3-3 Slopes of Lines

Main Ideas:

- Find slopes of lines.
- Use slope to identify parallel and perpendicular lines.

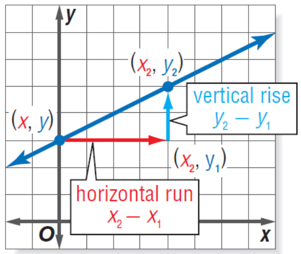
New Vocabulary:

- slope
- rate of change

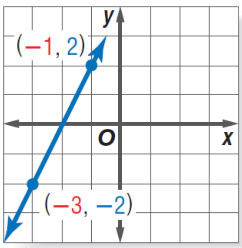
We are already familiar with the idea of slope of a line, because we used it a lot in algebra.

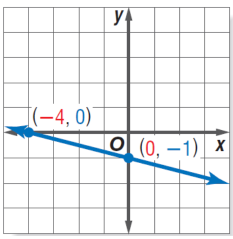
To recap:

$$\text{slope} = \frac{\text{vertical rise}}{\text{horizontal run}}$$



When we are looking at a line graphed to scale on a coordinate plane, we can count both the rise and the run, and find the slope directly.





If we are only given the coordinates of the points, we must apply the slope formula to calculate the slope of the corresponding line:

Slope Formula

The formula to find the slope of line on the coordinate plane with coordinates (x_1, y_1) and (x_2, y_2) is as follows:

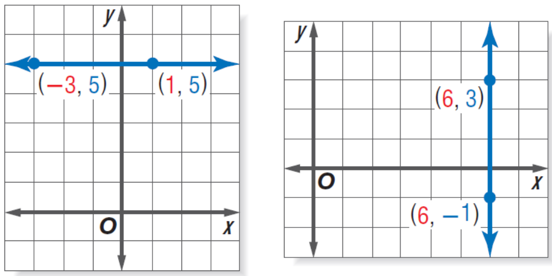
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Find the slopes of:

1A. the line containing
(−6, −2) and (3, −5)

1B. the line containing
(8, −3) and (−6, −2)

Special Lines, Special Slopes



Postulates about slope

- Two nonvertical lines have the same slope if and only if they are parallel.
- Two nonvertical lines are perpendicular if and only if he product of their slopes is -1. (The slopes are *negative reciprocals* of each other.)

"if and only if"

Why is this special?

When a postulate or other statement contains "if and only if," then we know that we are able to apply the rule **in both directions** - forward and backward.

So...if two nonvertical lines are parallel, then they have the same slope **AND** if two nonvertical lines have the same slope, then they are parallel.

How do we apply these postulates?
What kind of questions might we be asked?

1) **Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither*.**

$A(-2, -5), B(4, 7), C(0, 2), D(8, -2)$

Find the slopes of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

slope of $\overleftrightarrow{AB} = \frac{7-(-5)}{4-(-2)}$

$= \frac{12}{6}$ or 2

slope of $\overleftrightarrow{CD} = \frac{-2-2}{8-0}$

$= -\frac{4}{8}$ or $-\frac{1}{2}$

The product of the slopes is $2\left(-\frac{1}{2}\right)$ or -1. So, $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$.

Try it:

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither*.

$A(-8, -7), B(4, -4), C(-2, -5), D(1, 7)$

We also use the slope relationships in order to graph particular lines.

Graph the line that contains $P(-2, 1)$ and is perpendicular to $\overleftrightarrow{JK}$ with $J(-5, -4)$ and $K(0, -2)$.

First, find the slope of $\overleftrightarrow{JK}$.

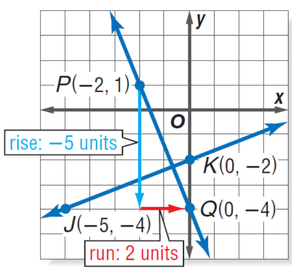
$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{-2-(-4)}{0-(-5)}$ Substitution

$= \frac{2}{5}$ Simplify.

The slope of the line perpendicular to line *JK* is the negative reciprocal of the slope of line *JK*, so the slope of the line perpendicular is $-\frac{5}{2}$.

Graph the line. Start at $(-2, 1)$.
Move down 5 units and then move right 2 units.
Label the point Q. Draw $\overleftrightarrow{PQ}$.



Parallel lines have the **same slope**, so use the slope of the original line for the new line!

Graph the line that contains Q(5,1) and is parallel to $\overleftrightarrow{MN}$ with M(-2, 4) and N(2,1).