11.2 Arithmetic Series

defn: an indicated *sum* of the terms of a *sequence*

Sequence	***Series***
-9, -3, 3	-9 + (-3) + 3
8, 11, 14, 17, 20	8 + 11 + 14 + 17 + 20

notation: S_n represents the sum of the first ***n*** terms of a series
(ex: S_4 is the sum of the first four terms)

To find the sum of a series, use

$$S_n = \frac{n}{2}(2a_1 + (n-1)d) \quad \text{OR} \quad S_n = \frac{n}{2}(a_1 + a_n)$$

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Find S_n for the arithmetic series:
$a_1 = 4$, $a_n = 100$, $n = 25$

$S_{25} = \frac{25}{2}(4+100)$

$S_{25} = \frac{25}{2}(104)$ → 52

$S_{25} = 1300$

Example: Find the first 4 terms of an arithmetic series in which $a_1 = 14$ $a_n = 29$ $S_n = 129$

1) Find n: $$S_n = \frac{n}{2}(a_1 + a_n)$$

2) Find d: $$a_n = a_1 + (n-1)d$$

3) Use d to find the terms of the series

Example: Find the sum of the first 20 even numbers beginning with *2*

$n = 20 \quad a_1 = 2 \quad d = 2$

Formula 1

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

Formula 2

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Sigma Notation: concise notation for a series

last value of n

$$\sum_{n=3}^{7}(2n+1)$$

formula for the series

first value of n

WS2
pg. 1 $a_n = a_1 + (n-1)d$

1) $a_{18} = -3367$ (a_1)
$a_{39} = -7567$ (a_c)
$n = 22$

$-7567 = -3367 + (22-1)d$
$+3367 \quad +3367$
$4200 = 21d$
$d = 200$

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15) $a_1 = \frac{16}{3}$ $a_n = 12$ $n = 6$

Evaluate the series

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_n = \frac{6}{2}\left(\frac{16}{3} + 12\frac{36}{3}\right)$

$S_n = \frac{6}{2}\left(\frac{52}{3}\right)$

$S_n = 52$

19) $11 + 17 + 23 + 29, \ldots$ $n = 16$

$a_1 = 11$
$n = 16$
$d = 6$ → common difference

$S_n = \frac{16}{2}(2(11) + (16-1)6)$

$S_n = 8(22 + (15 \cdot 6))$

$S_n = 8(22 + 90)$

$S_n = 8(112)$

$S_n = 896$

21) $\sum_{n=4}^{11}(10n - 18)$

a_1, $n=4$: $10(4) - 18 = 22$

a_8, $n=11$: $10(11) - 18 = 92$

$n = 8$
count from 4-11

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_n = \frac{8}{2}(22 + 92)$

$S_n = 4(114)$

$S_n = 456$

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22) $\sum_{m=5}^{11}\left(\frac{1}{6} + \frac{4}{3}m\right)$

7 terms → count from 5 to 11

$a_1 =$, $m = 5$: $\frac{1}{6} + \frac{4}{3}(5) = \frac{41}{6}$

$a_7 =$, $m = 11$: $= \frac{1}{6} + \frac{4}{3}(11) = \frac{89}{6}$

Now find the sum → $a_1 = \frac{41}{6}$ $a_n = \frac{89}{6}$ $n = 7$

$S_n = \frac{7}{2}\left(\frac{41}{6} + \frac{89}{6}\right)$

$S_n = \frac{7}{2}\left(\frac{65}{3}\right)$

$S_n = \frac{455}{6}$

Evaluate: $\sum_{n=1}^{7}(3n)$

Method 1

$$\sum_{n=1}^{7}(3n) = 3(1) + 3(2) + 3(3) + 3(4) + 3(5) + 3(6) + 3(7)$$

$$\sum_{n=1}^{7}(3n) = 3 + 6 + 9 + 12 + 15 + 18 + 21$$

$$\sum_{n=1}^{7}(3n) = 84$$

Method 2 - Use $S_n = \frac{n}{2}(a_1 + a_n)$

$a_1 =$ $a_n =$

$$\sum_{n=1}^{7}(3n) = S_7 = \frac{7}{2}(3 + 21)$$

$$\sum_{n=1}^{7}(3n) = 84$$

$$\sum_{n=5}^{12}(4-3n)$$

WS1

35) $a_1 = -31$ $\qquad a_n = a_1 + (n-1)d$

$a_6 = 19$

$19 = -31 + (6-1)d$
$+31$

$50 = 5d$
$d = 10$

$-31, \underline{-21}, \underline{-11}, \underline{-1}, \underline{9}, 19$

37) $a_1 = 1$
$a_7 = -2$
$n = 7$

$-2 = 1 + (7-1)d$
$-3 = 6d$
$\frac{-3}{6}$
$-\frac{1}{2} = d$

$1, \underline{\frac{1}{2}}, \underline{0}, \underline{\frac{-1}{2}}, \underline{-1}, \underline{\frac{-3}{2}}, -2$

38) $a_1 = \frac{-2}{5}$ $\quad a_7 = \frac{38}{5}$ $\quad n = 7$

$\frac{38}{5} = -\frac{2}{5} + (7-1)d$
$+\frac{2}{5}$

$\frac{40}{5}$ $\quad 8 = 6d$
$\frac{8}{6} = d$
$\frac{4}{3} = d = \frac{20}{15}$

$\frac{-2}{5},$
$-\frac{6}{15}, \underline{\frac{14}{15}}, \underline{\frac{34}{15}}, \underline{\frac{54}{15}}, \underline{\frac{74}{15}}, \underline{\frac{94}{15}}, \frac{38}{5}$
$+\frac{20}{15}$

$\frac{54}{15} \div 3 = \frac{18}{5}$