Then find the sum of these areas.
3. Find the diameter of the top of the container and use it to find the perimeter or circumference of that face.
4. Multiply the circumference by the height of the container. What does this product represent?
5. Add the product from Exercise 4 to the sum of the areas of the two circular bases.
6. Compare your answers from Exercises 2 and 5.
7. **MAKE A CONJECTURE** Write a method for finding the area of all the surfaces of a cylinder given the measures of its height and the diameter of one of its bases.

7-7

Surface Area of Prisms and Cylinders

Main IDEA

Find the lateral and total surface area of prisms and cylinders.



Standard 7MG2.1

Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

Standard 7MG3.5 Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.

NEW Vocabulary

lateral face
lateral surface area
total surface area



Vocabulary Link

Lateral

Everyday Use situated on, directed toward, or coming from the side

Lateral face

Math Use a face of a solid that is not a base

MINI Lab

STEP 1

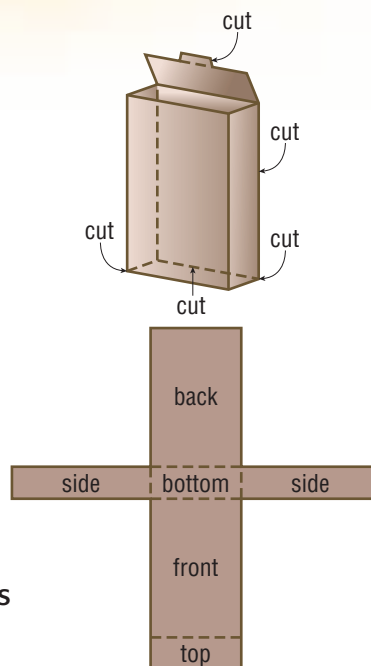
Use an empty box with a tuck-in lid. Measure and record the height of the box and the perimeter of the top or bottom face.

STEP 2

Label the top, bottom, front, back, and side faces using a marker.

STEP 3

Open the lid and make 5 cuts as shown. Then open the box and lay it flat to form a net of the box. Measure and record the dimensions of each face.



1. Find the area of each face. Then find the sum of these areas.
2. Multiply the perimeter of a base by the height of the box. What does this product represent?
3. Add the product from Exercise 2 to the sum of the areas of the two bases.
4. Compare your answers from Exercises 1 and 3.

In the Mini Lab, you found the area of each surface, or face, of a box. A **lateral face** of a solid is any flat surface that is *not* a base. The **lateral surface area** of a solid is the sum of the areas of its lateral faces. The **total surface area** of a solid is the sum of the areas of all its surfaces.

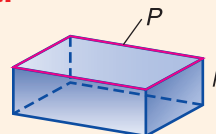
KEY CONCEPTS

Lateral Surface Area of a Prism

Words

The lateral area L of a prism is the perimeter P of the base times the height h of the prism.

Model



Symbols

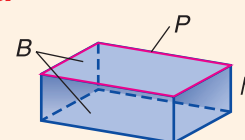
$$L = Ph$$

Total Surface Area of a Prism

Words

The total surface area S of a prism is the lateral surface area L plus the area of the two bases $2B$.

Model



Symbols

$$S = L + 2B \text{ or } S = Ph + 2B$$

STUDY TIP

Bases of Rectangular Prisms

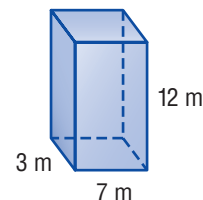
For the examples and exercises in this book, assume that the top and the bottom faces of a rectangular prism are its bases.

EXAMPLES

Surface Areas of a Prism

- 1** Find the lateral and total surface areas of the rectangular prism.

The bases of this prism are rectangles that are 3 meters wide and 7 meters long. Begin by finding the perimeter and area of one base.



Perimeter of Base

$$P = 2\ell + 2w$$

$$P = 2(7) + 2(3) \text{ or } 20$$

Area of Base

$$B = \ell w$$

$$B = 7(3) \text{ or } 21$$

Use this information to find the lateral and total surface areas.

Lateral Surface Area

$$L = Ph$$

$$L = 20(12) \text{ or } 240$$

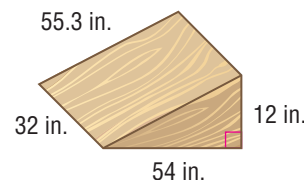
Total Surface Area

$$S = L + 2B$$

$$S = 240 + 2(21) \text{ or } 282$$

The lateral surface area is 240 square meters, and the total surface area of the prism is 282 square meters.

- 2 SKATEBOARDING** A wedge skateboarding ramp is built in the shape of a triangular prism. You plan to paint all surfaces of the ramp. Find the total surface area to be painted.



Estimate $S = (60 + 50 + 10)(30) + 60(10) \text{ or } 4,200 \text{ in}^2$.

The bases of the prism are triangles with side lengths of 12 inches, 54 inches, and 55.3 inches. Find the perimeter and area of one base.

Perimeter of Base

$$P = 55.3 + 12 + 54$$

$$P = 121.3$$

Area of Base

$$B = \frac{1}{2}bh$$

$$B = \frac{1}{2}(54)(12) \text{ or } 324$$

Use this information to find the total surface area.

$$S = Ph + 2B$$

$$S = 121.3(32) + 2(324)$$

$$S = 4,529.6$$

Total surface area of prism

$$P = 121.3, h = 32, \text{ and } B = 324.$$

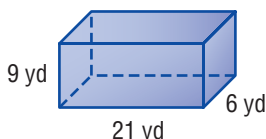
Simplify.

The surface area is 4,529.6 square inches. **Compare to the estimate.**

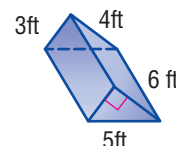
CHECK Your Progress

Find the lateral and total surface areas of each prism.

a.



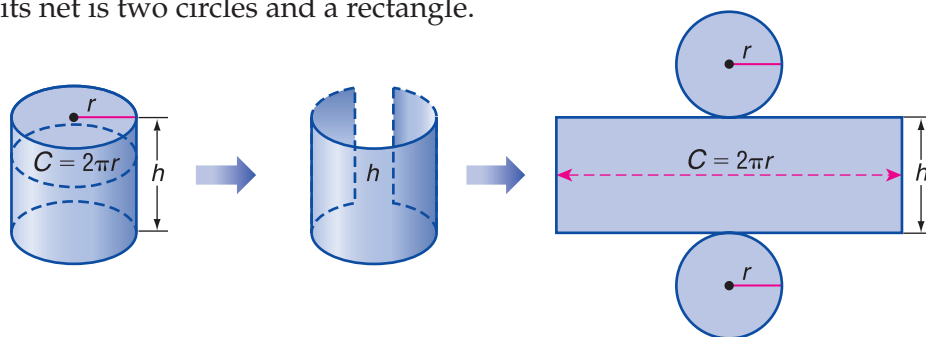
b.



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You can find the total surface area of a cylinder by finding the area of its two bases and adding the area of the curved surface. The lateral area of a cylinder is the area of the curved surface. If you unfold a cylinder, its net is two circles and a rectangle.



Model	Net	Area
2 circular bases	2 congruent circles with radius r	$2(\pi r^2)$ or $2\pi r^2$
1 curved surface	1 rectangle with width h and length $2\pi r$	$2\pi r \cdot h$ or $2\pi rh$

STUDY TIP

Cylinders The formulas for the lateral and total surface areas of cylinders are similar to those of prisms.

Prism: $L = Ph$

For cylinders, the base is a circle, so its perimeter is the circumference.

Prism: $S = L + 2B$

For cylinders, the base B is a circle with area πr^2 .

Just as with prisms, you can use the measures of a cylinder to find the lateral and total surface areas of a cylinder.

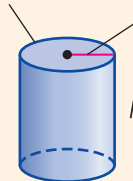
KEY CONCEPTS

Lateral Surface Area of a Cylinder

Words The lateral area L of a cylinder with height h and radius r is the circumference of the base times the height.

Symbols $L = 2\pi rh$

Model circumference of base = $2\pi r$

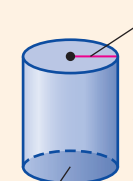


Total Surface Area of a Cylinder

Words The surface area S of a cylinder with height h and radius r is the lateral area plus the area of the two bases.

Symbols $S = L + 2\pi r^2$ or $S = 2\pi rh + 2\pi r^2$

Model



area of a base = πr^2

EXAMPLES

Surface Areas of Cylinders

- 3** Find the lateral area and the total surface area of the cylinder. Round to the nearest tenth.

Lateral Surface Area

$$L = 2\pi rh$$

$$L \approx 2(3.14)(2)(3)$$

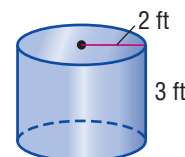
$$L \approx 37.7$$

Total Surface Area

$$S = L + 2\pi r^2$$

$$S \approx 37.7 + 2(3.14)(2)^2$$

$$S \approx 62.8$$



The lateral area is about 37.7 square feet, and the surface area of the cylinder is about 62.8 square feet.



4 LABELS Find the area of the label on the can of vegetables shown at the right.

Since the label covers the lateral surface of the can, you only need to find the can's lateral surface area.

Estimate $L = 2\pi rh$

$$L \approx 2(3)(2)(5) \quad \pi \approx 3, r = 1.75 \approx 2, h = 5$$

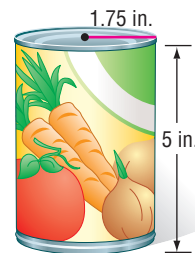
$$L \approx 60 \text{ in}^2$$

$$L = 2\pi rh \quad \text{Lateral surface area of cylinder}$$

$$L \approx 2(3.14)(1.75)(5) \quad \pi = 3.14, r = 1.75, h = 5$$

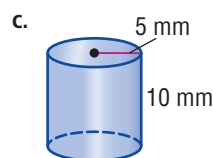
$$L \approx 55.0 \quad \text{Simplify.}$$

The area of the label is about 55 square inches. **Compare to the estimate.**



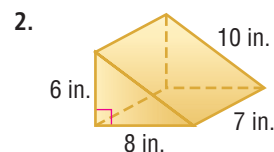
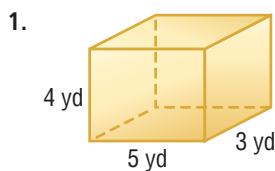
CHECK Your Progress

Find the lateral and total surface areas of each cylinder. Round to the nearest tenth.

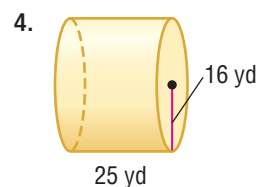
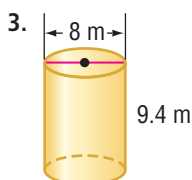


CHECK Your Understanding

Examples 1, 2 Find the lateral and total surface areas of each solid. Round to the nearest tenth if necessary.
(p. 387)



Example 3
(p. 388)



Example 4
(p. 389)

5. **CONTAINERS** Frozen orange juice often comes in cylindrical cardboard containers with metal lids. Find the area of the cardboard portion of the orange juice container shown.



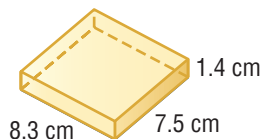
Exercises

HOMEWORK HELP

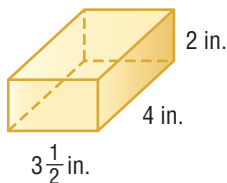
For Exercises	See Examples
6, 7	1
8, 9, 13	2
10, 11	3
12, 13	4

Find the lateral and total surface areas of each solid. Round to the nearest tenth if necessary.

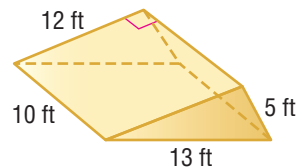
6.



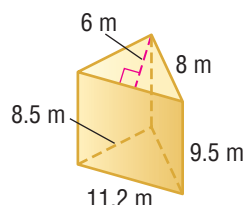
7.



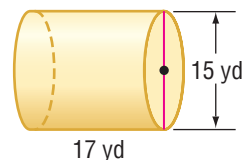
8.



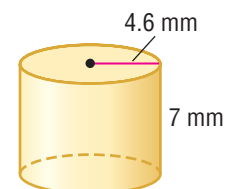
9.



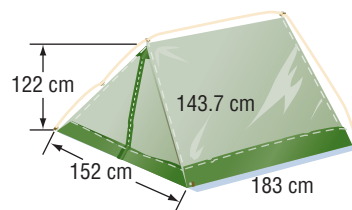
10.



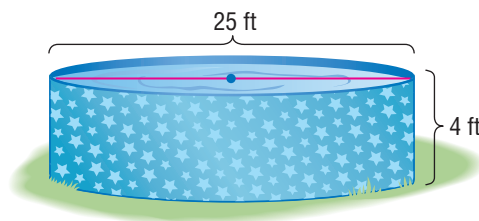
11.



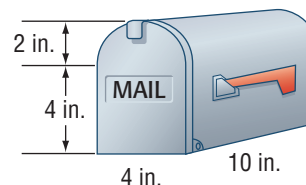
12. **CAMPING** A manufacturer makes nylon tents like the one shown. How much material is needed to make the tent?



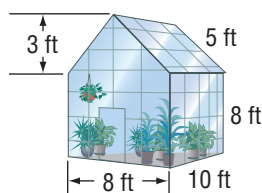
13. **POOL** A vinyl liner covers the inside walls and bottom of the swimming pool. Find the area of this liner to the nearest square foot.



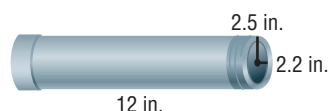
14. A rectangular prism has length 12 centimeters and width 4 centimeters. If its surface area is 467 square centimeters, what is the height of the prism?
15. **MANUFACTURING** Find the amount of metal needed to construct the mailbox at the right to the nearest tenth of a square inch.



16. **GARDENING** The door of the greenhouse below has an area of 4.5 square feet. How many square feet of plastic are needed to cover the roof and sides of the greenhouse?



17. **PLUMBING** A hollow piece of a cylindrical pipe is shown. Find the total surface area of the pipe, including the interior.



EXTRA PRACTICE

See pages 695, 714.

Math online

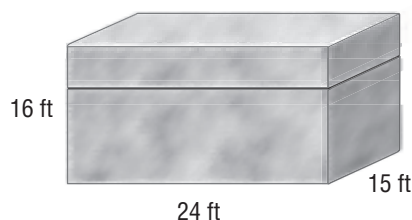
Self-Check Quiz at
ca.gr7math.com

**H.O.T. Problems**

18. **REASONING** Determine whether the following statement is *true* or *false*.
If *false*, give a counterexample.
*If two rectangular prisms have the same volume,
then they also have the same surface area.*
19. **CHALLENGE** Will the surface area of a cylinder increase more if you double the height or double the radius? Explain your reasoning.
20. **NUMBER SENSE** If you double the edge length of a cube, explain how this affects the surface area of the prism.
21. **WRITING IN MATH** Explain the difference between lateral area and surface area.

**STANDARDS PRACTICE**

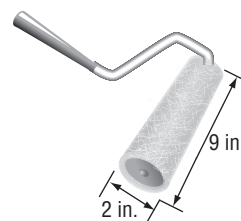
22. Molly is painting the rectangular toy chest shown in the diagram below.



If Molly paints only the outside of the toy chest, what is the total surface area, in square inches, she will paint?

- A 330 in² C 5,760 in²
B 399 in² D 1,968 in²

23. A paint roller like the one shown is used for painting.



To the nearest tenth, how many square inches does a single rotation of the paint roller cover?

- F 18.0 H 56.5
G 28.3 J 113.1

Spiral Review

Find the volume of each solid. Round to the nearest tenth if necessary.

(Lesson 7-6)

24. rectangular pyramid: length, 14 m; width, 12 m; height, 7 m
25. cone: diameter, 22 cm; height, 24 cm
26. **HEALTH** The inside of a refrigerator in a medical laboratory measures 17 inches by 18 inches by 42 inches. You need at least 8 cubic feet to refrigerate some samples from the lab. Is the refrigerator large enough for the samples? Explain your reasoning. (Lesson 7-5)

GET READY for the Next Lesson

PREREQUISITE SKILL Multiply. (Lesson 2-3)

27. $\frac{1}{2} \cdot 2.8$

28. $\frac{1}{2} \cdot 10 \cdot 23$

29. $\frac{1}{2} \cdot 2.5 \cdot 16$

30. $\frac{1}{2} \left(3\frac{1}{2} \right) (20)$

Extend 7-7

Measurement Lab Net of a Cone

Main IDEA

Make a net of a cone.



Standard 7MG3.5

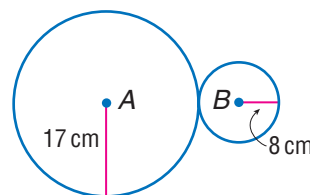
Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.

Standard 7MR2.5 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

A cone is a three-dimensional figure with one circular base. The lateral surface is part of a larger circle. So that the edges match, the circumference of the base is equal to *part* of the circumference of the larger circle.

ACTIVITY Make a Net of a Cone

STEP 1 Use a compass to draw two circles slightly touching, one with a radius of 17 centimeters and one with a radius of 8 centimeters.



STEP 2 Think: What part of the circumference of A is equal to the circumference of B? Let x represent the part.

$$x(34\pi) = 16\pi \quad \begin{array}{l} \text{The circumference of A is } 34\pi. \\ \text{The circumference of B is } 16\pi. \end{array}$$

$$x \cdot \frac{34\pi}{34\pi} = \frac{16\pi}{34\pi} \quad \text{Divide each side by } 34\pi.$$

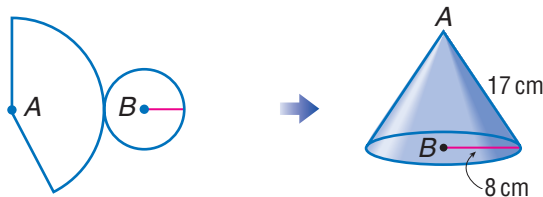
$$x \approx 0.47 \quad \text{Simplify.}$$

You need 0.47 of the circumference of A.

STEP 3 Find the size of the central angle to be cut from A.

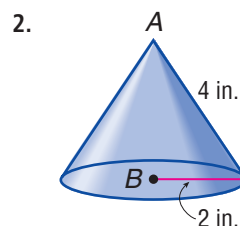
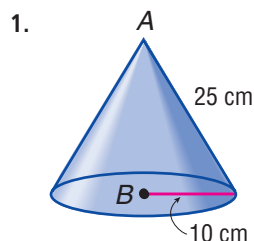
$$0.47 \cdot 360^\circ \approx 170^\circ$$

Cut a central angle of 170° from circle A and make a cone.



ANALYZE THE RESULTS

Find the central angle of each cone and then make a net and the cone.



7-8

Surface Area of Pyramids

Main IDEA

Find the lateral and total surface areas of pyramids.



Standard 7MG2.1

Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

NEW Vocabulary

regular pyramid
slant height

GET READY for the Lesson

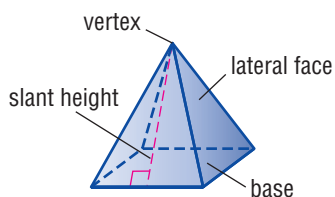
HISTORY In 1485, Leonardo da Vinci sketched a pyramid-shaped parachute in the margin of his notebook. In June 2000, using a parachute created with tools and materials available in medieval times, Adrian Nicholas proved da Vinci's design worked by descending 7,000 feet.

1. How many cloth faces does this pyramid have? What shape are they?
2. How could you find the total area of the material used for the parachute?

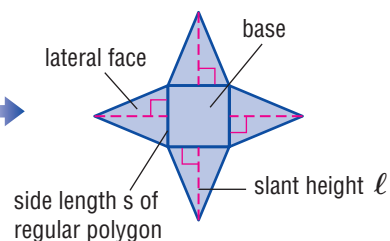


A **regular pyramid** is a pyramid with a base that is a regular polygon. The lateral faces of a regular pyramid are congruent isosceles triangles. At the top of the pyramid, these triangles meet at a common point called the vertex. The altitude or height of each lateral face is called the **slant height** of the pyramid.

Model of Regular Pyramid



Net of Regular Pyramid



STUDY TIP

Look Back

You can review **area of triangles** on page 693.

To find the lateral area L of a regular pyramid, look at its net. The lateral area of a pyramid is the sum of the areas of its lateral faces, which are all triangles.

The net of a square pyramid is a square and four triangles as shown above.

$$L = 4\left(\frac{1}{2}s\ell\right) \quad \text{Area of the lateral faces}$$

$$L = \frac{1}{2}(4s)\ell \quad \text{Commutative Property of Multiplication}$$

$$L = \frac{1}{2}P\ell \quad \text{The perimeter of the base } P \text{ is } 4s.$$

The total surface area of a regular pyramid is the lateral surface area plus the area of the base.



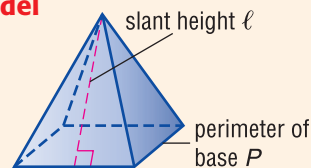
KEY CONCEPTS

Lateral Surface Area of a Pyramid

Words The lateral surface area L of a regular pyramid is half the perimeter P of the base times the slant height ℓ .

Symbols $L = \frac{1}{2}P\ell$

Model

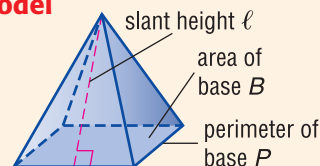


Total Surface Area of a Pyramid

Words The total surface area S of a regular pyramid is the lateral area L plus the area of the base B .

Symbols $S = L + B$ or $S = \frac{1}{2}P\ell + B$

Model



EXAMPLE

Surface Areas of a Pyramid

- 1** Find the lateral and total surface areas of the triangular pyramid.

$$L = \frac{1}{2}P\ell$$

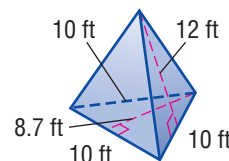
$$S = L + B$$

$$L = \frac{1}{2} \cdot 30 \cdot 12 \quad S = 180 + 43.5 \quad B = \frac{1}{2} \cdot 10 \cdot 8.7$$

$$L = 180$$

$$S = 223.5$$

The lateral and total surface areas are 180 and 223.5 square feet.



CHECK Your Progress

- a. Find the lateral and total surface areas of a pyramid with a slant height of 18 meters and a square base with 11-meter sides.



Real-World EXAMPLE

- 2 ARCHITECTURE** Use the information at the left to find the lateral surface area of the Pyramid of the Sun if it has a slant height of 132.5 meters.

$$L = \frac{1}{2}P\ell$$

Lateral surface area of a pyramid

$$L = \frac{1}{2} \cdot 894 \cdot 132.5$$

$$P = 223.5(4) \text{ or } 894 \text{ and } \ell = 132.5$$

$$L = 59,227.5$$

Simplify.

The lateral area of the pyramid is 59,227.5 square meters.



CHECK Your Progress

- b. **AWARDS** A music award is a square pyramid with a 6-inch-long base and a 13-inch slant height. Find the award's total surface area.



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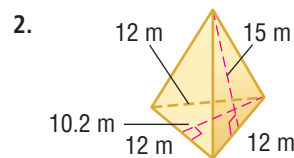
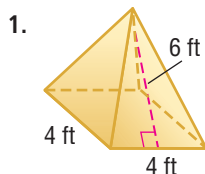
Real-World Link

The Pyramid of the Sun in Teotihuacán, Mexico, was built in the second century, A.D. It is about 71 meters tall, and its square base has side lengths of 223.5 meters.

Source: infoplease.com

CHECK Your Understanding

Example 1 Find the lateral and total surface areas of each regular pyramid. Round to the nearest tenth if necessary.
(p. 394)



Example 2
(p. 394)

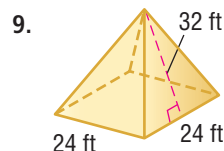
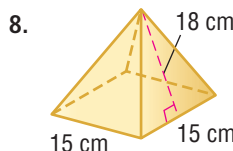
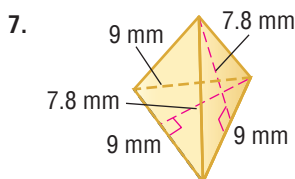
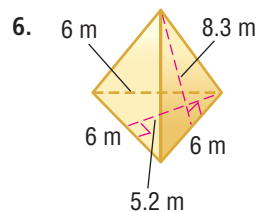
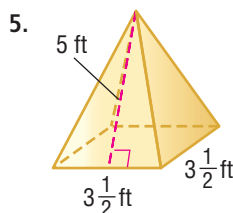
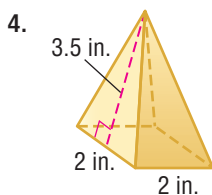
3. **HISTORY** Refer to the lesson opener. Each face of the parachute has a base of about 12 yards and a height of about 17 yards. Find the amount of material needed to make the parachute.

Exercises

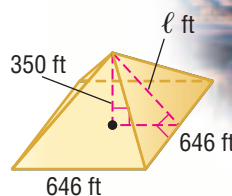
HOMEWORK HELP

For Exercises	See Examples
4–9	1
10, 11	2

Find the lateral and total surface area of each regular pyramid. Round to the nearest tenth if necessary.



10. **ARCHITECTURE** The Transamerica Pyramid in San Francisco is shaped like a square pyramid. It has a slant height of 856.1 feet and each side of its base is 145 feet. Find the lateral area of the building.
11. **ROOFS** A pyramid-shaped roof has a slant height of 16 feet and its square base is 40 feet wide. How much roofing material is needed to cover the roof?
12. A square pyramid has a lateral area of 107.25 square centimeters and a slant height of 8.25 centimeters. Find the length of each side of its base.
13. **GLASS** The Luxor Hotel in Las Vegas, Nevada, is a pyramid-shaped building standing 350 feet tall and covered with glass. Its base is a square with each side 646 feet long. Find the surface area of the glass on the Luxor. (*Hint: Use the Pythagorean Theorem to find the pyramid's slant height ℓ .*)



EXTRAPRACTICE

See pages 696, 714.

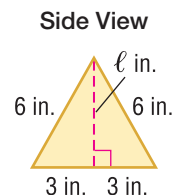
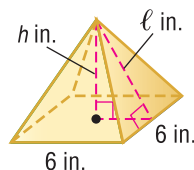
Math  online

Self-Check Quiz at
ca.gr7math.com

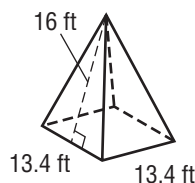
**H.O.T. Problems**

CHALLENGE For Exercises 14–16, use the drawings of the pyramid shown, in which the lateral faces are equilateral triangles.

14. Find the measure of the slant height ℓ .
15. Use the slant height to find the height h of the pyramid.
16. Find the volume and surface area of the pyramid.
17. **OPEN ENDED** Draw a square pyramid, giving measures for its slant height and base side length. Then find its lateral area. Justify your answer.
18. **WRITING IN MATH** Explain how the slant height and the height of a pyramid are different.

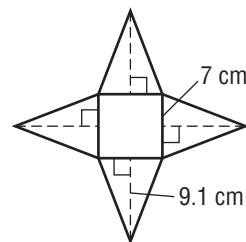
**STANDARDS PRACTICE**

19. Which is the best estimate for the surface area of the pyramid?



- A 107 ft^2
 B 180 ft^2
 C 429 ft^2
 D 608 ft^2

20. The net of a paperweight is shown below. Which is closest to the lateral surface area of the paperweight?



- F 32 cm^2 H 127 cm^2
 G 49 cm^2 J 176 cm^2

Spiral Review

21. **GEOMETRY** Find the surface area of a cylinder that has a diameter of 22 feet and a height of 7.5 feet. (Lesson 7-7)
22. **MOUNTAINS** A student is creating a clay model of a mountain shaped like a cone. If the mountain is 4 feet tall and the radius of the base is 2 feet, what is the volume of clay needed to make the mountain? Round to the nearest tenth if necessary. (Lesson 7-6)

GET READY for the Next Lesson

PREREQUISITE SKILL Solve each proportion. (Lesson 4-2)

23. $\frac{16}{n} = \frac{12}{40}$

24. $\frac{3}{5} = \frac{x}{8}$

25. $\frac{a}{13} = \frac{7}{39}$

26. $\frac{10}{26} = \frac{30}{w}$

Explore 7-9

Spreadsheet Lab Similar Solids

Main IDEA

Investigate the relationships between the surface areas and volumes of similar solids.

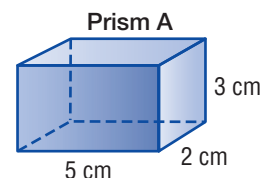


Standard 7MG2.3 Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.

Standard 7MR2.2 Apply strategies and results from simpler problems to more complex problems.

ACTIVITY

- Find the surface area and volume of the prism at the right. Then find the surface areas and volumes of similar prisms with scale factors of 2, 3, and 4.



Similar Prisms.xls							
	A	B	C	D	E	F	G
1	Prism	Scale Factor	Length	Width	Height	Surface Area	Volume
2	A	1	5	2	3	62	30
3	B	2	10	4	6	248	240
4	C	3	15	6	9	558	810
5	D	4	20	8	12	992	1920

The spreadsheet evaluates the formula $2 \cdot C3 \cdot D3 + 2 \cdot C3 \cdot E3 + 2 \cdot D3 \cdot E3$.

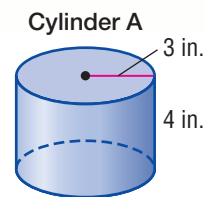
The spreadsheet evaluates the formula $C5 \cdot D5 \cdot E5$.

ANALYZE THE RESULTS

- What is the ratio of the surface area of prism B to the surface area of prism A? of prism C to prism A? of prism D to prism A?
- How are the answers to Exercise 1 related to the scale factors?
- What is the ratio of the volume of prism B to the volume of prism A? of prism C to prism A? of prism D to prism A?
- How are the answers to Exercise 3 related to the scale factors?
- MAKE A PREDICTION** If the dimensions of prism E are 5 times that of prism A, predict the ratio of the surface areas of prism E to prism A.
- Explain how you can use the ratio in Exercise 5 to predict the surface area of prism E. Find the surface area using the spreadsheet.
- MAKE A PREDICTION** If the dimensions of prism E are 5 times that of prism A, predict the ratio of the volumes of prism E to prism A.
- Explain how you can use the ratio in Exercise 6 to predict the volume of prism E. Find the volume using the spreadsheet.

ACTIVITY

- 2** Find the surface area and volume of the cylinder at the right. Then find the surface areas and volumes of similar cylinders with scale factors of 2, 3, and 4.



Similar Cylinders.xls							
	A	B	C	D	E	F	G
1	Cylinder	Scale Factor	Radius	Height	Surface Area	Volume	
2	A	1	3	4	131.9	113.1	
3	B	2	6	8	527.8	904.78	
4	C	3	9	12	1,187.5	3053.6	
5	D	4	12	16	2,111.2	7238.2	

The spreadsheet evaluates the formula $2\pi r^2 + 2\pi rh$ for cylinder C.

The spreadsheet evaluates the formula $\pi r^2 h$ for cylinder D.

Spreadsheet Notation in Microsoft® Excel®, the expression $\text{PI}()$ gives the value for π .

Spreadsheet Notation The expression C5^2 squares the value in cell C5.

ANALYZE THE RESULTS

- What is the ratio of the surface areas of cylinder B to cylinder A? of cylinder C to cylinder A? of cylinder D to cylinder A?
- How are the answers to Exercise 9 related to the scale factors of each cylinder?
- What is the ratio of the volume of cylinder B to the volume of cylinder A? of cylinder C to cylinder A? of cylinder D to cylinder A?
- How are the answers to Exercise 11 related to the scale factors of each cylinder?
- MAKE A PREDICTION** If the dimensions of cylinder F are 6 times that of cylinder A, predict the ratio of the surface areas of cylinder F to cylinder A.
- Explain how you can use the ratio to predict the surface area of cylinder F. Find the surface area using the spreadsheet.
- MAKE A PREDICTION** If the dimensions of cylinder F are 5 times that of cylinder A, predict the ratio of the volumes of cylinder F to cylinder A.
- MAKE A CONJECTURE** If two solids A and B are similar and the scale factor relating solid A to solid B is $\frac{a}{b}$, write expressions for the ratios of their surface areas and volumes.

7-9

Similar Solids

Main IDEA

Find dimensions, surface area, and volume of similar solids.



Standard 7MG2.3
Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.

NEW Vocabulary

similar solids

REVIEW Vocabulary

scale factor the ratio of corresponding measures of two similar figures
(Lesson 4-5)

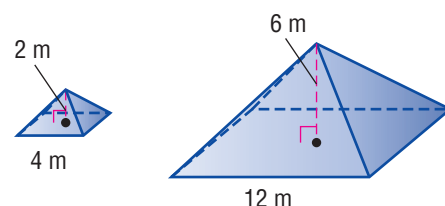
GET READY for the Lesson

The model car at the right is $\frac{1}{43}$ the size of the original car.

1. If the model car is 4.2 inches long, 1.6 inches wide, and 1.3 inches tall, what are the dimensions of the original car?
2. Make a conjecture about the radius of the wheel of the original car compared to the model.



The pyramids at the right have the same shape. The ratios of their corresponding linear measures, such as length, width, or height, are $\frac{6}{2}$ or 3 and $\frac{12}{4}$ or 3. We say that 3 is the scale factor.

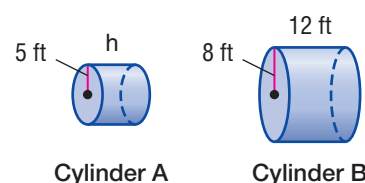


These pyramids are called **similar solids** because they have the same shape, their corresponding linear measures are proportional, and their corresponding faces are similar polygons. If you know two solids are similar, you can use a proportion to find a missing measure.

EXAMPLE Find Missing Linear Measures

- 1 The cylinders at the right are similar. Find the height of cylinder A.

Since the two cylinders are similar, the ratios of their corresponding linear measures are proportional.



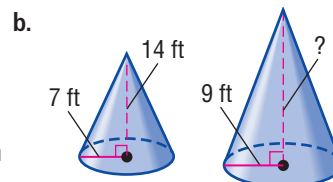
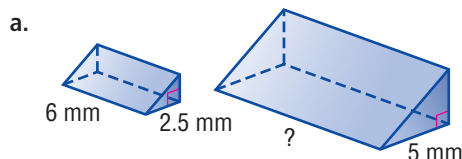
Words	$\frac{\text{radius cylinder A}}{\text{radius cylinder B}}$ is proportional to $\frac{\text{height cylinder A}}{\text{height cylinder B}}$	
Variable	Let h represent the height of cylinder A.	
Equation	$\frac{5}{8}$	$= \frac{h}{12}$

$$\begin{aligned}\frac{5}{8} &= \frac{h}{12} && \text{Write the proportion.} \\ 5 \cdot 12 &= 8 \cdot h && \text{Find the cross products.} \\ \frac{5 \cdot 12}{8} &= \frac{8 \cdot h}{8} && \text{Divide each side by 8.} \\ 7.5 &= h && \text{Simplify.}\end{aligned}$$

The height of cylinder A is 7.5 feet.

CHECK Your Progress

Find the missing measure for each pair of similar solids.



As you discovered in the Geometry Lab prior to this lesson, the surface areas and volumes of similar solids are proportional.

KEY CONCEPTS

Ratios of Similar Solids

Ratios of Surface Area

Words If two solids are similar, the ratio of their surface areas is proportional to the square of the scale factor between them.

Symbols $\frac{\text{surface area of solid A}}{\text{surface area of solid B}} = \left(\frac{a}{b}\right)^2$

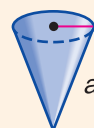
Ratios of Volumes

Words If two solids are similar, the ratio of their volumes is proportional to the cube of the scale factor between them.

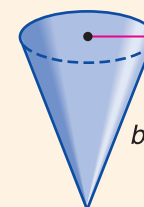
Symbols $\frac{\text{volume of solid A}}{\text{volume of solid B}} = \left(\frac{a}{b}\right)^3$

Model

Solid A



Solid B



EXAMPLE

Find Surface Area of a Similar Solid

- 2** The pyramids at the right are similar. Find the total surface area of pyramid B.

The ratio of the measures of pyramid A to pyramid B is $\frac{a}{b} = \frac{8}{12}$ or $\frac{2}{3}$.

$$\frac{\text{surface area of pyramid A}}{\text{surface area of pyramid B}} = \left(\frac{a}{b}\right)^2$$

$$\frac{224}{S} = \left(\frac{2}{3}\right)^2$$

$$\frac{224}{S} = \frac{4}{9}$$

$$224 \cdot 9 = 4S$$

$$\frac{224 \cdot 9}{4} = \frac{4S}{4}$$

$$504 = S$$

Write a proportion.

Substitute the known values.
Let S represent the surface area.

$$\left(\frac{2}{3}\right)^2 = \frac{2}{3} \cdot \frac{2}{3} \text{ or } \frac{4}{9}$$

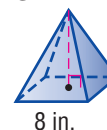
Find the cross products.

Divide each side by 4.

Simplify.

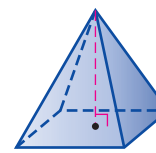
The surface area of pyramid B is 504 square inches.

Pyramid A
 $S = 224 \text{ in}^2$



8 in.

Pyramid B



12 in.

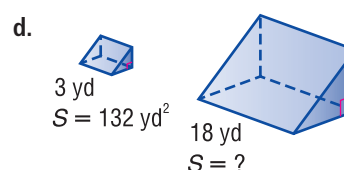
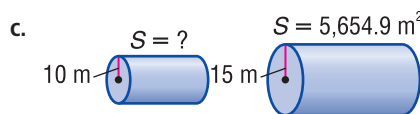
STUDY TIP

Scale Factor

Remembering that area is expressed in *square* units can help you remember to *square* the scale factor when working with surface areas of similar solids.

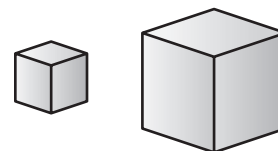
CHECK Your Progress

Find the missing measure for each pair of similar solids. Round to the nearest tenth if necessary.



STANDARDS EXAMPLE Find Volume of a Similar Solid

- 3 A cube has a volume of 27 cubic feet. Suppose the dimensions are doubled. What is the volume of the new cube?



- A 13.5 ft³ B 54 ft³ C 108 ft³ D 216 ft³

Test-Taking Tip

Scale Factors

When the lengths of all dimensions of a solid are multiplied by a scale factor x , then the surface area is multiplied by x^2 and the volume is multiplied by x^3 .

Read the Item

You know that the cubes are similar, the ratio of the side lengths $\frac{a}{b}$ is $\frac{1}{2}$, and the volume of the smaller cube is 27 cubic feet.

Solve the Item

Since the volumes of similar solids have a ratio of $\left(\frac{a}{b}\right)^3$ and $\frac{a}{b} = \frac{1}{2}$, replace a with 1 and b with 2 in $\left(\frac{a}{b}\right)^3$.

$$\frac{\text{volume of smaller cube}}{\text{volume of larger cube}} = \left(\frac{a}{b}\right)^3$$

$$\frac{27}{V} = \left(\frac{1}{2}\right)^3$$

$$\frac{27}{V} = \frac{1}{8}$$

$$27 \cdot 8 = V \cdot 1$$

$$216 = V$$

Write a proportion.

Substitute the known values. Let V represent the volume of the larger cube.

$$\left(\frac{1}{2}\right)^3 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$$

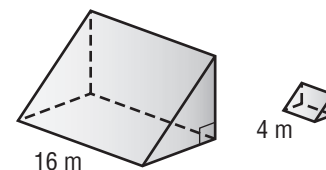
Find the cross products.

Multiply.

So, the volume of the larger cube is 216 cubic feet. The answer is D.

CHECK Your Progress

- e. A triangular prism has a volume of 896 cubic meters. If the prism is reduced to one-fourth its original size, what is the volume of the new prism?



- F 14 m³ H 64 m³
G 56 m³ J 224 m³

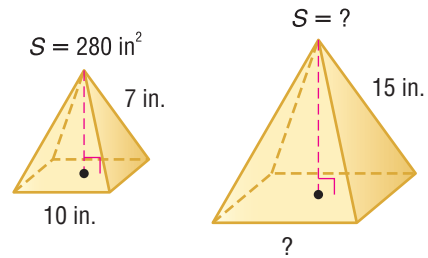


CHECK Your Understanding

Examples 1, 2
(pp. 399–400)

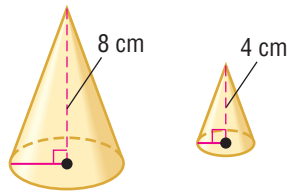
For Exercises 1 and 2, use the two similar pyramids shown. Round to the nearest tenth if necessary.

- Find the missing side length.
- Find the missing surface area.



Example 3
(p. 401)

3. **STANDARDS PRACTICE** A cone has a volume of 134.4 cubic centimeters. Suppose that the dimensions are reduced to half their current value. What is the volume of the resulting cone?



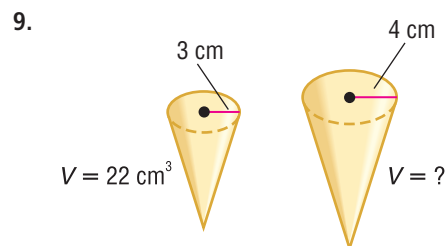
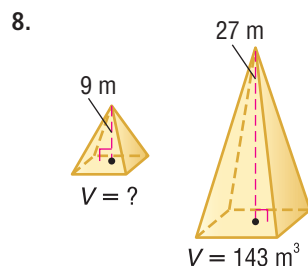
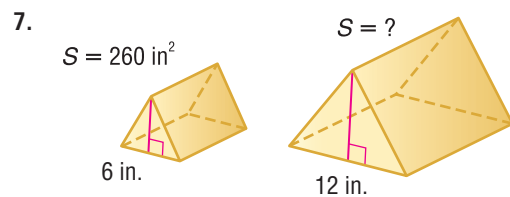
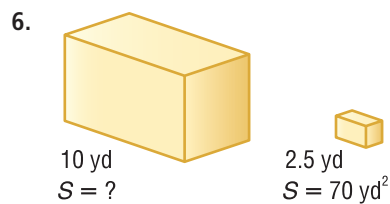
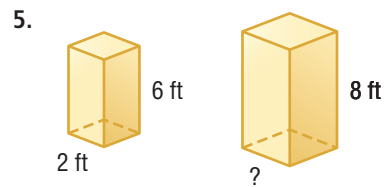
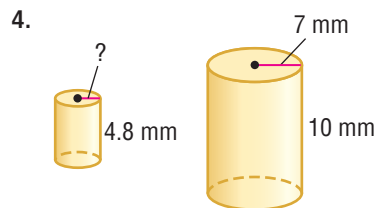
- A 8.4 cm^3 B 16.8 cm^3 C 33.6 cm^3 D 67.2 cm^3

Exercises

HOMEWORK HELP

For Exercises	See Examples
4, 5, 10	1
6, 7	2
8, 9, 11, 23, 24	3

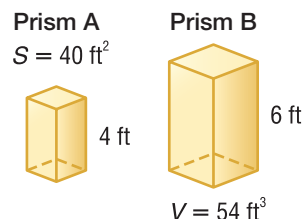
Find the missing measure for each pair of similar solids. Round to the nearest tenth if necessary.



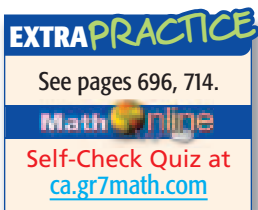
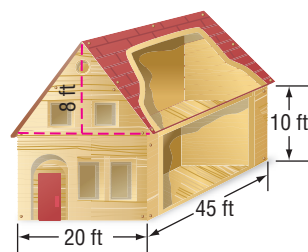


10. **ARCHITECTURE** The model of a high-rise apartment building is 25.2 inches tall. On the model, 2 inches represents 45 feet. What is the height of the building?
11. **ART** In art class, Rueben made two similar cylindrical containers. One was 4 inches tall, and the other was 8 inches tall. If the volume of the smaller container is 16.7 cubic inches, find the volume of the larger container.

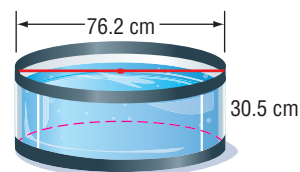
For Exercises 12–14, use the two similar prisms at the right.



12. Write the ratio of the surface areas and the ratio of the volumes.
13. Find the surface area of prism B.
14. Find the volume of prism A.
15. The surface areas of two similar solids are 36 square yards and 144 square yards. Find the ratio of their linear measures.
16. **HOBBIES** Darcy is building a doll house similar to her family's house. If the doll house will be $\frac{1}{20}$ the size of her actual house, what will be the lateral surface area of her doll house, not including the roof? Round to the nearest tenth.



17. **AQUARIUMS** A zoo has three cylindrical aquariums. The smallest is $\frac{3}{4}$ the size of the one shown, while the largest is $1\frac{1}{2}$ times larger. Determine the volumes of the three aquariums. Round to the nearest tenth.



H.O.T. Problems

18. **CHALLENGE** The ratio of the surface areas of two similar pyramids is $\frac{1}{25}$. What is the ratio of the volumes of the pyramids? Explain your reasoning.
19. **OPEN ENDED** Draw and label two cones that are similar. Explain why they are similar.
20. **SELECT A TECHNIQUE** Ruby is packing two similar boxes. The smaller box is 9 inches long and 12 inches tall, and the larger box is 18 inches long and 24 inches tall. Which of the following techniques might Ruby use to determine how much greater the volume of the larger box is? Justify your selection(s). Then use the technique(s) to solve the problem.

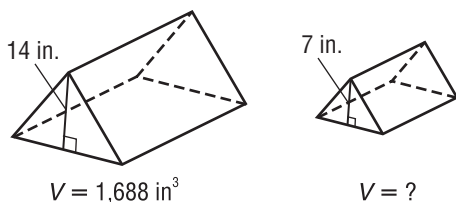
mental math

number sense

estimation

21. **REASONING** True or False? All spheres are similar. Explain your reasoning.
22. **WRITING IN MATH** Refer to the application at the beginning of the lesson. Write a real-world problem involving a model car. Then solve your problem.

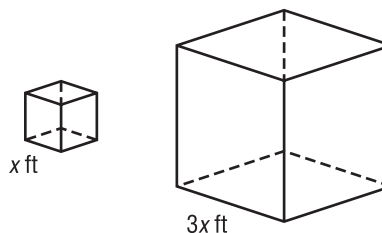
23. The triangular prisms shown are similar.



Find the volume of the smaller prism.

- A 211 in³
- B 844 in³
- C 3,376 in³
- D 6,752 in³

24. The dimensions of two cubes are shown below.



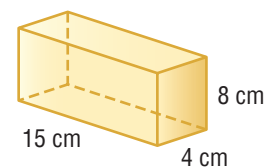
The volume of the smaller cube is 125 cubic feet. Find the volume of the larger cube.

- F 375 ft³
- G 3,375 ft³
- H 5,125 ft³
- J 15,625 ft³

Spiral Review

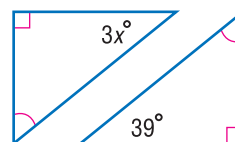
25. **HISTORY** The great pyramid of Khufu in Egypt was originally 481 feet high, and had a square base measuring 756 feet on a side and slant height of about 611.8 feet. What was its lateral surface area? Round to the nearest tenth. (Lesson 7-8)

26. **MEASUREMENT** Find the lateral surface and total surface area of the rectangular prism at the right. (Lesson 7-7)



27. **GEOMETRY** Graph parallelogram $QRST$ with vertices $Q(-3, 3)$, $R(2, 4)$, $S(3, 2)$, and $T(-2, 1)$. Then graph the image of the figure after a reflection over the x -axis, and write the coordinates of its vertices. (Lesson 6-6)

28. **ALGEBRA** Find the value of x in the two congruent triangles. (Lesson 6-3)
29. **MONEY** A \$750 investment earned \$540 in 6 years. Find the simple interest rate. (Lesson 5-9)



Cross-Curricular Project

Math and Architecture

Under construction It's time to complete your project. Use the information and data you have gathered about floor covering costs and loan rates to prepare a Web page or brochure. Be sure to include a labeled scale drawing with your project.



Cross-Curricular Project at ca.gr7math.com

Study Guide and Review

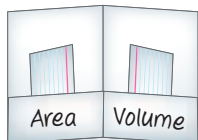


Download Vocabulary
Review from ca.gr7math.com

FOLDABLES Study Organizer

GET READY to Study

Be sure the following
Key Concepts are noted
in your Foldable.



Key Concepts

Circles (Lesson 7-1)

- Circumference: $C = \pi d$ or $C = 2\pi r$
- Area: $A = \pi r^2$

Volume (Lessons 7-5 and 7-6)

- Prism: $V = Bh$
- Cylinder: $V = Bh$ or $V = \pi r^2 h$
- Pyramid: $V = \frac{1}{3}Bh$
- Cone: $V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$

Surface Area (Lessons 7-7 and 7-8)

- Prism
Lateral Surface Area: $L = Ph$
Total Surface Area: $S = L + 2B$
- Pyramid
Lateral Surface Area: $L = P\ell$
Total Surface Area: $S = L + B$
- Cylinder
Lateral Surface Area: $L = 2\pi rh$
Total Surface Area: $S = L + 2B$

Similar Solids (Lesson 7-9)

- If two solids are similar with a scale factor of $\frac{a}{b}$, then the surface areas have a ratio of $\left(\frac{a}{b}\right)^2$ and the volumes have a ratio of $\left(\frac{a}{b}\right)^3$.

Key Vocabulary

base (p. 369)	net (p. 385)
center (p. 352)	pi (p. 352)
chord (p. 352)	plane (p. 368)
circle (p. 352)	polyhedron (p. 368)
circumference (p. 352)	prism (p. 368)
complex figure (p. 363)	pyramid (p. 369)
complex solid (p. 375)	radius (p. 352)
cone (p. 381)	regular pyramid (p. 393)
cylinder (p. 374)	similar solids (p. 399)
diameter (p. 352)	slant height (p. 393)
edge (p. 368)	solid (p. 368)
face (p. 368)	total surface area (p. 386)
lateral face (p. 386)	vertex (p. 368)
lateral surface area (p. 386)	volume (p. 373)

Vocabulary Check

State whether each sentence is *true* or *false*.
If *false*, replace the underlined word or
number to make a true sentence.

1. The flat surface of a prism is called a face.
2. Circumference is the distance around a circle.
3. The measure of the space occupied by a solid is called the total surface area.
4. A cylinder is a figure that has two parallel, congruent circular bases.
5. A solid is any two-dimensional figure.
6. The side of a prism is called a vertex.
7. The radius is the distance across a circle through its center.

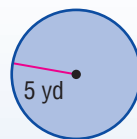
Lesson-by-Lesson Review

7-1 Circumference and Area of Circles (pp. 352–357)

Find the circumference and area of each circle. Round to the nearest tenth.

8. radius: 18 in. 9. diameter: 6 cm
10. **LANDSCAPING** Bill is planting a circular flowerbed. What is the area of the flowerbed if the diameter is 30 feet?

Example 1 Find the circumference and area of the circle.



The radius is 5 yards.

$$C = 2\pi r$$

$$C \approx 2 \cdot 3.14 \cdot 5$$

$$C \approx 31.4 \text{ yd}$$

$$A = \pi r^2$$

$$A \approx 3.14 \cdot 5^2$$

$$A \approx 78.5 \text{ yd}^2$$

7-2 PSI: Solve a Simpler Problem (pp. 360–361)

Solve. Use the *solve a simpler problem* strategy.

11. **GEOGRAPHY** The total area of Arizona is 114,006 square miles. Of that, about 42% of the land is desert. About how many square miles of Arizona's land is *not* covered by desert?

Example 2 A total of 450 students were surveyed. If 60% of the students voted to hold a carnival, find the number of students who voted for the carnival.

Find 10% of 450 and use the result to find 60% of 450.

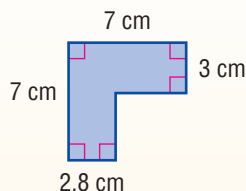
10% of 450 = 45; so 60% is 6×45 or 270.

So, 270 students voted for the carnival.

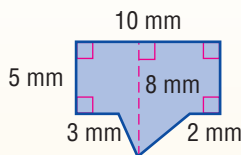
7-3 Area of Complex Figures (pp. 363–367)

Find the area of each figure. Round to the nearest tenth if necessary.

12.

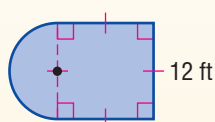


13.

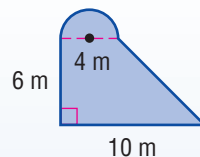


14.

BASKETBALL Travis is going to paint part of a basketball court as shown. What is the area of the court?



Example 3 Find the area of the complex figure.



Area of semicircle

$$A \approx \frac{1}{2} \cdot 3.14 \cdot 2^2$$

$$A \approx 6.3$$

Area of trapezoid

$$A = \frac{1}{2}(6)(4 + 10)$$

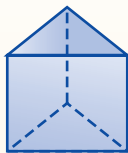
$$A = 42$$

The area is about $6.3 + 42$ or 48.3 square meters.

7-4 Three-Dimensional Figures (pp. 368–372)

Identify each solid. Name the number and shapes of the faces. Then name the number of edges and vertices.

15.

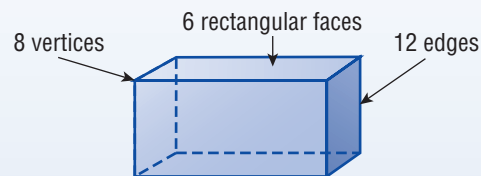


16.



17. **CRYSTALS** Kelli found a crystal in the shape of a pentagonal pyramid. How many faces, edges, and vertices does the crystal have?

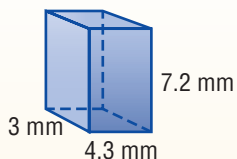
Example 4 Name the number and shapes of the faces of a rectangular prism. Then name the number of edges and vertices.



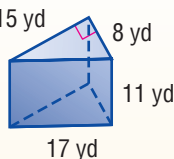
7-5 Volume of Prisms and Cylinders (pp. 373–378)

Find the volume of each solid.

18.



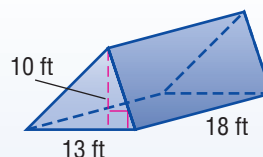
19.



20. **FOOD** A can of green beans has a diameter of 10.5 centimeters and a height of 13 centimeters. Find its volume.

Example 5 Find the volume of the solid.

The base of this prism is a triangle.



$$V = Bh$$

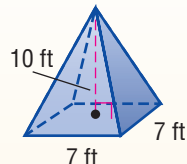
$$V = \left(\frac{1}{2} \cdot 13 \cdot 10\right) 18$$

$$V = 1,170 \text{ ft}^3$$

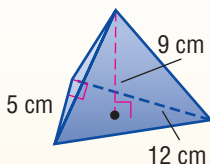
7-6 Volume of Pyramids and Cones (pp. 380–384)

Find the volume of each solid. Round to the nearest tenth if necessary.

21.



22.

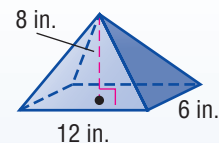


23. cone: diameter, 9 yd; height, 21 yd

24. **ICE CREAM** A waffle cone is five inches tall. The opening of the cone has a radius of 1.5 inches. What is the volume of ice cream that the cone can hold?

Example 6 Find the volume of the pyramid.

The base B of the pyramid is a rectangle.



$$V = \frac{1}{3}Bh$$

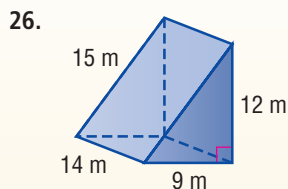
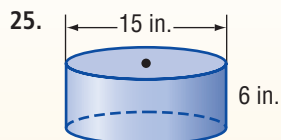
$$V = \frac{1}{3}(12 \cdot 6)8$$

$$V = 192 \text{ in}^3$$

7-7

Surface Area of Prisms and Cylinders (pp. 386–391)

Find the surface area of each solid.
Round to the nearest tenth if necessary.



Example 7 Find the surface area of the cylinder.



Surface area of a cylinder

$$S = 2\pi r^2 + 2\pi rh$$

$$S \approx 2(3.14)(8)^2 + 2(3.14)(8)(11)$$

$$r = 8 \text{ and } h = 11$$

$$S \approx 954.6 \text{ mm}^2$$

Simplify.

7-8

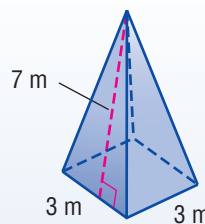
Surface Area of Pyramids (pp. 393–396)

27. **ARCHITECTURE** A hotel shaped like a square pyramid has a slant height of 92.5 meters and each side of its base is 183.5 meters long. What is the lateral surface area of the pyramid?

Example 8 Find the total surface area of the square pyramid.

$$A = \frac{1}{2}bh \quad \text{Area of triangle}$$

$$A = \frac{1}{2}(3)(7) \text{ or } 10.5$$

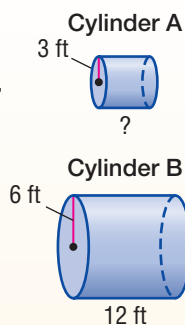


The total lateral area is $4(10.5)$ or 42 square meters. The area of the base is $3(3)$ or 9 square meters. So, the total surface area is $42 + 9$ or 51 square meters.

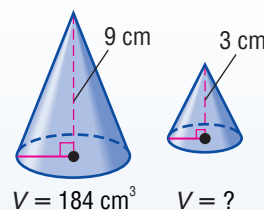
7-9

Similar Solids (pp. 399–404)

28. Cylinders A and B are similar. If the total surface area of cylinder A is 84 square feet, what is the total surface area of cylinder B?



Example 9 Two similar cones are shown at the right. Find the volume of the smaller cone.



$$\frac{\text{volume of smaller cone}}{\text{volume of larger cone}} = \left(\frac{a}{b}\right)^3 \quad \text{Write a proportion.}$$

$$\frac{V}{184} = \frac{1}{27} \quad \frac{1}{27} = \left(\frac{1}{3}\right)^3$$

$$V \cdot 27 = 184 \cdot 1 \quad \text{Find the cross products.}$$

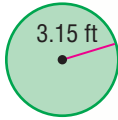
$$\frac{27V}{27} = \frac{184}{27} \quad \text{Divide each side by 27.}$$

$$V = 6.8 \text{ cm}^3 \quad \text{Simplify.}$$

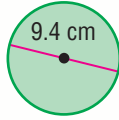
Practice Test

Find the circumference and area of each figure. Round to the nearest tenth if necessary.

1.



2.

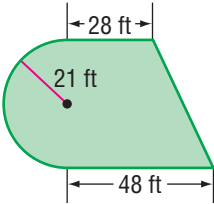


3.  **STANDARDS PRACTICE** A jogger ran around a circular track two times. If the track has a radius of 25 yards, about how far did the jogger run?

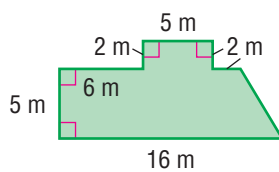
A 314 yd C 78.5 yd
B 157 yd D 50 yd

Find the area of each figure. Round to the nearest tenth if necessary.

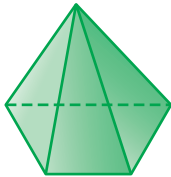
4.



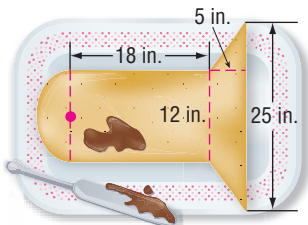
5.



6. **GEOMETRY** Identify the solid. Name the number and shapes of its faces. Then name its number of edges and vertices.

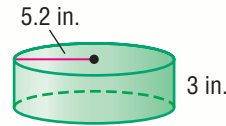


7. **CAKE DECORATION** Mrs. Lee designed the flashlight birthday cake shown below. If one container of frosting covers 250 square inches of cake, how many containers will she need to frost the top of this cake? Explain.

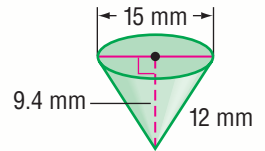


Find the volume of each solid. Round to the nearest tenth.

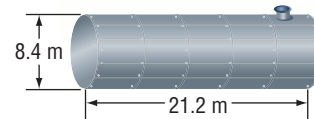
8.



9.

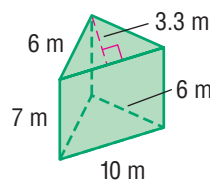


10. **FUEL** The fuel tank is made up of a cylinder. What is the volume of the tank? Round to the nearest tenth.

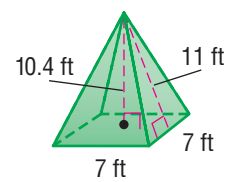


Find the volume and the total surface area of each solid. Round to the nearest tenth if necessary.

11.

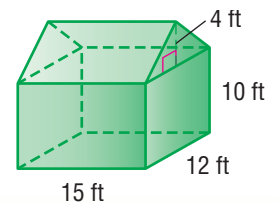


12.



13.  **STANDARDS PRACTICE** Find the volume of the solid.

F 2,160 ft³
G 2,520 ft³
H 3,600 ft³
J 7,200 ft³

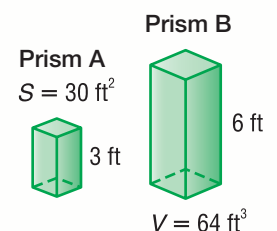


For Exercises 14–16, use the two similar prisms.

14. Write the ratio of the surface areas.

15. Find the total surface area of prism B.

16. Find the volume of prism A.



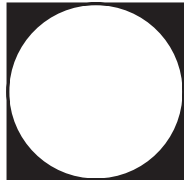
California Standards Practice

Cumulative, Chapters 1–7



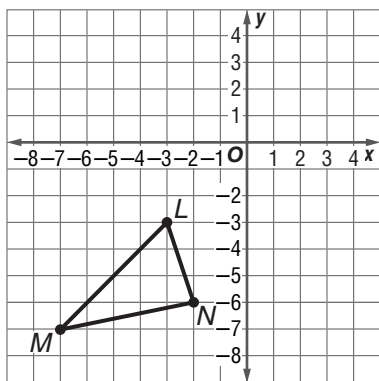
Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

- 1 The figure shows a circle inside a square.



Which procedure should be used to find the area of the shaded region?

- A Find the area of the square and then subtract the area of the circle.
 - B Find the area of the circle and then subtract the area of the square.
 - C Find the perimeter of the square and then subtract the circumference of the circle.
 - D Find the circumference of the circle and then subtract the perimeter of the square.
- 2 If $\triangle LMN$ is translated 7 units up and 2 units to the right, what are the coordinates of point L' ?



- F $(-1, 4)$
- G $(7, 2)$
- H $(2, 7)$
- J $(4, -1)$

- 3 What is the surface area of the shoe box?

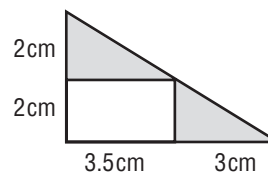


- A 200 in^2
- B 224 in^2
- C 400 in^2
- D 448 in^2

TEST-TAKING TIP

Question 3 Most standardized tests will include any commonly used formulas at the front of the test booklet, but it will save you time to memorize many of these formulas. For example, you should memorize that the surface area of a prism is $2\ell h + 2\ell w + 2hw$.

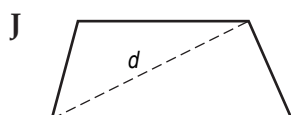
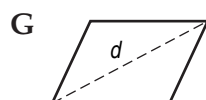
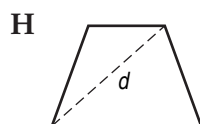
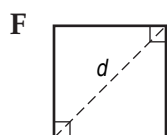
- 4 What is the area of the shaded region in the figure below?



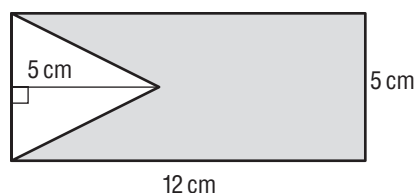
- F 6.5 cm
 - G 13 cm
 - H 7 cm
 - J 26 cm
- 5 Martin and his two brothers equally shared the cost of a new computer game with a list price of \$35. They received a 25% discount on the video game and paid 5.5% sales tax on the discounted price. Find the approximate amount that each of the brothers paid toward the cost of the game.

- A \$14.77
- B \$11.73
- C \$9.23
- D \$8.42

- 6 Suppose you know the side lengths of each figure below. Which one would contain enough information to let you find the length of diagonal d ?

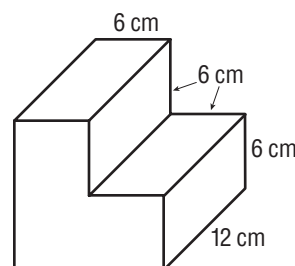


- 7 An isosceles triangle is removed from a rectangle as shown in the figure below. Find the area of the remaining part of the rectangle.



- A 60 cm^2 C 47.5 cm^2
B 55 cm^2 D 35 cm^2
- 8 Susan has two similar rectangular packages. The dimensions of the first box is three times that of the second package. How many times greater is the volume of the first package than of the second package?
- F 81 H 9
G 27 J 3

- 9 A stackable block shown below is made of wood. The height and width of each section is 6 cm. The length is 12 cm.



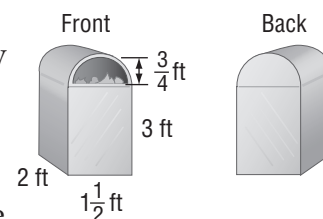
What is the volume, in cubic centimeters, of the wood used to create this block?

- A 2,592 C 432
B 1,296 D 30

Pre-AP

Record your answers on a sheet of paper.
Show your work.

- 10 The diagrams show the design of the trash cans in the school cafeteria.



- Find the volume of each trash can to the nearest tenth.
- The tops and sides of the cans need to be painted. Find the surface area of each can to the nearest tenth.
- The paint used by the school covers 200 square feet per gallon. How many trash cans can be covered with 1 gallon of paint?

NEED EXTRA HELP?

If You Missed Question...	1	2	3	4	5	6	7	8	9	10
Go to Lesson...	7-1	6-7	7-7	7-3	5-8	3-5	7-3	7-5	7-5	7-7
For Help with Standards...	MG2.2	MG3.2	MG2.1	MG2.2	NS1.7	MG3.3	MG2.2	MG2.3	MG2.3	MG2.1