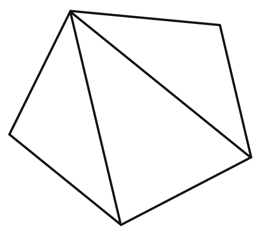
Interior Angles of Polygons

Where did Poly go? I don't know...

The <u>interior angles of polygons</u> starts with triangles... Oh no, not more triangles... Well, just for an explanation...

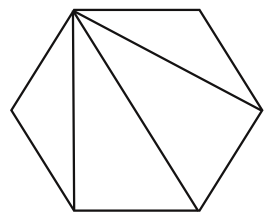
Pentagon



How many sides? ________

How many triangles? _________

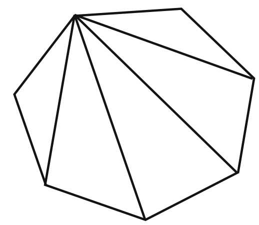
Hexagon



How many sides? ________

How many triangles? ________

Heptagon



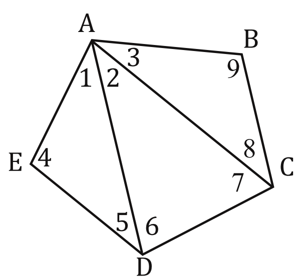
How many sides? ________

How many triangles?_________

Now, if I have a polygon with n (some number of) sides, how many triangles are in the polygon?

I'll give you this one... it's n-2.

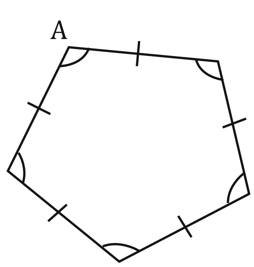
Now look at this one...



Okay, now stay with me here.... The measure of the interior angles of this pentagon is m∠EAB+m∠ABC+m∠BCD+m∠CDE+m∠DEA.
The m∠EAB=m∠1+m∠2+m∠3 and m∠ABC=m∠9,
m∠BCD=m∠7+m∠8, m∠CDE=m∠1+m∠2, and m∠DEA=m∠4.
So, that means that the sum of the measure of the interior angles of the pentagon is equal to the sum of the measures of the interior angles of all of the triangles! How many triangles are there? n-2 of course! n=5 because that's how many sides there are so there are 5-2 or 3 triangles (as you can see). That means the sum of the interior angles of the pentagon is 3*180°, which is 540°. That gives us this formula...

Sum of the interior angles of
any polygon= (n-2)180.

<u>Regular polygons</u>.... A regular polygon is actually quite a special thing. The word regular means equiangular and equilateral. Huh? In plain ole' English, that means a regular polygon's angles are all the same measurement, and its sides are all the same length. Like this...



If this polygon's interior angles all add to 540°, how could we find the measure of one angle, say angle A? Divide it by 5! (since there are 5 angles).
m∠A=540°/5=108°. So, that leads to a new formula...

One interior angle of a REGULAR polygon with n sides=$\frac{(n-2)180^\circ}{n}$

Let's check out some examples...

Ex. 1. What is the sum of the interior angles of a hexagon?

$n=6$ and $(n-2)180°$ so $(6-2)180=4*180=720°$

Ex. 2. What is measure of one interior angle of a regular hexagon?

$n=6$ and $\frac{(n-2)180°}{n}$ so $\frac{(6-2)180}{6}=\frac{4*180}{6}=\frac{720°}{6}=120°$

Ex. 3. How many sides does a polygon have if the sum of its interior angles = 2,340°?

$(n-2)180=2{,}340°$

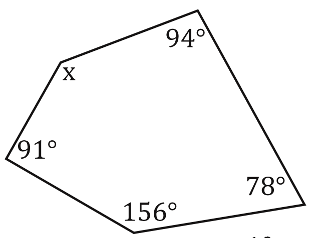
$\frac{(n-2)180}{180}=\frac{2{,}340°}{180}$

$n-2=13$

$+2 \quad +2$

$n=15$ sides

Ex. 4. Find the missing angle...



Step 1. Count the sides...

$n=5$

Step 2. Find the sum of the interior angles...

$(n-2)180°$

$(5-2)180$

$3*180$

$540°$

Step 3. Set up an equation and solve...

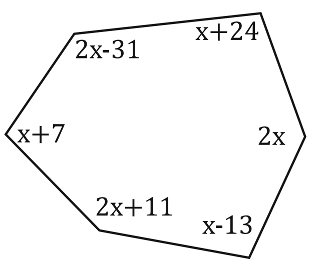
$94°+78°+156°+91°+x=540°$

$419°+x=540°$

$-419° \quad -419°$

$x=121°$

Ex. 5. Solve for x...



Step 1. Count the sides...

$n=6$

Step 2. Find the sum of the interior angles...

$(n-2)180°$

$(6-2)180$

$4*180$

$720°$

$x+7+2x-31+x+24+2x+x-13+2x+11=720°$

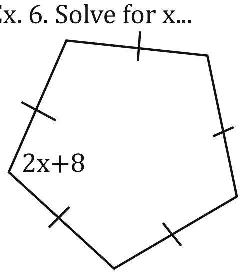
$9x-2=720°$

$+2 \quad +2°$

$\frac{9x}{9}=\frac{722°}{9}$

$x=80.22$

Ex. 6. Solve for x...



Step 1. Count the sides...

$n=5$

Step 2. Find the angle measure...

$\frac{(n-2)180°}{n}$

$\frac{(5-2)180}{5}$

$108°$

Step 3. set up an equation and solve...

$2x+8=108$

$-8 \quad -8$

$\frac{2x}{2}=\frac{100}{2}$

$x=50$