

Name \_\_\_\_\_  
 Algebra II/Trig WS 13-7

Period \_\_\_\_\_  
 Mrs. Jensen

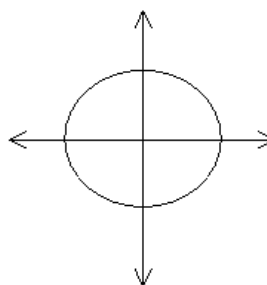
Solve *without* using a calculator?

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = x$$

Hint: You know you are looking for the angle that has a sine value of  $-\frac{\sqrt{3}}{2}$ . Where on the unit circle is sine equal to  $-\frac{\sqrt{3}}{2}$ ?

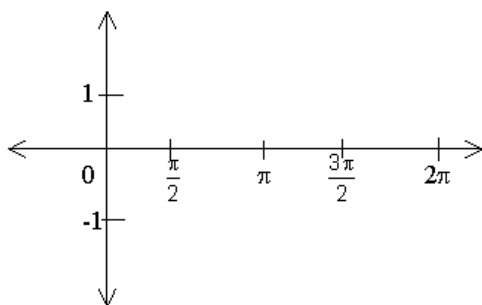
Did you find TWO angles that have sine equal to  $-\frac{\sqrt{3}}{2}$ ? Good!

Now what is the answer to the equation?

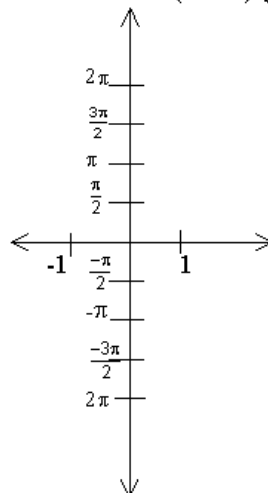


Normally, we can use the calculator inverse functions to find an unknown angle. But we should be able to find the common angles (30, 45, 60, 90, 180, 270, 360) without using the calculator.

First examine the sine graph:



And the inverse ( $\sin^{-1}$ ) graph



The domain of the sine function must be restricted so that the inverse sine is also a function. Therefore, we will only use the following angles when solving sine equations:

$$\frac{-\pi}{2} \leq x \leq \frac{\pi}{2}, \text{ where } x \text{ is the angle in radians}$$

The same holds true for cosine and tangent functions. The summary of the restricted domains for the functions is:

$$y = \sin x \text{ iff } y = \sin x \text{ and } \frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$y = \cos x \text{ iff } y = \cos x \text{ and } 0 \leq x \leq \pi$$

$$y = \tan x \text{ iff } y = \tan x \text{ and } \frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$$

You must check the restriction to determine the unknown angle.

The inverse of the function is called the Arc- function and can be symbolized by

Arcsin or  $\sin^{-1}$   
Arccos or  $\cos^{-1}$   
Arctan or  $\tan^{-1}$

Four examples:

1.  $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

2.  $\cos^{-1}(-1)$

3.  $\arcsin\left(\frac{1}{2}\right)$

4.  $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

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*Solve the following without using a calculator (degrees):*

5. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = x$	6. $x = \sin^{-1}\frac{\sqrt{3}}{2}$	7. $x = \arctan\sqrt{3}$
8. $x = \tan^{-1}(-1)$	9. $x = \arccos\left(-\frac{1}{2}\right)$	10. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

*Solve the following USING a calculator (nearest degrees):*

11. $\cos^{-1}0.9347$	12. $\tan^{-1}12$	13. $\sin^{-1}(-0.9)$
14. $\cos^{-1}(-1)$	15. $\arccos\left(\frac{1}{2}\right)$	16. $\sin^{-1}0$

Finally, functions and inverse functions can be combined.

Example:  $\tan\left(\sin^{-1}\frac{1}{2}\right)$

$$\begin{array}{c} \downarrow \\ \tan(30^\circ) \\ \downarrow \\ \frac{\sqrt{3}}{3} \end{array}$$

WORK INSIDE-OUT  $\sin^{-1}\frac{1}{2}$

What angle has sine =  $\frac{1}{2}$  ?

NOW FIND  $\tan 30^\circ$

$$\text{SO... } \tan\left(\sin^{-1}\frac{1}{2}\right) = \frac{\sqrt{3}}{3}$$



**LOOK** at your final answer – were you supposed to end up with an angle measure or a trig value of an angle?

Your turn:

17.  $\sin(\cos^{-1}1)$

18.  $\sin^{-1}\left(\cos\frac{\pi}{3}\right)$

19.  $\sin^{-1}(\tan 45^\circ)$

20.  $\cos\left(2\sin^{-1}\frac{\sqrt{3}}{2}\right)$

21.  $\cos^{-1}\left(\sin\frac{\pi}{6}\right)$

22.  $\tan\left(\tan^{-1}\frac{1}{2}\right)$



