

Trig Identities List

Pythagorean Identities	$\sin^2 \theta + \cos^2 \theta = 1$ $\tan^2 \theta + 1 = \sec^2 \theta$ $\cot^2 \theta + 1 = \csc^2 \theta$
Quotient Identities	$\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\cot \theta = \frac{\cos \theta}{\sin \theta}$
Reciprocal Identities	$\sec \theta = \frac{1}{\cos \theta}$ $\csc \theta = \frac{1}{\sin \theta}$ $\cot \theta = \frac{1}{\tan \theta}$
Sum and	

$$1 + \sec^2 \theta \sin^2 \theta = \sec^2 \theta$$
$$1 + \frac{1}{\cos^2 \theta} \sin^2 \theta = \sec^2 \theta$$

$$1 + \tan^2 \theta = \sec^2 \theta$$
$$\sec^2 \theta = \sec^2 \theta$$

$$\begin{array}{ccc} (\sin \theta) & = & (\sin \theta) \\ -1 & & -1 \end{array}$$

Using identities to verify trigonometric identities

Verifying means simplifying one or both sides of an equation separately until they are exactly the same.

Technically, since you are not sure if these are equations, you are not allowed to perform any operations across the equal sign.

Verify:

$$\csc\theta \cos\theta \tan\theta = 1$$

$$\frac{1}{\sin\theta} \cdot \cos\theta \cdot \frac{\sin\theta}{\cos\theta} \stackrel{?}{=} 1$$

$$1 = 1 \checkmark$$

Change to
 $\frac{\sin\theta}{\cos\theta}$

$$2 + 2 = 4$$

$$4 = 4 \checkmark$$

$$\csc\theta + \sec\theta = \frac{1 + \cot\theta}{\cos\theta}$$

$$\csc\theta + \sec\theta = \frac{1}{\cos\theta} + \frac{\cot\theta}{\cos\theta}$$

$$\csc\theta + \sec\theta = \sec\theta + \frac{\frac{\cos\theta}{\sin\theta}}{\cos\theta}$$

$$\csc\theta + \sec\theta = \sec\theta + \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta}$$

$$\csc\theta + \sec\theta = \sec\theta + \csc\theta$$

$$\sec\theta + \csc\theta = \sec\theta + \csc\theta$$

$$\tan^2\theta = (\sec\theta - 1)(\sec\theta + 1) \quad \text{FOIL}$$

$$\tan^2\theta = \sec^2\theta - 1$$

$$\tan^2\theta = \tan^2\theta \quad \checkmark$$

$$\tan^2 \theta \csc^2 \theta = \underbrace{1 + \tan^2 \theta}$$

$$\frac{\cancel{\sin^2 \theta}}{\cos^2 \theta} \cdot \frac{1}{\cancel{\sin^2 \theta}} = \sec^2 \theta$$

$$\sec^2 \theta = \sec^2 \theta$$

$$8. \cos^2 \theta + \tan^2 \theta \cos^2 \theta = 1$$

$$I \quad \cos^2 \theta + \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \cos^2 \theta = 1$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$1 = 1$$

II

$$\cos^2 \theta (1 + \tan^2 \theta) = 1$$

$$\cos^2 \theta (\sec^2 \theta) = 1$$

$$\cos^2 \theta \frac{1}{\cos^2 \theta} = 1$$

$$1 = 1$$

$$9) \cot \theta (\cot \theta + \tan \theta) = \csc^2 \theta$$

$$\cot^2 \theta + \cot \theta \tan \theta = \csc^2 \theta$$


$$\cot^2 \theta + \cot \theta \frac{1}{\cot \theta} = \csc^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

$$\checkmark \csc^2 \theta = \csc^2 \theta$$

$$\begin{aligned}
 12) \quad \frac{1 - \cos \theta}{1 + \cos \theta} &= (\csc \theta - \cot \theta)^2 && \text{change to sin/cos} \\
 &= \left(\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2 \\
 \frac{1 - \cos \theta}{1 + \cos \theta} &= \left(\frac{1 - \cos \theta}{\sin \theta} \right)^2 && \text{square the term} \\
 \frac{1 - \cos \theta}{1 + \cos \theta} &= \frac{1 - 2\cos \theta + \cos^2 \theta}{\sin^2 \theta} && \text{factor the numerator \& use Pythag Identity on denominator} \\
 \frac{1 - \cos \theta}{1 + \cos \theta} &= \frac{(1 - \cos \theta)(1 - \cos \theta)}{1 - \cos^2 \theta} \\
 \frac{1 - \cos \theta}{1 + \cos \theta} &= \frac{(1 - \cos \theta)(1 - \cos \theta)}{(1 - \cos \theta)(1 + \cos \theta)} \\
 \frac{1 - \cos \theta}{1 + \cos \theta} &= \frac{1 - \cos \theta}{1 + \cos \theta} && \text{cancel like terms in numer. and denom.} \\
 &&& \text{ta da!!!}
 \end{aligned}$$

To Do's

- 1) Change functions to sine/cosine 
- 2) Use a common denominator to combine terms
- 3) Use a conjugate to square the terms
(then use Pythagorean Identities)
- 4) Factor or FOIL
- 5) Start on the more complicated side
- 6) Start over if the problem gets too chaotic
- 7) Use identities to change the form

Do Not Perform Operations Across = Sign!

