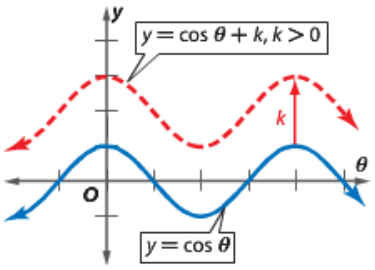
2 Vertical Translations Recall that the graph of $y = x^2 + 5$ is the graph of the parent function $y = x^2$ shifted up 5 units. Similarly, graphs of trigonometric functions can be translated vertically through a **vertical shift**.

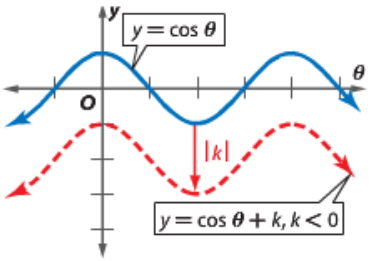
KeyConcept Vertical Shift

Words The vertical shift of the functions $y = a \sin b\theta + k$, $y = a \cos b\theta + k$, and $y = a \tan b\theta + k$ is k.

Models



If $k > 0$, the shift is k units up.



If $k < 0$, the shift is $|k|$ units down.

Examples

$y = \sin \theta + 4$ The vertical shift is 4 units up.

$y = \tan \theta - 3$ The vertical shift is 3 units down.

Find the vertical shift for the following:

$y = 3\sin\theta - 1$

VS: -1

$y = \boxed{-1} + 3\sin\theta$

$y = \cos(2\theta + \pi) + 2$

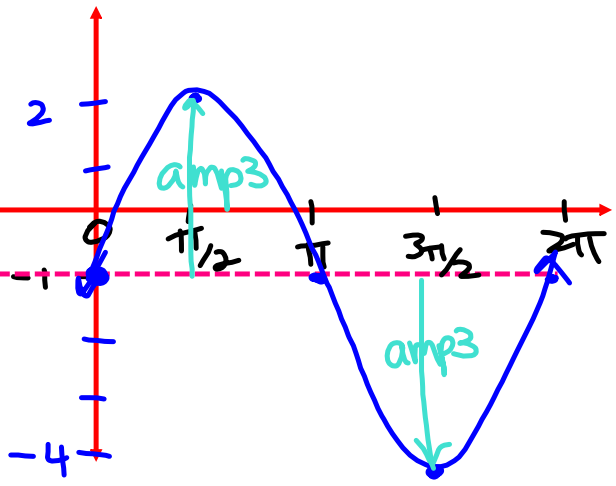
VS: 2

$y = \frac{1}{2}\cos\frac{1}{2}\theta - 3$

VS: -3

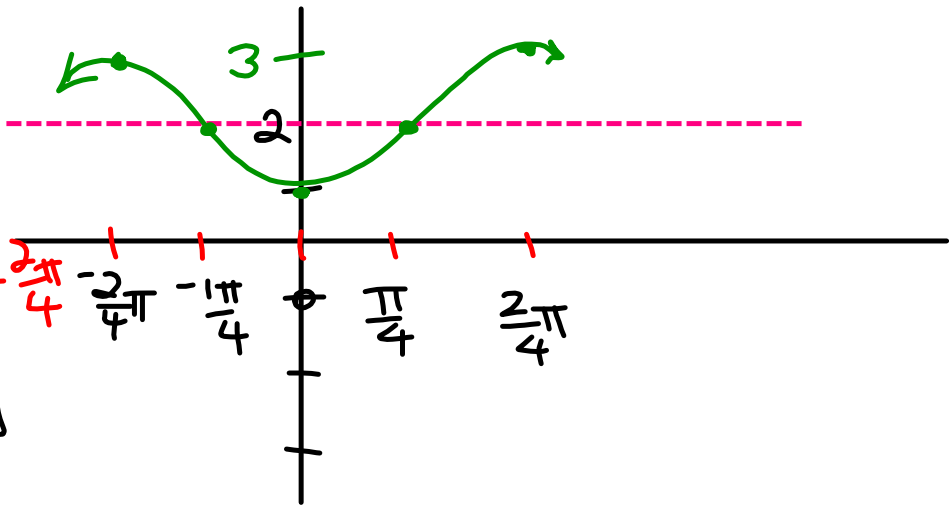
$$y = 3\sin\theta - 1$$

amp: 3
per: 2π
QP: $\frac{\pi}{2}$
PS: None
VS: -1



$$y = |\cos(2\theta + \pi) + 2$$

amp 1
per: π
QP: $\frac{\pi}{4}$
PS: $-\frac{\pi}{2}$ or $\frac{3\pi}{4}$
VS: 2



$$y = \frac{1}{2}\cos\frac{1}{2}\theta - 3$$

amp $\frac{1}{2}$

per: $\frac{2\pi}{\frac{1}{2}} \Rightarrow 4\pi$

QP: π

PS: $\emptyset$

VS: -3