7.1 Polynomial Functions

Objectives:

- Evaluate polynomial functions
- Identify general shapes of graphs and polynomial functions

A ***polynomial*** is a monomial or a sum of monomials

A ***polynomial in one variable*** contains only ***1 variable***

$$5x^3 - 3x^2 + x - 10$$

Recall

The ***degree of the polynomial*** is the greatest exponent of its variable. The ***leading coefficient*** is the coefficient of the term with the highest degree.

For the polynomial above, the degree is ***3***

and the leading coefficient is ***5***

Recall

Evaluate a polynomial function

function notation f(x)

For $f(x) = 5x^3 - 3x^2 + x - 10$, find $f(2)$

This means: find the value of function *f* when x = 2

Plugging in ***2 for x*** will evaluate the polynomial for ***x = 2***

$f(2) = 5(2)^3 - 3(2)^2 + 2 - 10$

$f(2) = 5(8) - 3(4) + 2 - 10$

$f(2) = 40 - 12 + 2 - 10$

$f(2) = 20$ (2, 20)

The value of function *f* when ***x = 2*** is 20.

You may have to evaluate functions for variables and algebraic expressions.

If $\mathbf{f(x)} = 2x^4 - x^3 + 3x$

a) Find $\mathbf{f(y^2)}$

Plug y^2 in for x in the polynomial

$f(y^2) = 2(y^2)^4 - (y^2)^3 + 3(y^2)$

$f(y^2) = 2y^8 - y^6 + 3y^2$

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b) Find $f(2x - 1)$

Plug $2x - 1$ in for x in the polynomial

$f(2x - 1) = 2(2x-1)^4 - (x-1)^3 + 3(x-1)$

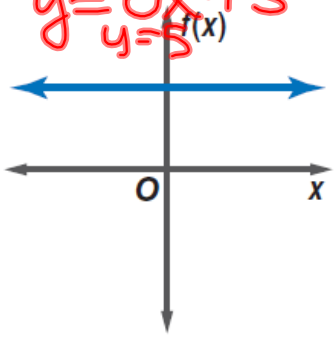
7.1

II Graphs of Polynomial Functions
p. 348

The diagram shows the general graphs of several polynomial functions below. These graphs show the maximum number of times the graph may intersect the x-axis (max # of zeros). Pay attention to the degree of the polynomial and its # of solutions.

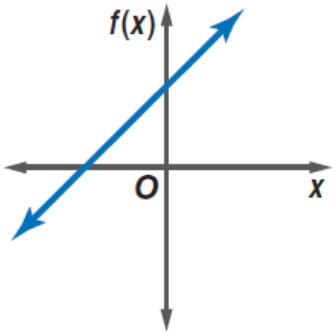
Constant function
Degree 0

$y = 0x + 5$
$y = 5$



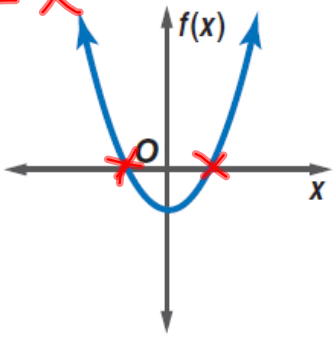
Linear function
Degree 1

$y = mx + b$



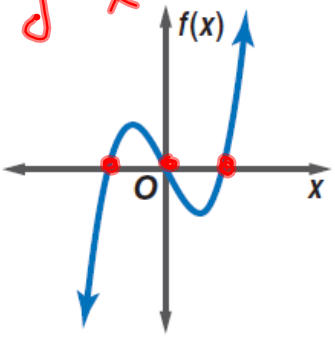
Quadratic function
Degree 2

$y = x^2$

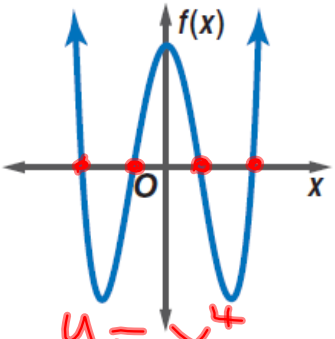


Cubic function
Degree 3

$y = x^3$

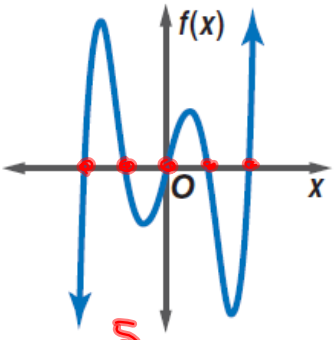


Quartic function
Degree 4



$y = x^4$

Quintic function
Degree 5



$y = x^5$

END BEHAVIOR

The end behavior is the behavior of the graph as x approaches positive infinity or negative infinity

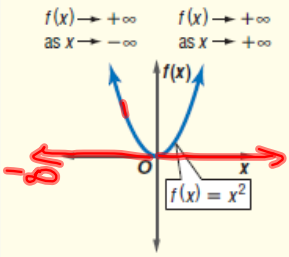
This is represented by $\rightarrow +\infty$ "*approaches positive infinity*"

or

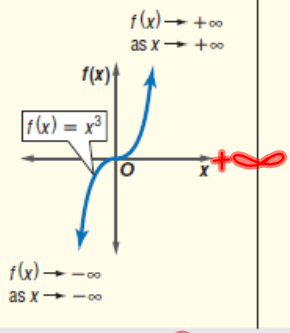
$\rightarrow -\infty$ "*approaches negative infinity*"

Concept Summary — End Behavior of a Polynomial Function

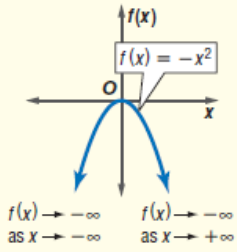
Degree: even
Leading Coefficient: positive
End Behavior:



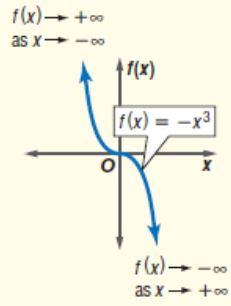
Degree: odd
Leading Coefficient: positive
End Behavior:



Degree: even
Leading Coefficient: negative
End Behavior:



Degree: odd
Leading Coefficient: negative
End Behavior:



$x \rightarrow +\infty$
$f(x) \rightarrow +\infty$

$x \rightarrow -\infty$
$f(x) \rightarrow -\infty$

Finding the zeros of the function:

Even degree function: May or may not intersect the x-axis

- If the graph intersects the x-axis in **2 places**, it has 2 real zeros.
- If the graph **does not intersect** the x-axis, the roots are *imaginary* and cannot be determined from the graph.
- If the graph is **tangent** to the axis, there are 2 zeros that are the same.

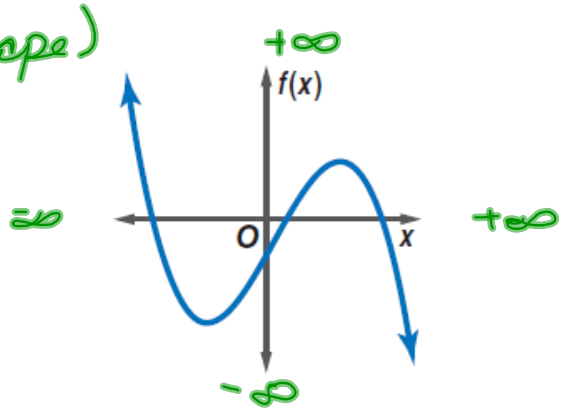
Odd degree function always crosses the x-axis at least once, and therefore always has at least one real zero.

For the following graph,
1) describe the end behavior
2) determine whether it is odd or even degree
3) state the number of real zeros

degree 3 (look at shape)

x-int?

3 real zeros



* as $x \to +\infty$
$f(x) = -\infty$

* as $x \to -\infty$
$f(x) \to +\infty$

e polynomial is the greatest exponent of
ading coefficient is the coefficient of
ghest degree.

lynomial above, the degree is ***3***
and the leading coefficient is ***5***