2-6 Algebraic Proof

The following are properties of numbers (algebra) that you studied last year in Algebra. We will use them to logically prove statements to be true, based on deductive reasoning such as we have begun to explore.

Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.

Examples:

Symmetric Property: $a + b = d$ can be written as $d = a + b$, if that is a more clear way for us to read it.

Transitive Property: If I know that $AB = FG$ and $FG = QR$, then I also know that $AB = QR$.

Addition and Subtraction Properties	If $a = b$, then $a + c = b + c$ and $a - c = b - c$.

Examples:

Addition Property: I can add the same number to both sides of an equation, and the sides will still be equal - -

$$2a - 3 = 5$$

$$2a - 3 + 3 = 5 + 3$$

Subtraction Property: I can subtract the same number from both sides of an equation, and the sides will still be equal - -

$$2a + 3 = 5$$

$$2a + 3 - 3 = 5 - 3$$

Multiplication and Division Properties	If $a = b$, then $a \cdot c = b \cdot c$ and if $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$.

Examples:

Multiplication Property: I can multiply both sides of the equation by the same number, and the sides will still be equal - -

$$\frac{1}{2}(x-1) = 4$$

$$2 \cdot \frac{1}{2}(x-1) = 4 \cdot 2$$

Division Property: I can divide both sides of the equation by the same number, and the sides will still be equal - -

$$3x = 9$$

$$\frac{3x}{3} = \frac{9}{3}$$

Substitution Property	If $a = b$, then a may be replaced by b in any equation or expression.

Examples:

Substitution Property: When I simplify the equation, ***having shown my work for addition, subtraction, multiplication*** or ***division properties***, I am using the substitution property because I am substituting the answer for the work.

1) $\frac{2x}{2} = \frac{2}{2}$

$x = 1$

2) $-10 + 10 + 2b = 3 - 10$

$2b = -7$

I am also using the substitution property when I combine like terms on one side of the =.

$$2t + 4 - t + 16 = 3t + 4 - 4t$$
$$t + 20 = -t + 4$$

Distributive Property	$a(b + c) = ab + ac$

Examples:

Distributive Property: When I simplify the equation by distributing a number outside the parenthesis to all the values inside the parenthesis, I am using the distributive property.

1) $2(5 - 3a) - 4(a + 7) = 92$

$10 - 6a - 4a - 28 = 92$

2) $3(x - 1) = 4(1 + x)$

$3x - 3 = 4 + 4x$

We use these properties of equality (they show that the sides are still equal) to justify ***every step*** when we solve an equation.

Doing this creates a deductive argument that we call a proof because we are using the argument to **prove** that the statement is true.

Try it: Fill in the blanks to justify each step of the solution.

Question: **Solve $6x + 2(x - 1) = 30$.**

Answer:

Algebraic Steps	**Properties**
$6x + 2(x - 1) = 30$	____________
$6x + 2x - 2 = 30$	____________
____________	Substitution
____________	Addition Property
$8x = 32$	____________
$\frac{8x}{8} = \frac{32}{8}$	____________
$x = 4$	____________

Here's my answer:

Question: **Solve $6x + 2(x - 1) = 30$.**

Answer:

Algebraic Steps	**Properties**
$6x + 2(x - 1) = 30$	Given
$6x + 2x - 2 = 30$	Distributive Property
$8x - 2 = 30$	Substitution
$8x - 2 + 2 = 30 + 2$	Addition Property
$8x = 32$	Substitution
$\frac{8x}{8} = \frac{32}{8}$	Division Property
$x = 4$	Substitution

Before you ask!!!

YES, you must write the word PROPERTY for all of the properties besides substitution. If you write it for substitution as well, you will be developing a very good habit.

Another note: The first line reason is always "Given," because the first line of any proof is to write the information we have been given. This is a reason why I always ask you to *write the question!*

Your turn:

Complete each proof.

1. Given: $\frac{4x + 6}{2} = 9$
Prove: $x = 3$

Statements	Reasons
a. $\frac{4x+6}{2} = 9$	**a.** ________
b. $\_\left(\frac{4x+6}{2}\right) = 2(9)$	**b.** Multiplication Property
c. $4x + 6 = 18$	**c.** ________
d. $4x + 6 - 6 = 18 - 6$	**d.** ________
e. $4x =$ ________	**e.** Substitution
f. $\frac{4x}{4} =$ ________	**f.** Division Property
g. ________	**g.** Substitution

2. Given: $4x + 8 = x + 2$
Prove: $x = -2$

Statements	Reasons
a. $4x + 8 = x + 2$	**a.** ________
b. $4x + 8 - x = x + 2 - x$	**b.** ________
c. $3x + 8 = 2$	**c.** Substitution
d. ________	**d.** Subtraction Property
e. ________	**e.** Substitution
f. $\frac{3x}{3} = \frac{-6}{3}$	**f.** ________
g.	**g.** Substitution

We also use properties to work with segment measures and angle measures, since they are real numbers!

Property	Segments	Angles
Reflexive	$AB = AB$	$m\angle 1 = m\angle 1$
Symmetric	If $AB = CD$, then $CD = AB$.	If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.
Transitive	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

Try it:

State the property that justifies each statement.

1. If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.

2. If $m\angle 1 = 90$ and $m\angle 2 = m\angle 1$, then $m\angle 2 = 90$.

3. If $AB = RS$ and $RS = WY$, then $AB = WY$.

4. If $AB = CD$, then $\frac{1}{2}AB = \frac{1}{2}CD$.

5. If $m\angle 1 + m\angle 2 = 110$ and $m\angle 2 = m\angle 3$, then $m\angle 1 + m\angle 3 = 110$.

6. $RS = RS$

7. If $AB = RS$ and $TU = WY$, then $AB + TU = RS + WY$.

8. If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

Got it?

1. Symmetric Property
2. Substitution Property
3. Transitive Property
4. Multiplication Property
5. Substitution Property
6. Reflexive Property
7. Addition Property
8. Transitive Property

Your turn:

State the property that justifies each statement.

1. If $80 = m\angle A$, then $m\angle A = 80$.

2. If $RS = TU$ and $TU = YP$, then $RS = YP$.

3. If $7x = 28$, then $x = 4$.

4. If $VR + TY = EN + TY$, then $VR = EN$.

5. If $m\angle 1 = 30$ and $m\angle 1 = m\angle 2$, then $m\angle 2 = 30$.

Now, we can put all the pieces together and write a proof from beginning to end:

Write a two-column proof for the following.

If $\overline{PQ} \cong \overline{QS}$ and $\overline{QS} \cong \overline{ST}$, then $PQ = ST$.