

Notes: Physics I - Chapter 12 – Simple Harmonic Motion (SHM), Vibrations, and Waves

Many objects vibrate or oscillate (guitar strings, tuning forks, pendulum, atoms within a molecule and atoms within a crystal, ocean waves, earthquake waves, etc.). An understanding of simple harmonic motion will lead to an understanding of wave motion in general.

Simple harmonic motion (SHM) is repetitive motion over the same path; when a vibration or oscillation is caused by a linear restoring force that follows Hooke's law.

- **Restoring force** – acts in direction of returning a mass to the equilibrium position that direction is why it is negative.
- **Hooke's law** – force required to stretch or compress a spring is directly proportional to the displacement (valid anytime the coils do not touch and the elastic limit is not exceeded)

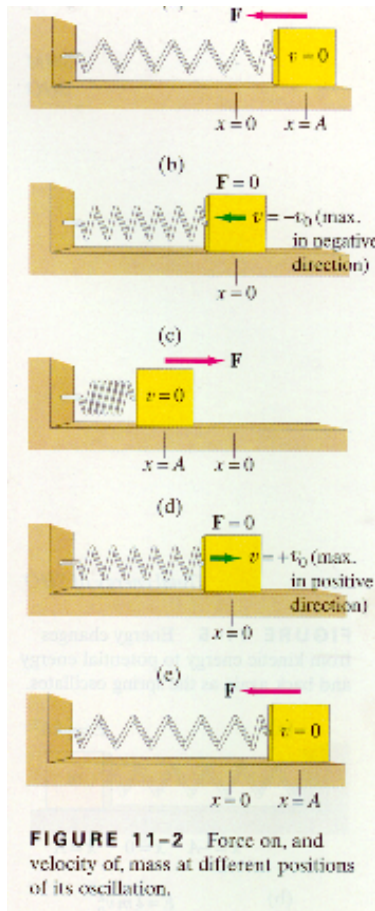
$$F_s = -kx$$

- x = displacement from equilibrium position
- k = spring constant; "stiffness" of spring; larger value of k the "stiffer" the spring
- $(-)$ indicates that the force acts in the direction opposite the displacement
- **NOTE** that the **FORCE** involved is not a constant, but varies with position. Therefore, *the acceleration is not constant* either. Constant force and constant acceleration equations do not apply.
- **Elastic Potential Energy**

$$U_s = 1/2 kx^2$$

For an oscillating mass, the potential energy (U_s) is a maximum at the amplitudes and the velocity and kinetic energy ($KE = 1/2 mv^2$) are maximum as the spring races through the equilibrium position...SO, when Force and Acceleration are max., the velocity is zero!

- The following diagram illustrates the main concepts related to simple harmonic motion.



Stretch the spring a distance $x = A$ (amplitude) and release. The mass m accelerates as F_s pulls it to the equilibrium position ($x = 0$) **force and acceleration are a maximum at maximum displacement, while velocity is zero!**

m passes equilibrium position ($x = 0$) with considerable speed. **at $x = 0$, F_s decreases to zero, BUT speed is at its maximum**

m moves back in the opposite direction until F_s acts to slow m as it moves to the left and m stops momentarily at $x = -A$; **once again force and acceleration are at maximum**

repeats motion symmetrically between $x = A$ and $x = -A$

Key Terms:

Amplitude – (A) maximum displacement

Period – (T) time to complete one cycle

Frequency – (f) cycles per second

Hertz – unit of frequency; 1 Hertz = 1 cycle/ second; $1 \text{ Hz} = 1 \text{ sec}^{-1}$

$T = 1/f$ and $f = 1/T$ they are more than inverses of each other - they are reciprocals!

The period, T , depends on m & k but NOT (strange as it may seem) on the *AMPLITUDE*.

This is due to the fact that the force increases proportionally to increases in amplitude causing the mass to initially accelerate faster. Therefore, the object has a greater max speed and is able to travel a greater distance in the same amount of time.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

You should be able to explain the following according to Newton's laws and the laws of motion, etc.

- **As m increases T increases.** This can be explained using Newton's 2nd law which shows that acceleration is inversely proportional to mass. As mass increases, acceleration decreases, thereby increasing the time required to travel the same distance. **Note that mass must quadruple in order to double T since it is a squared relationship.**
- **As k increases (stiffer spring) T decreases.** This can also be explained using Newton's 2nd law which shows that acceleration is directly proportional to net force. As k increases, the net force increases, and therefore a increases allowing the object to travel the same distance in a shorter time. **Note the squared relationship as with mass.**
- **Example #1:** An ideal spring with a spring constant of $k = 500 \text{ N/m}$ is placed on a horizontal frictionless surface. One end of the spring is fixed, and the other end is fastened to a mass of 2.0 kg .
 - Calculate the amount of work required to stretch the spring a distance of 0.10 m .

The mass is released.

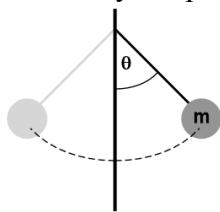
- Calculate the maximum speed of the mass.

The amplitude of the oscillation is now increased:

- State whether the period of the oscillation increases, decreases, or remains the same. Justify your answer.

The motion of a simple pendulum at small angles resembles simple harmonic motion.

- The restoring force is a component of the weight of the object which changes with position. The restoring force is the component of weight tangent to the arc ($F = -mg \sin\theta$).
- For small angles, the pendulum's motion can be considered to be simple harmonic motion. Although the restoring force is not directly proportional to the displacement and therefore the motion is not truly simple harmonic, for angles less than 15° the difference is less than 1 percent and is approximately simple harmonic.



- **The period, T , of a simple pendulum depends upon length and gravity.**

$$T_p = 2\pi\sqrt{\frac{l}{g}}$$

Example #2: Calculate the gravitational acceleration in the Sea of Serenity on the moon if a 0.750 m long pendulum has a period of 4.26s.

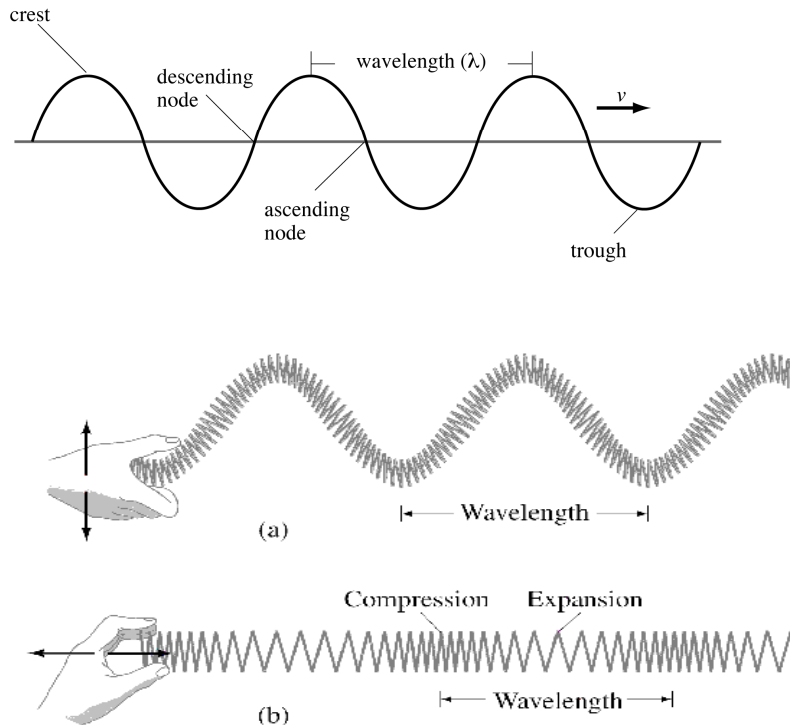
Wave motion:

- For the first time we consider the motion of something that is not matter. We will consider the motion of **energy** propagated *through* matter
- **Mechanical waves** (water, sound, waves on a rope, etc.) *require a material medium in order to transmit energy*
- **Electromagnetic waves** (light, radio waves, X-rays, etc.) *do not require a known medium and they travel at the speed of light “c”* ($c = 3.00 \times 10^8$ m/s) **in a vacuum.**
- **Pulse wave** is a single, nonperiodic disturbance. **Periodic wave** is repetitive since its source continues to vibrate.

Two types of waves:

- **Transverse waves** – causes particles in the medium to vibrate perpendicularly to the motion of the wave
- **Longitudinal waves** (often called compression, pressure, or density waves) – causes the particles of a medium to move parallel to the direction of the wave; ex. sound, fluids (usually)

• Diagrams of waves



In the figure above, letter “b” is a diagram of a longitudinal wave.

Calculating speed of a wave: $v = \lambda f$ *Speed or velocity = frequency times wavelength*

Example #3: The red light emitted by a He-Ne laser has a wavelength of 633nm in air and travels at 3.00×10^8 m/s. Find the frequency of this light wave.

Example #4: Calculate the wavelength for a radio wave in air for the KERA radio station broadcasting at 90.1 MHz (Megahertz).

*****The Speed of waves is determined by the MEDIUM, if medium changes speed can change.

*****Frequency of waves is determined by the SOURCE

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Calculate the wavelength of radio waves

- Speed of a wave depends only on the properties of the medium. The speed of a wave is constant for any given medium. For example, the speed of sound is typically faster in liquids than in gases, and typically the fastest in solids. But in a given medium, all sound waves travel at the same speed. For example, at a concert, sound waves from different instruments reach your ears at the same moment, even when the frequencies of the sound waves are different.
- **When frequency increases, wavelength must decrease in order for speed to remain constant. Speed of a given wave only changes when the wave travels from one medium into a medium with different properties!**

Example #5: Calculate the speed in air if a tuning fork produces a 256Hz sound with a wavelength of 1.35m.

Example #6: What frequency would this tuning fork produce in water if its speed changes to 1500m/s?

Example #7: Calculate the wavelength for the wave in example #6.

Wave interference:

- **Superposition principle** – the displacement of a medium caused by two or more waves is the algebraic sum of the displacements caused by the individual waves
- **Reflection of waves** – When reflected from a rigid barrier, a wave will return inverted and 180 degrees out of phase with the original wave. **BUT** when reflected

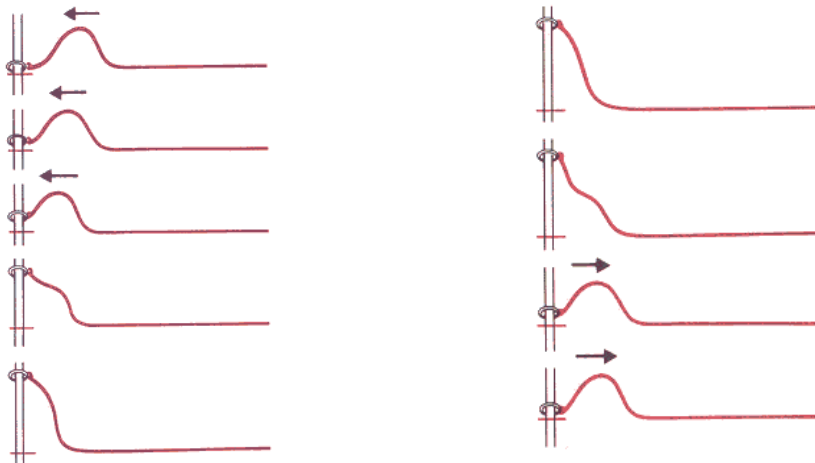
from a moveable or flexible barrier, the wave returns upright and in phase with the original wave. When the reflected wave and the original or incident wave encounter each other and overlap, the superposition principle is observed and interference happens. (Demonstrate with slinky and coiled spring)

Reflection of waves:

- *A wave reflected at fixed boundary will be inverted relative to the original wave. (see below)* On the left, the progression is toward the boundary...
then after reflection...



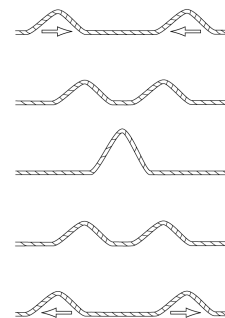
- On the left, the progression is toward the boundary...
then after reflection...



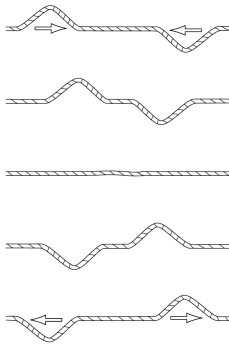
- *A wave reflected at a free boundary will be on the same side as the original wave. (see above)*

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Constructive interference – occurs when wave displacements are in the same direction; amplitude of resultant wave is larger

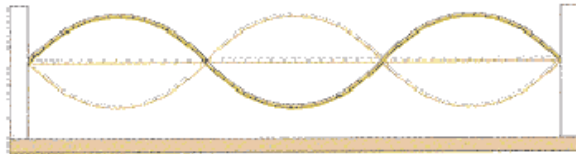


- **Destructive interference** – occurs when amplitudes are in opposite directions; amplitude of resultant wave is smaller

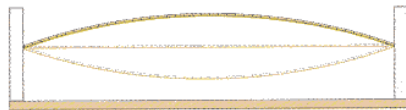
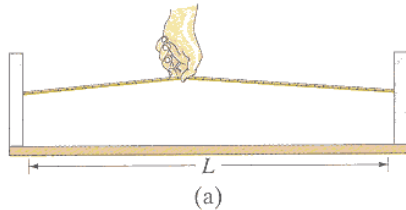


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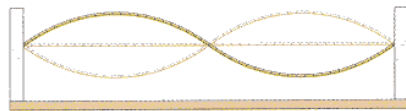
Standing waves—wave pattern that results when two waves of the same frequency, wavelength, and amplitude travel in opposite directions and interfere; destructive interference occurs at the nodes and constructive at the antinodes



- Only certain frequencies of vibrations produce standing wave patterns as we will see in more detail next chapter.



Fundamental or first harmonic, f_1



First overtone or second harmonic, $f_2 = 2f_1$



Second overtone or third harmonic, $f_3 = 3f_1$

HW Practice Problems: Ch. 12 p. 469-473 #'s 5, 8-10, 13, 18-22, 25, 27, 28, 31, 33, 35, 37, 40, 45, 48, 50, and 57; Read p. 466-467 also!