Graphing Sine and Cosine Horizontal Translation

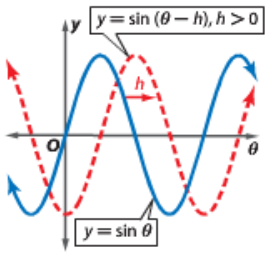
$y = a\sin(bx + c) + d$ $y = a\cos(bx + c) + d$

1 Horizontal Translations Recall that a *translation* occurs when a figure is moved from one location to another on the coordinate plane without changing its orientation. A horizontal translation of a periodic function is called a **phase shift**.

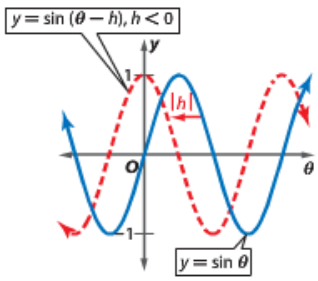
KeyConcept Phase Shift

Words The phase shift of the functions $y = a \sin b(\theta - h)$, $y = a \cos b(\theta - h)$, and $y = a \tan b(\theta - h)$ is h, where $b > 0$.

Models



If $h > 0$, the shift is h units to the right.



If $h < 0$, the shift is $|h|$ units to the left.

Examples

$y = \cos(\theta - 90°)$ The phase shift is 90° to the right.

$y = \tan(\theta + 30°)$ The phase shift is 30° to the left.

Find the phase shi for the following equaons:

$y = 3\sin(2\theta + \pi)$

$2\theta + \pi = 0$

$2\theta = -\pi$

$\theta = -\frac{\pi}{2}$

left (−)

$y = 2\sin(\theta - \frac{\pi}{2})$

$\theta - \frac{\pi}{2} = 0$

$\theta = \frac{\pi}{2}$

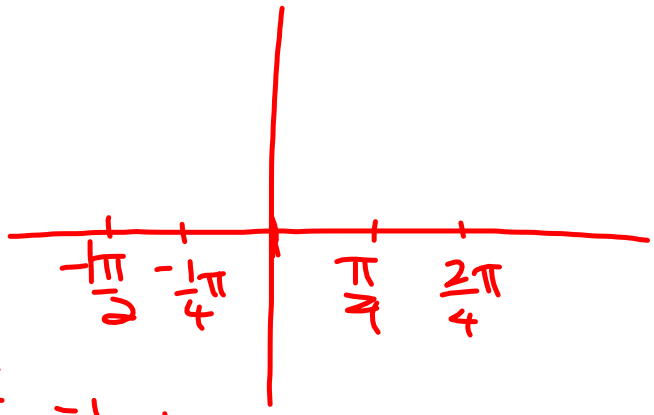
Right

$y = \cos(2\theta + \frac{\pi}{3})$

$2\theta + \frac{\pi}{3} = 0$

$\frac{1}{2}2\theta = -\frac{\pi}{3}\frac{1}{2}$

$\theta = -\pi/6$



QP $\frac{\pi}{4}$

$-\frac{1}{2}+\frac{1}{4}$

$-\frac{1}{4}+\frac{1}{4}=0$

$y = 3\sin(2\theta + \pi)$

amp: 3

per: $\frac{2\pi}{2} = \pi$

QP: $\frac{\pi}{4}$

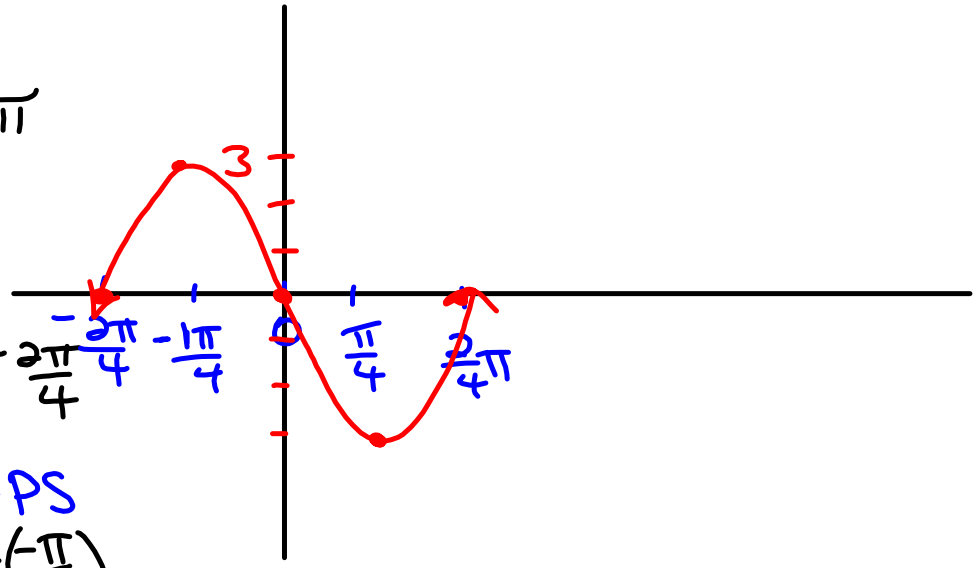
PS: $-\frac{\pi}{2}$ OR $-\frac{2\pi}{4}$

START

END: Per + PS

$\pi + (-\frac{\pi}{2})$

$\frac{\pi}{2}$



$$y = 2\sin(\theta - \frac{\pi}{2})$$

amp: 2

per: 2π

QP: $\frac{\pi}{2}$

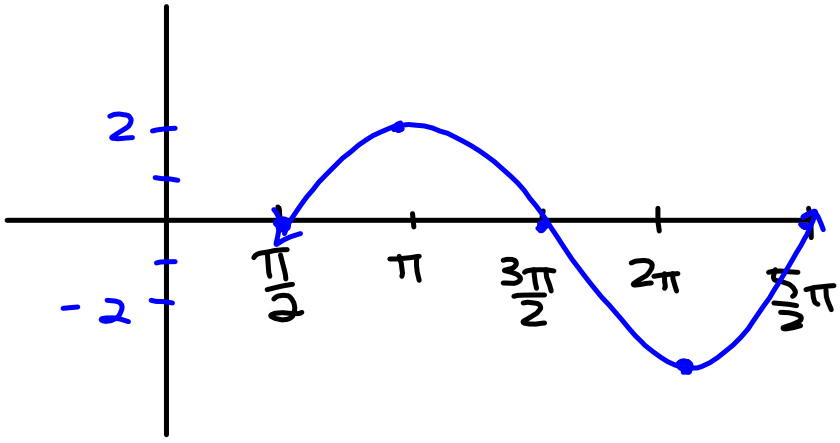
PS: $\frac{\pi}{2}$

START $\frac{\pi}{2}$

END PS + Per

$\frac{\pi}{2} + 2\pi$

$\frac{5\pi}{2}$



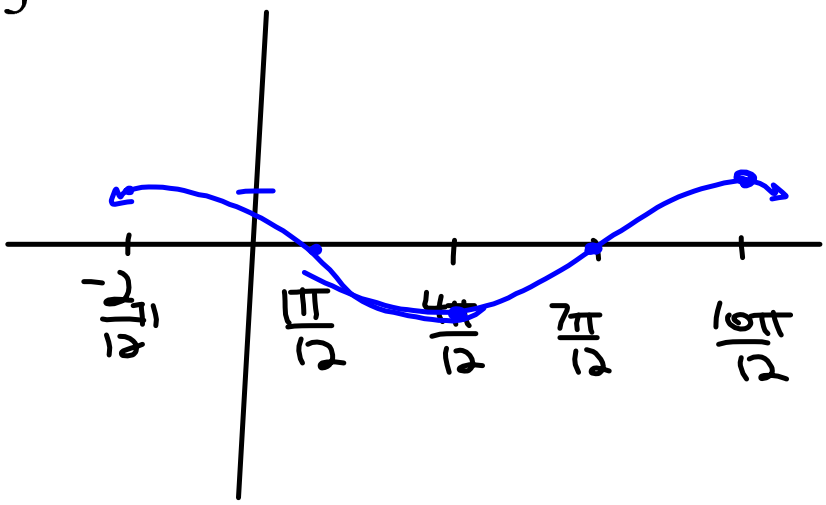
$$y = \cos(2\theta + \frac{\pi}{3})$$

amp 1

per: π

QP: $\frac{\pi}{4} \Rightarrow \frac{3\pi}{12}$

PS: $\frac{-\pi}{6} \Rightarrow \frac{-2\pi}{12}$



END
PS + Per
$-\pi/6 + \pi$
$5\pi/6$