

Def. Conditional Statements

Statements that are in the **IF-THEN** form.

Conditionals are represented by the following:

If **p** then **q**

Example:

If it is raining outside then you need an umbrella

Try it:

Write the following statements in conditional form.

- 1) "Free phone with every 2-year service agreement"
- 2) "Sign up for a 6-month fitness plan and get 6 months free!"
- 3) Get \$1500 cash back when you buy a new car.

Got it?

- 1) **If** you sign a 2-year service agreement, **then** you will get a free phone.
- 2) **If** you sign up for a 6-month fitness plan, **then** you will get 6 months free.
- 3) **If** you buy a new car, **then** you will get \$1500 cash back.

Your turn: Write the following as if-then statements.

- 1) Going to the movies will cost you \$8.00.
- 2) You need to study for the test in order to earn a high grade.
- 3) You receive a free pizza with 12 coupons.
- 4) Collect 6 caps and win a free soda.
- 5) Recycling paper earns the school money.

Parts of a conditional

- **Hypothesis:** the portion following the word **IF**
 - > ("If" is not part of the hypothesis.)
- **Conclusion:** the portion following the word **THEN**
 - > ("Then" is not part of the conclusion.)

Try it: Identify the hypothesis and conclusion of each conditional.

- 1) If points A , B , and C lie on line k , then they are collinear.
- 2) If a polygon has seven sides, then it is a hexagon.
- 3) If it snows more than 6 inches, school may be closed.

Got it?

1) *Hypothesis*: points A , B , and C lie on line k .

Conclusion: points A , B , and C are collinear.

NOTICE! Do not use the pronouns when you re-write the conclusion. Use the specific nouns used in the conclusion.

2) *Hypothesis*: A polygon has seven sides.

Conclusion: The polygon is a heptagon.

3) *Hypothesis*: It snows more than 6 inches.

Conclusion: School may be closed.

Truth value: whether a statement is true or false.

We say, "The truth value of a statement can be either true or false."

The hypothesis, the conclusion, and/or the entire statement, can be true **or** false.

SCHOOL Determine the truth value of the following statement for each set of conditions.

If you get 100% on your test, then your teacher will give you an A.

a. You get 100%; your teacher gives you an A.

The hypothesis is true since you got 100%, and the conclusion is true because the teacher gave you an A. Since what the teacher promised is true, the conditional statement is true.

b. You get 100%; your teacher gives you a B.

The hypothesis is true, but the conclusion is false. Because the result is not what was promised, the conditional statement is false.

c. You get 98%; your teacher gives you an A.

The hypothesis is false, and the conclusion is true. The statement does not say what happens if you do not get 100% on the test. You could still get an A. It is also possible that you get a B. In this case, we cannot say that the statement is false. Thus, the statement is true.

Try it: Determine the truth value of the following statement for the given set of conditions.

1) If you get 100% on your test, then your teacher will give you an A.

You get 85%; your teacher gives you a B.

Answer: The hypothesis is false, but (the same as in part c of the example) we cannot say that the statement is false. Therefore, the conditional statement is true.

Truth tables: we can consolidate our data from the previous example, showing the truth value (true or false) of each portion of the conditional - the hypothesis (p), the conclusion (q) and the conditional itself $p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

We can use the truth table to help us decide on the truth value for given statements.

Another set:

Determine the truth value of the following statement for each set of conditions.

If Yukon rests for 10 days, his ankle will heal.

- 1) Yukon rests for 10 days, and he still has a hurt ankle.
- 2) Yukon rests for 3 days, and he still has a hurt ankle.
- 3) Yukon rests for 10 days, and he does not have a hurt ankle anymore.
- 4) Yukon rests for 7 days, and he does not have a hurt ankle anymore.

How'd you do?

- 1) False, since the hypothesis is true and the conclusion is false.
- 2) True, because the hypothesis and the conclusion are both false.
- 3) True, because the hypothesis and the conclusion are both true.
- 4) False, since the hypothesis is false, but the conclusion is true.

Related Conditionals: other statements based on a given conditional statement

We use such methods as switching the order of the hypothesis (p) and the conclusion (q); negating the hypothesis ($\sim p$) - read it "not p " and the conclusion ($\sim q$) - read it "not q "; or by doing both.

Related Conditionals

Statement	Formed by	Symbols	Examples
Conditional	given hypothesis and conclusion	$p \rightarrow q$	If two angles have the same measure, then they are congruent.
Converse	exchanging the hypothesis and conclusion of the conditional	$q \rightarrow p$	If two angles are congruent, then they have the same measure.
Inverse	negating both the hypothesis and conclusion of the conditional	$\sim p \rightarrow \sim q$	If two angles do not have the same measure, then they are not congruent.
Contrapositive	negating both the hypothesis and conclusion of the converse statement	$\sim q \rightarrow \sim p$	If two angles are not congruent, then they do not have the same measure.

If a given conditional is true, the converse and inverse are not necessarily true. However, the contrapositive of a true conditional is always true, and the contrapositive of a false conditional is always false. Likewise, the converse and inverse of a conditional are either both true or both false.

Statements with the same truth values are said to be **logically equivalent**. So, a conditional and its contrapositive are logically equivalent as are the converse and inverse of a conditional. These relationships are summarized in the table below.

p	q	Conditional $p \rightarrow q$	Converse $q \rightarrow p$	Inverse $\sim p \rightarrow \sim q$	Contrapositive $\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

This is the example from the top of p. 94 in the text.

Write the converse, inverse, and contrapositive of the following statement. Determine whether each statement is *true* or *false*. If a statement is false, give a counterexample.

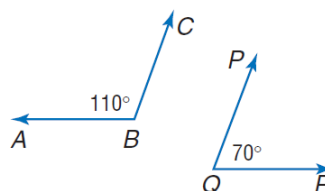
Linear pairs of angles are supplementary.

First, write the conditional in if-then form.

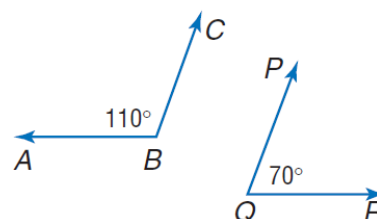
Conditional: If two angles form a linear pair, then they are supplementary.
The conditional statement is true.

Write the converse by switching the hypothesis and conclusion.

Converse: If two angles are supplementary, then they form a linear pair.
The converse is false. $\angle ABC$ and $\angle PQR$ are supplementary, but are not a linear pair.



Inverse: If two angles do not form a linear pair, then they are not supplementary. The inverse is false. $\angle ABC$ and $\angle PQR$ do not form a linear pair, but they are supplementary. The inverse is formed by negating the hypothesis and conclusion of the conditional.



The contrapositive is formed by negating the hypothesis and conclusion of the converse.

Contrapositive: If two angles are not supplementary, then they do not form a linear pair. The contrapositive is true.

Your turn:

Write the conditional, converse, inverse and contrapositive of the statement "All squares are rectangles." Determine whether each statement is *true* or *false*. If a statement is false, give a counterexample.

Got it?

Conditional: If a shape is a square, then it is a rectangle.

Converse: If a shape is a rectangle, then it is a square. **FALSE** (Counterexample: A rectangle can have measures 4 in. and 2 in. It is not a square.)

Inverse: If a shape is not a square, then it is not a rectangle. **FALSE** (Counterexample: A four-sided polygon with side lengths 2, 2, 4 and 4 is not a square, but it would be a rectangle.)

Contrapositive: If a shape is not a rectangle, then it is not a square. **TRUE**

Try some more:

Text p. 95 # 9, 10

(be sure to check #9 in the back of the book)

Homework:

Section 2-3 # 11 - 41 odds, skip 25