13.3 Sum and Difference Formulas

To find exact values of some uncommon angles, we can substitute a sum or difference of common angles that we know the value of such as:
30, 45, 60, 90, 120, 135, 150, 180, 210, ...

to find sin 75
we don't know the values of 75
how can you break up 75 into common angles?

30 + 45 225 -150

120 - 45 135 - 60

Sine of a Sum or Difference

$\sin(A + B) = \sin A\cos B + \cos A\sin B$
$\sin(A - B) = \sin A\cos B - \cos A\sin B$

Use the formulas to break up 75 into a sum:

$\sin(75) = \sin(30 + 45) = \sin 30\cos 45 + \cos 30\sin 45$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

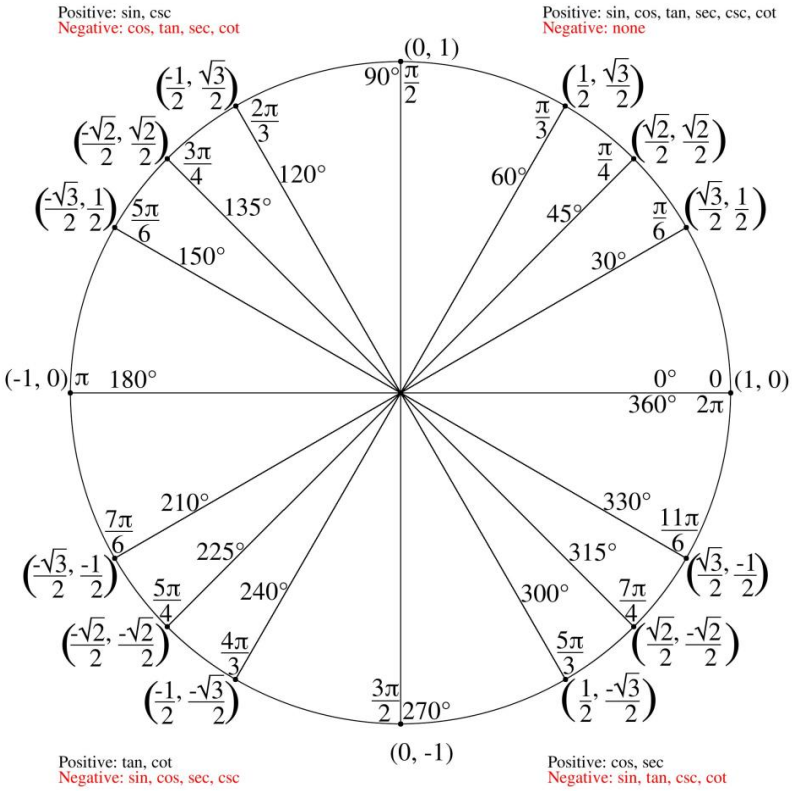
$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

Alternately, breaking up 75 using a difference:

$\sin(75) = \sin(225 - 150) = \sin 225\cos 150 - \cos 225\sin 150$

$= \left(\frac{-\sqrt{2}}{2}\right)\left(\frac{-\sqrt{3}}{2}\right) + \left(\frac{-\sqrt{2}}{2}\right)\left(\frac{-1}{2}\right)$

SAME ANSWER $= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$



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$\sin 195^\circ$

Cosine of a Sum or Difference

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\cos(-75)$

Verify the following
(using sum and difference formulas)

$\cos(180 - \theta) = -\cos\theta$

Cosine of a Sum or Difference

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\cos(A - B) = \cos A\cos B + \sin A\sin B$

Verify the following

$$\sin(\theta + 30) + \cos(\theta + 60) = \cos\theta$$