Complex Numbers

The imaginary unit: i

Pure imaginary numbers: $6i$, $-2i$

$i = \sqrt{-1}$

Square roots of negative numbers:

$\sqrt{-27}$

$\sqrt{27}\sqrt{-1}$

$3\sqrt{3} \cdot i$

$3i\sqrt{3}$

$\sqrt{-18}$

$\sqrt{-1}\sqrt{18}$

$i\sqrt{9}\sqrt{2}$

$3i\sqrt{2}$

$\sqrt{-125}$

$\sqrt{-1}\sqrt{125}$

$i\sqrt{25} \cdot \sqrt{5}$

$5i\sqrt{5}$

Powers of I

Definitions:	$\sqrt{-1} = i$
	$i^2 = -1$

Look for the pattern **that is formed when i is raised to the powers 1, 2, 3, 4, and so on.**

i^1	$= i$	$= i$	$= i$
i^2	$= i\,i$	$= -1$	$= -1$
i^3	$= i^2 i$	$= (-1)i$	$= -i$
i^4	$= i^2\, i^2$	$= (-1)(-1)$	$= 1$
i^5	$= i^4 i$	$= (1)\,i$	$= i$
i^6	$= i^4 i^2$	$= (1)\,i^2$	$= -1$
i^7	$= \cancel{i^4} \cdot i^3$	$= i^2 i$	$= -i$
i^8	$= i^4 \cdot i^4$	$= 1$	$= 1$
i^9	$= \cancel{i^8} \cdot i$	$= i$	$= i$
i^{10}	$= \cancel{i^8}\, i^2$	$= i \cdot i$	$= -1$
i^{11}	$= \cancel{i^8}\, i^3$	$= i^2 i$	$= -i$
i^{12}	$= 1$	$= 1$	$= 1$

$(i^4)^3$

Now simplify:

highest multiple of 4

1. i^{17} $\cancel{i^{16}} \cdot i$ i	**2.** i^{22} $\cancel{i^{20}} \cdot i^2$ -1	**3.** i^{38} $\cancel{i^{36}} \cdot i^2$ -1
4. i^{40} $(i^4)^{10} = 1$	**5.** i^{67} $\cancel{i^{64}} \cdot i^3$ $i^2 \cdot i$ $-i$	**6.** i^{90} $\cancel{i^{88}} \cdot i^2$ -1
7. i^{152} $(i^4)^{38} = 1$	**8.** $3i \bullet 6i^3$ $18i^4$ $18(1)$ 18	**9.** $2i \bullet (4i^3)^2$ $2i \cdot 16i^6$ $32i^7$ $32\cancel{i^4}\, i^3$ $32i^2 i$ $32(-1)i$ $-32i$

For any positive real number b,

$$\sqrt{-(b^2)} = \sqrt{b^2} \bullet \sqrt{-1} = bi$$

Example:

$$\sqrt{-24} = \sqrt{24} \bullet \sqrt{-1}$$
$$= \sqrt{4 \bullet 6} \bullet i$$
$$= 2i\sqrt{6}$$

Practice
Simplify

10. $\sqrt{-256}$ $\sqrt{-1}$	**11.** $\sqrt{-80}$	**12.** $\sqrt{-4}\bullet\sqrt{-9}$
13. $\sqrt{-\frac{25}{121}}$ $\sqrt{-1}\sqrt{\frac{25}{121}}$ $\frac{5}{11}i$	**14.** $\sqrt{-6}\bullet\sqrt{-12}$ $\sqrt{-1}\sqrt{6}\sqrt{-1}\sqrt{12}$ $i^2\sqrt{72}$ $-6\sqrt{2}$	**15.** $(7-3i)(8+4i)$ $56+28i-24i-12i^2$ $56+4i-12(-1)$ $68+4i$
16. $\left(-3\sqrt{-5}\right)^2$ $(-3i\sqrt{5})^2$ $9\cdot i^2\cdot 5$ -45	**17.** $\left(2\sqrt{-8}\right)\left(3\sqrt{-2}\right)$ $i2\sqrt{8}\cdot 3i\sqrt{2}$ $6i^2\sqrt{16}$ -24	**18.** $\left(2i\bullet 3i^2\right)^2$ $(6i^3)^2$ $36i^6$ -36
19. $(3i)^3\bullet(2i)^2$ $27i^3\cdot 4i^2$ $108i^5$ $\cancel{i^4}\cdot i$ $108i$	**20.** $-\sqrt{15}\bullet\sqrt{-15}$ $-\sqrt{15}\cdot i\sqrt{15}$ $-15i$	**21.** $\left(\sqrt{-13}\right)^2$ $(\sqrt{-1}\sqrt{13})^2$ $i^2\cdot 13$ -13

Equate complex numbers
set real parts equal to each other
and
set imaginary parts equal to each other

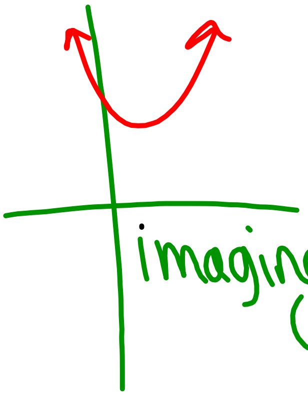
Find the value of x and y that make the equation true: $2x + yi = 14 - 3i$

Real parts $2x = 14$ $yi = -3i$ Imaginary parts

$\boxed{x = 7 \qquad y = -3}$

$\sqrt{\text{Square Root Property}} = \sqrt{x^2}$

Solve: $x^2+36=0$



$\sqrt{x^2}=\sqrt{-36}$

$x=\pm\sqrt{-1}\sqrt{36}$

$x=\pm 6i$

imaginary (non Real) solution

Solve ~~5x2 + 20 = 0~~

$5y^2+20=0$

$y^2+4=0$

$\sqrt{y^2}=\sqrt{-4}$

$y=\pm i\sqrt{4}$

$y=\pm 2i$

$\sqrt{x^2}=\pm$

2

6i

2+3i

6-12i

Complex Number $a + bi$,

where a and b are real numbers,

i is the imaginary unit,

a is called the real part, and

bi is called the imaginary part

Add and Subtract Complex Numbers

$(3 + 5i) + (2 - 4i)$

add real parts

Add imaginary parts

$5 + i$

Multiply Complex Numbers

In an AC circuit, the voltage E, current I and impedance Z are related by the formula E = I(Z)

Find the voltage in a circuit with current 1 + 4j amps and impedance 3-6j ohms.

j is imaginary

$E = ((1+4j)(3-6j))$

$3 - 6j + 12j - 24j^2$

$3 + 6j + 24(+1)$

$27 + 6j$

complex form

$a+bi$

Divide Complex Numbers