Simplifying Radicals

Perfect Squares

A special category of numbers is the set of numbers whose square root is a whole number, for example, 9 is a perfect square – the square root of 9 is 3 because $3 \times 3 = 9$.

It is helpful to memorize the perfect squares, so that we will be able to recognize them when we do calculations.

Perfect Squares

$1 = 1^2$

$4 = 2^2$

$9 = 3^2$

$16 = 4^2$

$25 = 5^2$

$36 = 6^2$

$49 = 7^2$

$64 = 8^2$

$81 = 9^2$

$100 = 10^2$

$121 = 11^2$

$144 = 12^2$

$169 = 13^2$

$196 = 14^2$

$225 = 15^2$

How do the questions look?

When we see something like $\sqrt{25}$, we want to ask ourselves, "What is the simplified form?"

We know that 25 is a perfect square, so we know that 5^2=25; the inverse tells us that $\sqrt{25} = 5$.

Sample Questions

1. $\sqrt{49}$
2. $\sqrt{64}$
3. $\sqrt{100}$
4. $\sqrt{1}$
5. $\sqrt{25}$
6. $\sqrt{9}$
7. $\sqrt{121}$
8. $\sqrt{144}$
9. $\sqrt{81}$
10. $\sqrt{4}$
11. $\sqrt{169}$
12. $\sqrt{196}$
13. $\sqrt{16}$
14. $\sqrt{36}$

But it's not always so easy:

More often than not, we have radicals that ***are not*** perfect squares by themselves, but may contain perfect squares as factors. We use those factors to help us "simplify the radical."

Factors

Remember, **factors** are the numbers we multiply together to get another number.

What are the factors of 18?

What are the factors of 72?

When we find factors that are perfect squares, we can pull their square roots out of the radical, and make the number under the radical smaller (or **simpler**).

Steps

- Complete the prime factorization of the number under the radical.
- Find any double numbers.
- These are perfect squares!

$$\sqrt{180}$$

$$180 = 2 \times 2 \times 3 \times 3 \times 5$$

$$2 \times 2 \text{ and } 3 \times 3$$

$$2 \times 2 = \mathbf{4}$$

$$3 \times 3 = \mathbf{9}$$

- Write the factors including the perfect squares under the radical. (Don't lose the non-perfect squares!)
- Separate into their own radicals.
- Write the roots of the perfect squares and leave the other numbers under the radical.

$$\sqrt{4 \times 9 \times 5}$$

$$\sqrt{4} \times \sqrt{9} \times \sqrt{5}$$

$$2 \times 3 \times \sqrt{5}$$

- Multiply the numbers outside together, and the numbers inside with one another, getting rid of the × between the numbers and the radical.

$$2 \times 3 \times \sqrt{5}$$

$$6\sqrt{5}$$

Practice

Simplify:

1. $\sqrt{150}$

2. $\sqrt{125}$